The Nuclear Equation of State and the Symmetry Energy

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TNPI2016 - XV Conference on Theoretical Nuclear Physics in Italy,
Pisa, April 20th-22nd 2016.

STRENGTH: STructure and REactions of Nuclei: towards a Global THeory.

First let me recall that my reasearch
at Milano is framed in a national initiative called

STRENGTH: STructure and REactions of
Nuclei: towards a Global THeory

network research of main Italian theoretical groups working at the **INFN** in the fields of **low-energy nuclear structure and reactions**: Catania, LNS, Milano, Napoli, Padova and Pisa.

STRENGTH: STructure and REactions of Nuclei: towards a Global THeory.

→ **Our goal**: **develop a more quantitative and unified understanding of nuclear structure and reactions for stable and exotic nuclei that are studied nowadays in RIB facilities**

→ **In this talk**:

- The **symmetry energy**, which rules the **isovector channel** of the nuclear **Equation of State**, is not well contrained in nuclear effective models.
- I will **review** part of our work in which we have tried to understand **which nuclear observables are more sensitive** to the properties of the **symmerty energy**. Such an investigation is of **crucial importance for building more reliable EDFs with larger predictive power** and may help in setting the **basis for our goal**.

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- → **Conclusions**

INTRODUCTION

The Nuclear Many-Body Problem:

- → **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- → **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- → **Complex systems**: **spin, isospin, pairing, deformation,** ...
- → **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - → **different nuclear interactions in the medium** are found **depending** on the **approach**
  - → EoS and (recently) **few groups** in the world are able to perform extensive calculations for **light and medum mass nuclei**
- → Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

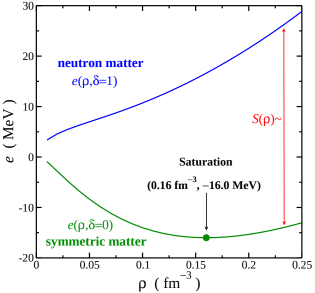
$$\begin{aligned}\mathcal{L}_{int} \quad &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma \quad + \bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ &- \bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} \quad - \bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\tau\Psi A^{(\rho)\mu} \\ &- e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}\end{aligned}$$

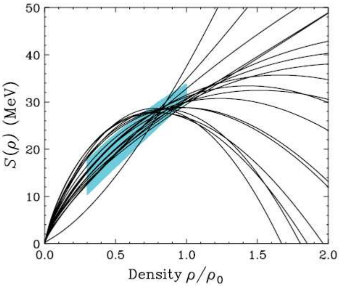
Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\mathrm{eff}}_{\mathrm{Nucl}} = V^{\mathrm{long-range}}_{\mathrm{attractive}} + V^{\mathrm{short-range}}_{\mathrm{repulsive}} + V_{\mathrm{SO}} + V_{\mathrm{pair}}$$

- $\rightarrow$ Fitted **parameters contain** (important) **correlations beyond the mean-field**
- $\rightarrow$ Nuclear energy functionals are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction**

The Nuclear Equation of State: Infinite System



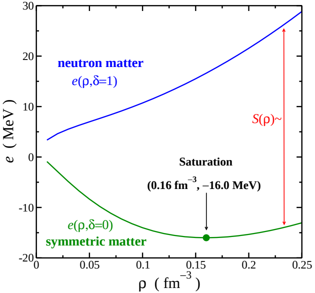


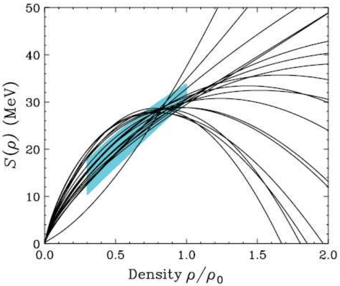
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

→ Nuclear Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System





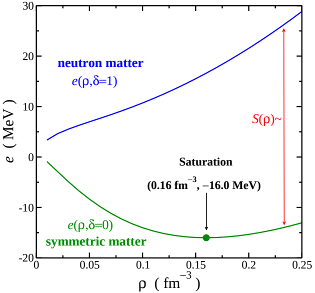
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

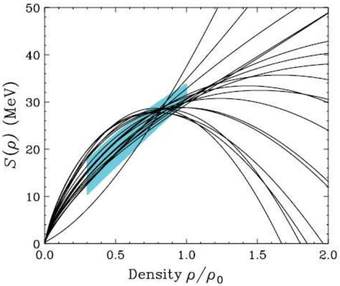
→ Nuclear Matter

→ Symmetric Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System





$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

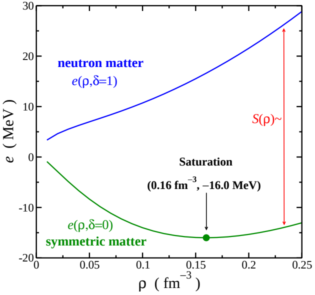
→ Nuclear Matter

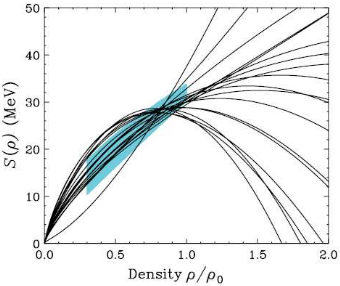
→ Symmetric Matter

→ Symmetry energy

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



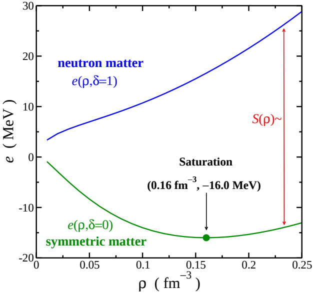


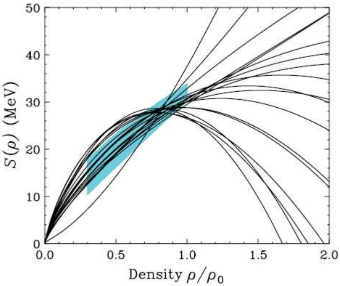
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3)\right)$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho-\rho_0}{3\rho_0}\right]$$

The Nuclear Equation of State: Infinite System





$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + \beta^2 \left(J + L x + \frac{1}{2} K_{sym} x^2 + \mathcal{O}(x^3) \right)$$

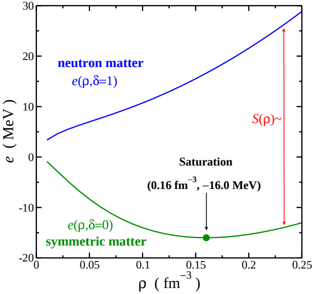
$\rightarrow \; S(\rho_0) = J$

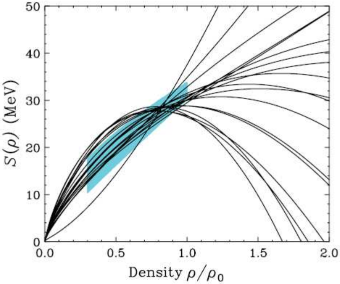
$$\rightarrow \; \frac{d}{d\rho} S(\rho) \Big|_{\rho_0} = \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2}$$

$$\rightarrow \; \frac{d^2}{d\rho^2} S(\rho) \Big|_{\rho_0} = \frac{K_{sym}}{9\rho_0^2}$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0} \right]$$

The Nuclear Equation of State: Infinite System





The uncertainties on $S(\rho)$ **impacts** on many nuclear physics and astrophysics **observables**.

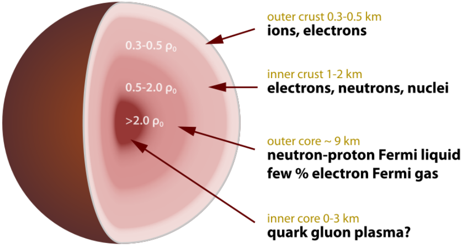
→ sym

ρ_0^2

$\left[\beta = \frac{\rho_n - \rho_p}{\quad};\quad x = \frac{\rho - \rho_0}{\quad}\right]$

The impact of the symmetry energy on nuclear and astrophyiscs observables

Relevance of the neutron star crust on the star evolution and dynamics (brief motivation)



→ The crust separates neutron star interior from the photosphere (X-ray radiation).

→ The thermal conductivity of the crust is relevant for determining the relation between observed X-ray flux and the temperature of the core.

→ Electrical resistivity of the crust might be important for the evolution of neutron star magnetic field.

→ Conductivity and resistivity depend on the structure and composition of the crust

→ Neutrino emission from the crust may significantly contribute to total neutrino losses from stellar interior (in some cooling stages).

→ A crystal lattice (solid crust) is needed for modelling pulsar glitches, enables the excitation of toroidal modes of oscillations, can suffer elastic stresses...

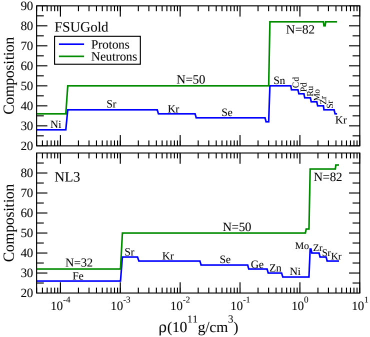
→ Mergers (binary systems that merge) may enrich the interstellar medium with heavy elements, created by a rapid neutron-capture process.

→ In accreting neutron stars, instabilities in the fusion light elements might be responsible for the phenomenon of X-ray bursts

Source: Pawel Haensel 2001

The symmetry energy and the structure and composition of a neutron star outer crust

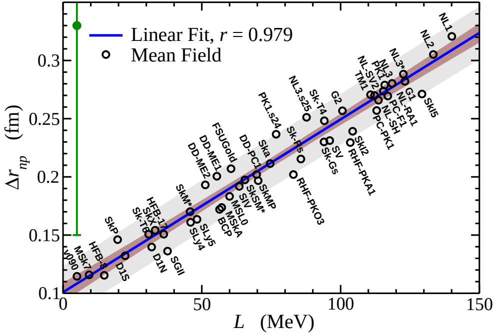
- → span **7** orders of magnitude in **denisty** (from **ionization** $\sim 10^4$ g cm to the **neutron drip** $\sim 10^{11}$ g cm)
- → it is organized into a **Coulomb lattice** of neutron-rich nuclei (ions) embedded in a relativistic **uniform electron gas**
- → $T \sim 10^6$ K ~ 0.1 keV $\rightarrow$ one can treat **nuclei and electrons at $T = 0$ K**
- → At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by ^{56}**Fe** nuclei.
- → As the **density increases**, the electronic contribution becomes important, it is energetically advantageous to lower its electron fraction by $e^- + (N, Z) \rightarrow (N+1, Z-1) + \nu_e$ and therefore $Z \downarrow$ with constant (approx) number of N
- → As the **density continues to increase, penalty energy from the symmetry energy** due to the neutron excess changes the composition to a dif ferent $N-$*plateau*
  $\frac{Z}{A} \approx \frac{Z_0}{A_0} - \frac{p_{F_e}}{8 a_{sym}}$ where $(A_0, Z_0) = {}^{56}\mathrm{Fe}_{26}$
- → The Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density is reached ($\mu_{drip} = m_n$).** $[M(N, Z) + m_n < M(N+1, Z)]$



Physical Review C **78**, *025807 (2008)*

The faster the symmetry energy increases with density **($L \uparrow$)**, the more exotic the composition of the outer crust.

The symmetry energy and the neutron skin in nuclei



Physical Review Letters **106**, *252501 (2011)*

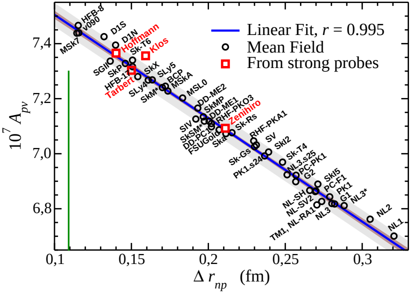
The faster the symmetry energy increases with density, the largest the size of the neutron skin in (heavy) nuclei.

$$\Delta r_{np} \sim \frac{1}{12}\frac{IR}{J}L$$

[Exp. from strongly interacting probes: 0.18 ± 0.03 **fm (***Physical Review C* **86** *015803 (2012)***)].**

The symmetry energy and parity violating electron scattering

(A_{pv}: relative difference between the elastic cross sections of right- and left-handed electrons)



Physical Review Letters **106**, 252501 (2011)

(Calculation at a fixed q equal to PREx)

- → Electrons interact by exchanging a γ **(couples to p)** or a Z_0 **boson (couples to n)**
- → Ultra-relativistic electrons, **depending on their helicity (±)**, will interact with the nucleus **seeing a slightly different potential**: **Coulomb ± Weak**
- → $A_{pv} \equiv \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \sim \frac{\text{Weak}}{\text{Coulomb}}$
- → Input for the calculation are the ρ_p and ρ_n **(main uncertainty)** and nucleon form factors for the e-m and the weak neutral current.
- → In PWBA for small momentum transfer:

$$A_{pv} \approx \frac{G_F q^2}{4\sqrt{2}\pi\alpha}\left(1 - \frac{q^2 \langle r_p^2 \rangle^{1/2}}{3F_p(q)} \Delta r_{np}\right)$$

The largest the size of the neutron distribution in nuclei, the smaller the elastic electron parity violating asymmetry. **[Exp. from ew probes:** 0.302 ± 0.175 **fm** (*Physical Review C* **85**, 032501 (2012))**].**

Isovector Giant Resonances (some considerations)

→ In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**

→ **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"

→ The **excitation energy (E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \rightarrow E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{S(\rho)}$$

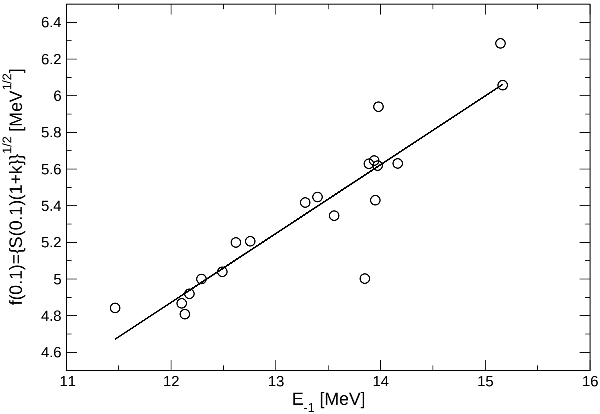
where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

→ The **dipole polarizability ($\alpha \sim \int \frac{\sigma_{\gamma-abs}}{\text{Energy}^2} \sim$ IEWSR)** measures the tendency of the nuclear charge distribution to be distorted, that is, from a **macroscopic** point of view

$$\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}$$

The symmetry energy and the Giant Dipole Resonance

($E_x \approx f(0.1) \propto \sqrt{S(0.1\text{fm}^{-3})}$)

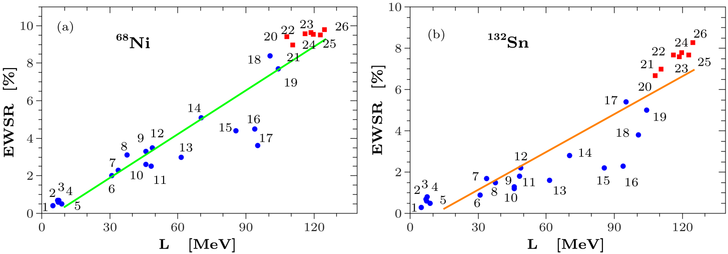


Physical Review C **77**, *061304 (2008)*

The faster the symmetry energy increases with density around saturation $\left[S(\rho_A) \approx J - L\frac{\rho_0 - \rho_A}{3\rho_0}\right]$, the smaller the excitation energy of the Giant Dipole Resonance (GDR).

The symmetry energy and the Pygmy Dipole Resonance

(Pygmy: low-energy excited state appearing in the dipole response of N . Z nuclei)

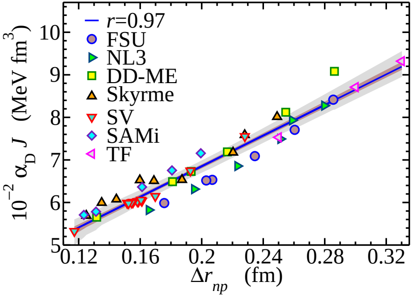


Physical Review C **81**, *041301 (2010)*

The faster the symmetry energy increases with density, the larger is the energy (E) times the probability (P) of exciting the Pygmy state ⇒ larger the Energy Weighted Sum Rule (EWSR) $\propto E \times P$.

WARNING: we lack of a clear understanding of the physical reason for this correlation

Dipole polarizability and the symmetry energy



Physical Review C **85** *041302 (2012);* **88** *024316 (2013);* **92***, 064304 (2015)*

Macroscopic model:

→ Using the **dielectric theorem:** m_{-1} moment can be computed from the expectation value of the Hamiltonian in the constrained (D dipole operator) ground state $\mathcal{H}' = \mathcal{H} + \lambda D$

→ Assuming the **Droplet Model** (heavy nucleus):

$$\alpha_D \approx \alpha_D^{bulk}\left[1 + \frac{1}{5}\frac{L}{J}\right] \text{ where}$$

$$\alpha_D^{bulk} \equiv \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J} \text{ (Migdal first derived)}$$

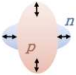
→ $$L \approx \frac{\alpha_D^{exp} - \alpha_D^{bulk}}{\alpha_D^{bulk}}\, 5J$$

By using the Droplet Model one can also find:

$$\alpha_D J \approx \frac{\pi e^2}{54} A\langle r^2\rangle \left[1 + \frac{5}{2}\frac{\Delta r_{np} - \Delta r_{np}^{coul} - \Delta r_{np}^{surf}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

For a fixed value of the symmetry energy at saturation, the faster the symmetry energy increases with density, the larger the dipole polarizability.

IV-IS GQRs and the symmetry energy



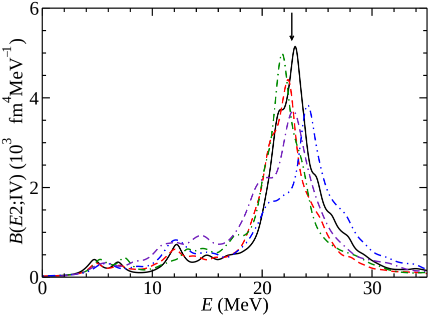
Within the Quantum Harmonic Oscillator approach

$$E_x^{IV} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m}\frac{V_{sym}\langle r^2\rangle}{(\hbar\omega_0)^2\langle r^4\rangle}}$$

and EDF calculations, one can deduce

$V_{sym} \approx 8(S(\rho_A) - S^{kin}(\rho_0))$

$S^{kin}(\rho_0) \approx \varepsilon_{F_0}/3$ (Non-Rel)

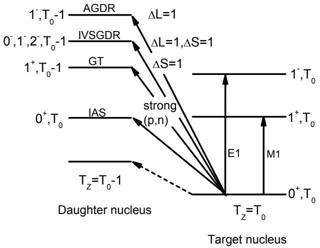


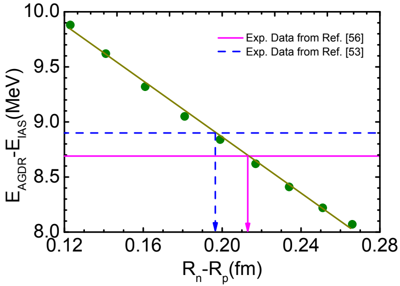
$$S(\rho_A) = \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$

$S(\rho = 0.1\ \mathbf{fm}^{-3}\ \textbf{for}\ ^{208}\mathbf{Pb}) \approx 23.3 \pm (0.6)_{exp} \pm \text{theory} \sim 10\%\ \mathbf{MeV}$

The faster the symmetry energy increase with density around saturation, the smallest the difference between the IS and IV excitation energy differences

E1 transitions in CER and the symmetry energy





Phys. Rev. C **92**, *034308 (2015)*

AGDR ($\Delta J^\pi = 1^-$ with $\Delta L = 1$ and $\Delta S = 0$) is the $T_0 - 1$ component of the charge-exchange of the GDR.

$$E_{AGDR} - E_{IAS} \approx 5\sqrt{\frac{5}{3}\frac{J}{I}}\,\frac{1+\gamma}{\alpha_H Z}\,\frac{\hbar c}{m\langle r^2\rangle^{1/2}}\left[\left(1-\frac{\varepsilon_{F\infty}}{3J}\right)I - \frac{3}{2}\left(\frac{\Delta R_{np} - \Delta R_{np}^{surf}}{\langle r^2\rangle^{1/2}}\right) - \frac{3}{7}I_C\right]$$

$$E_{AGDR} - E_{IAS} \approx \frac{\varepsilon}{\Delta E_C}\left(E_{IVGDR} - \varepsilon\right)\frac{m_0^{AGDR}}{m_0^{IVGDR}}$$

$$\Delta r_{np} \approx 0.21 \pm 0.01\ \mathbf{fm}$$

The faster the symmetry energy increases around saturation, the smaller the excitation energy of the IVGDR and the difference between the excitation energies of AGDR - IAS

CONCLUSIONS

Conclusions: EoS around saturation

→ The **isovector channel** of the nuclear effective interaction is **not well constrained by current experimental information.**

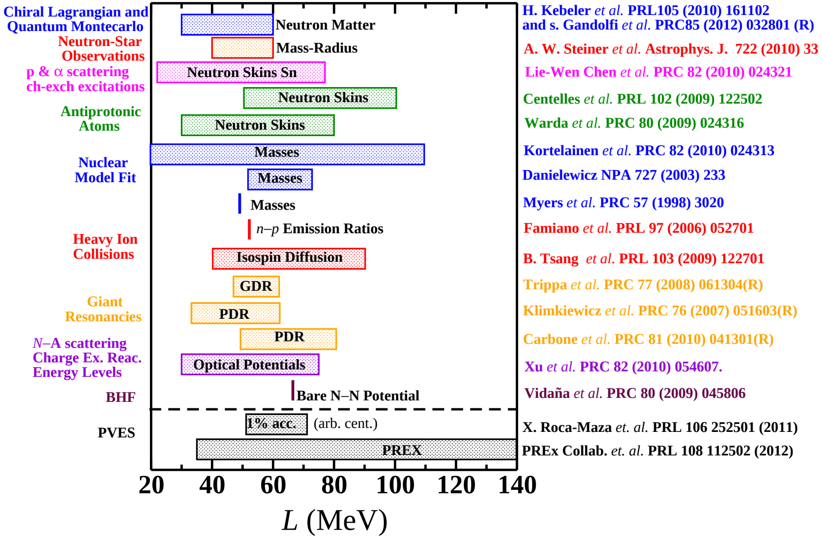
→ Many **observables available in current laboratories** are sensitive to the symmetry energy. **Problems**: **accuracy and model dependent analysis**. **Systematic experiments** may help.

→ **Exotic nuclei more sensitive** to the isovector properties (due to larger neutron excess). **Problems**: more **difficult to measure, accuracy and model dependent analysis**. **Systematic experiments** may help.

→ The most promising observables to constraint the symmetry energy are the neutron skin thickness and the dipole polarizability in medium and heavy nuclei.

Thank you for your attention!

Available constraints on L



AIP Conference Proceedings **1491**, *101 (2012)*