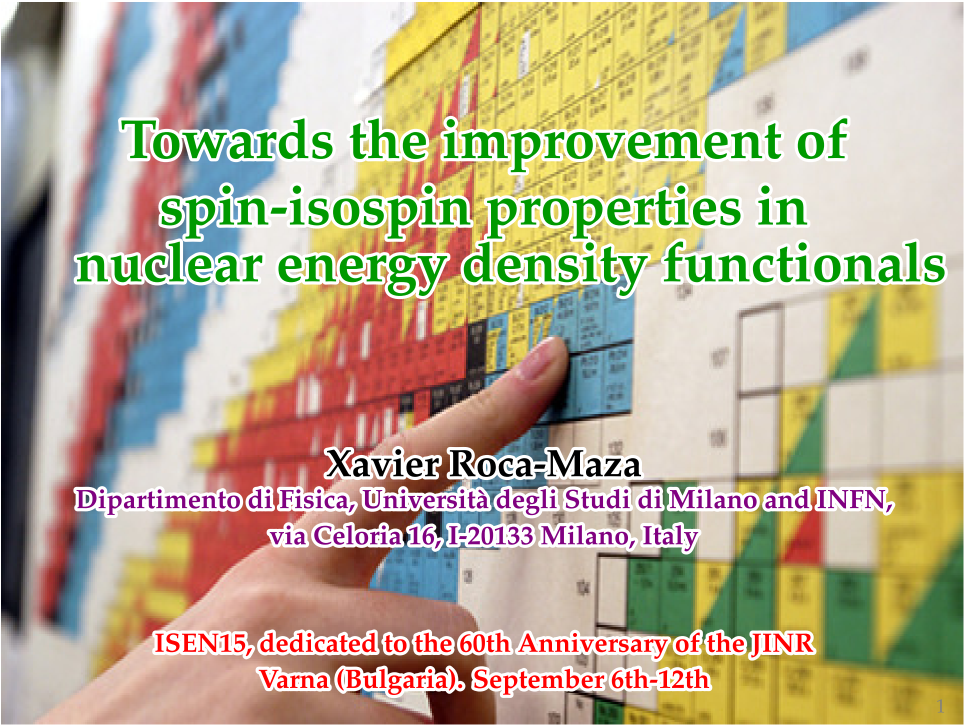

Towards the improvement of spin-isospin properties in nuclear energy density functionals

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- **Propose new fitting protocols**
  - **One based on empirical indications**
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Brief introduction

- **Density functional theory** is a successful approach to address the description of Quantum Many-Body systems, extensively used in physics, chemistry and material sciences.
- The **H(F)+RPA** method based on nuclear effective interactions of the **Skyrme, Gogny or Relativistic** types enables an effective description of the nuclear many-body problem and can be understood as an **approximate realization of an EDF**
- **One of the open problems that need to be better understood and solved in current EDFs is the ...**

accurate determination of **spin-isospin properties**

This **implies** (for example) an **accurate description** in charge-exchange excitations such as the **GTR**

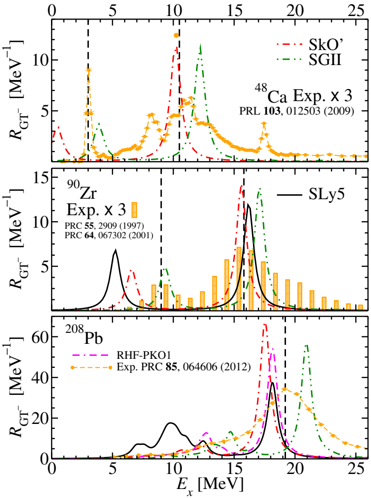
[GT transitions ...
govern electron capture during the core-colapse of supernovæ,
matrix el. are necessary for the study of double-β decay,
calibration of neutrino detectors ...]

Motivation: Gamow Teller Resonance I

The E_x is not properly described in H(F)+RPA

(Neither the strength → Beyond 1p-1h RPA effects)

- **SGII**[a] ⇒ earliest attempt to give a quantitative description of the GTR
- **SkO′**[b] ⇒ accurate in ground state finite nuclear properties and improves the GTR
- **PKO1**[c] ⇒ relativistic HF, reasonable GTR still not perfect.
- Relativistic H[d]: residual interaction modified *ad-hoc*

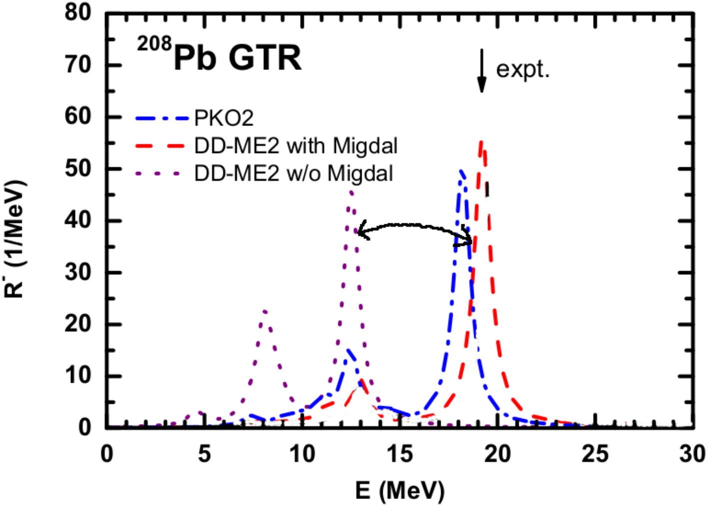


[a] N. Giai and H. Sagawa, Phys. Lett. B **106**, 379 (1981), [b] P.-G. Reinhard et al., Phys. Rev. C **60**, 014316 (1999), [c] H. Liang, N. Van Giai, and J. Meng, Phys. Rev. Lett. **101**, 122502 (2008), [d] N. Paar, T. Nikšić, D. Vretenar, and P. Ring, Phys. Rev. C 69, 054303

Motivation: Gamow Teller Resonance II

Exchange (Fock) effects on GTR in relativistic models

Effect of Migdal term → fitted to ^{208}Pb in RH

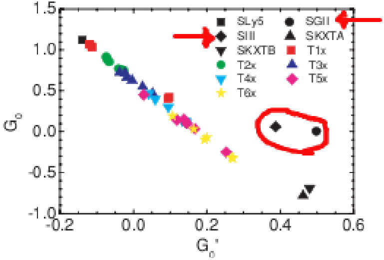


Motivation: which gs properties are important for describing the E_x^{GTR}?

The study[a] of the GTR and the spin-isospin Landau-Migdal parameter G_0' using several Skyrme sets,

- concluded that G_0' **is not the only important quantity** in determining the **excitation energy of the GTR**
- **spin-orbit splittings** also influences the GTR

- Empirical indications[b] suggest that $G_0' > G_0 > 0$
- **Not a very common feature** within available Skyrme forces[c]



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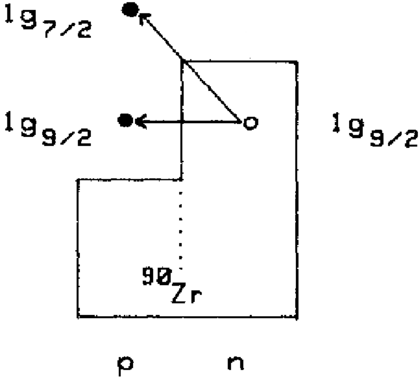
[a] M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C **65**, 054322 (2002); [b] T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005); T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999), [c] Li-Gang Cao, G. Colo, and H. Sagawa, Phys. Rev. C **81**, 044302 (2010)

Why spin-orbit splittings are important in E_x^{GTR}?

Schematic picture of single-particle transitions involved in the Gamow Teller Resonance of ^{90}Zr. Transitions excited by $\sigma\tau_-$ operator.



$$E_x^1 \approx \epsilon_{\pi 1g_{7/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^1 \qquad E_x^2 \approx \epsilon_{\pi 1g_{9/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^2$$

$$\Delta E_x \approx \Delta\epsilon_{\pi 1g} + \Delta\epsilon_{ph}$$

F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992)

We propose a new fitting protocol that help improving spin-isospin properties:

example with a Skyrme interaction

Fitting Protocol: Inspired on SLy5

χ^2 definition: $\chi^2 = \frac{1}{N_{\rm data}} \sum_i^{N_{\rm data}} \frac{(\mathcal{O}_i^{\rm theo.} - \mathcal{O}_i^{\rm data})^2}{(\Delta\mathcal{O}_i^{\rm data})^2}$

Landau-Migdal parameters in infinite nuclear matter G_0 and G_0' fixed to **0.15** and **0.35**, respectively, at ρ_0.

Table: Data and *pseudo*-data $\mathcal{O}_i$, adopted errors for the fit $\Delta\mathcal{O}_i$ and selected finite nuclei and EoS.

$\mathcal{O}_i$	$\Delta\mathcal{O}_i$	
B	1.00 MeV	^{40,48}Ca, ^{90}Zr, ^{132}Sn and ^{208}Pb
r_c	0.01 fm	^{40,48}Ca, ^{90}Zr and ^{208}Pb
ΔE_{SO}	$0.04\times\mathcal{O}_i$	$\pi 1g$ in ^{90}Zr and $\pi 2f$ in ^{208}Pb
$e_n(\rho)$	$0.20\times\mathcal{O}_i$	R. B. Wiringa *et al.*, PRC **38**, 1010 (1988)

Skyrme Aizu Milano interaction: SAMi

Parameter set and nuclear matter properties:

Table: SAMi parameter set and saturation properties with the estimated standard deviations inside parenthesis

	value(σ)			value(σ)	
t_0	−1877.75(75)	MeV fm^3	ρ_∞	0.159(1)	fm^{-3}
t_1	475.6(1.4)	MeV fm^5	e_∞	−15.93(9)	MeV
t_2	−85.2(1.0)	MeV fm^5	m^*_{IS}	0.6752(3)	
t_3	10219.6(7.6)	MeV fm$^{3+3\alpha}$	m^*_{IV}	0.664(13)	
x_0	0.320(16)		J	28(1)	MeV
x_1	−0.532(70)		L	44(7)	MeV
x_2	−0.014(15)		K_∞	245(1)	MeV
x_3	0.688(30)		G_0	0.15	(fixed)
W_0	137(11)		G_0'	0.35	(fixed)
W_0'	42(22)				
α	0.25614(37)				

Results

Gamow Teller Resonance in ^{48}Ca, ^{90}Zr and ^{208}Pb

$$\sum_{i=1}^{A} \sigma(i)\tau_{\pm}(i)$$

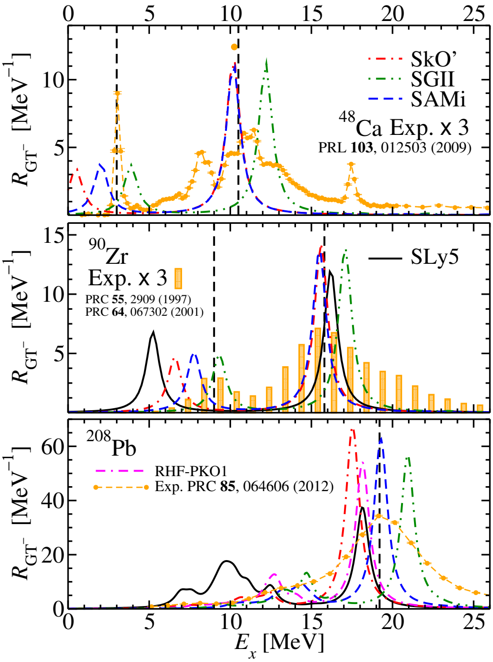


Figure: Gamow Teller strength distributions in ^{48}Ca (upper panel), ^{90}Zr (middle panel) and ^{208}Pb (lower panel) as measured in the experiment [T. Wakasa *et al.*, Phys. Rev. C **55**, 2909 (1997), K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009), A. Krasznaborkay *et al.*, Phys. Rev. C **64**, 067302 (2001), H. Akimune *et al.*, Phys. Rev. C **52**, 604 (1995) and T. Wakasa et al., Phys. Rev. C **85**, 064606 (2012)] and predicted by SLy5, SkO', SGII and SAMi forces.

Advantages and disadvantages of a RHF theory

Covariant density functional theory – RHF theory

- achieved **quantitative** description of $\mathrm{B}(\mathrm{N},\mathrm{Z})$ and $\langle r_{\mathrm{ch}}\rangle$ (*PLB* **640**, 150 (2006); *PRC* **76**, 034314 (2007); *EPL* **82**, 12001 (2008); *PRC* **81**, 024308 (2010))
- **effective mass splitting** in ANM can be described naturally (*PLB* **640**, 150 (2006))
- nuclear **spin-isospin resonances** can be described in a fully self-consistent way (*PRL* **101**, 122502 (2008); *PRC* **79**, 064316 (2009); *PRC* **85**, 064302 (2012))
- **improvement** on the descriptions of nuclear shell structures and their evolutions (*PRC* **76**, 034314 (2007); *EPL* **82**, 12001 (2008); *PLB* **680**, 428 (2009))

However ...

* RHF includes **non-local** potentials $v_{\mathrm{HF}}(\boldsymbol{r},\boldsymbol{r}')$
* RHF is much more complicated than RH theory.
* non negligible computational cost when improving the calculations and/or going beyond the mean-field

To construct RH functionals from RHF scheme

Therefore, it is desirable

- to find a covariant density functional based on only **local potentials**, yet **keeping the merits of the exchange terms**

Possible solution: construct RH functionals from RHF scheme

- **Fierz transformation** allow to **map** Fock terms into local Hartree terms **(for contact interactions)**
- but, masses of mesons are heavy ⇒ **zero-range approximation is reasonable in nuclei** (Skyrme, Relativistic point-coupling approaches, ...)

Fierz transformation: from α^{HF} to α^{H}

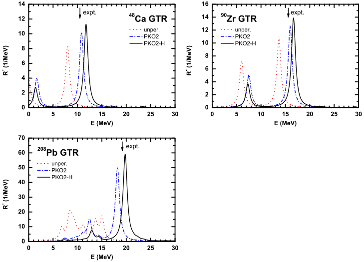
$$\begin{aligned}
\alpha_{S}^{H} &= +\frac{7}{8}\alpha_{S}^{HF} - \frac{4}{8}\alpha_{V}^{HF} - \frac{12}{8}\alpha_{tV}^{HF} \\
\alpha_{tS}^{H} &= -\frac{1}{8}\alpha_{S}^{HF} - \frac{4}{8}\alpha_{V}^{HF} + \frac{4}{8}\alpha_{tV}^{HF} \\
\alpha_{V}^{H} &= -\frac{1}{8}\alpha_{S}^{HF} + \frac{10}{8}\alpha_{V}^{HF} + \frac{6}{8}\alpha_{tV}^{HF} \\
\alpha_{tV}^{H} &= -\frac{1}{8}\alpha_{S}^{HF} + \frac{2}{8}\alpha_{V}^{HF} + \frac{6}{8}\alpha_{tV}^{HF} \\
\alpha_{T}^{H} &= -\frac{1}{16}\alpha_{S}^{HF} \\
\alpha_{tT}^{H} &= -\frac{1}{16}\alpha_{S}^{HF} \\
\alpha_{PS}^{H} &= -\frac{1}{8}\alpha_{S}^{HF} + \frac{4}{8}\alpha_{V}^{HF} + \frac{12}{8}\alpha_{tV}^{HF} \\
\alpha_{tPS}^{H} &= -\frac{1}{8}\alpha_{S}^{HF} + \frac{4}{8}\alpha_{V}^{HF} - \frac{4}{8}\alpha_{tV}^{HF} \\
\alpha_{PV}^{H} &= +\frac{1}{8}\alpha_{S}^{HF} + \frac{2}{8}\alpha_{V}^{HF} + \frac{6}{8}\alpha_{tV}^{HF} \\
\alpha_{tPV}^{H} &= +\frac{1}{8}\alpha_{S}^{HF} + \frac{2}{8}\alpha_{V}^{HF} - \frac{2}{8}\alpha_{tV}^{HF}
\end{aligned}$$

Check how good is the mapping in a practical case

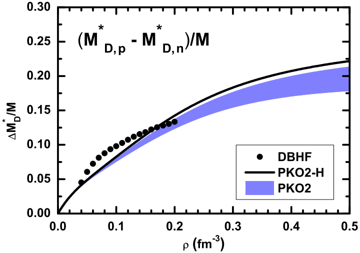
- **start** with an **available RHF** parametrization: PKO2 (which is based on a finte-range interaction)
- perform the **zero-range** reduction (if needed)
- perform the **Fierz transformation**

Compare observables sensitive to the Fock terms between the **original model** and the **localized** model, such as:

Gamow-Teller Resonance



Effective mass splitting between neutrons and protons



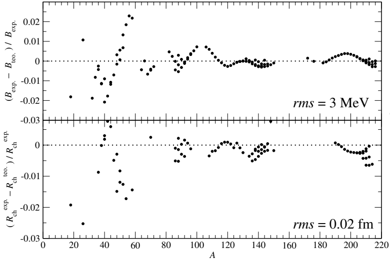
This opens the possibility for the development of new nuclear local covariant density functionals

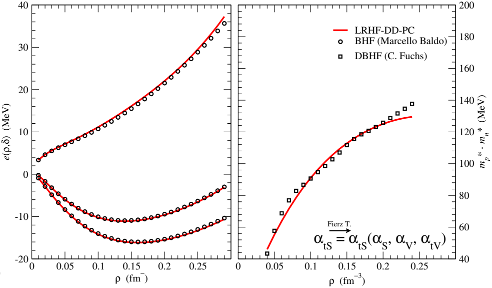
Preliminary test to build a localized model from RHF

Test to build a localized model: work in progress

Project:

- Build a **local CDF** including **all terms** in the Lagarngian **allowed by the symmetries** (S, V, tS, tV, T, tT, PS, tPS, PV, tPV terms)
- consider as **free parameters** the ones corresponding to the S, V **and** tV channels
- the **rest of the channels** will be determined by the **Fierz** transformations, within the same Hartee scheme, and in a fully **self-consistent manner.**





Conclusions:

- We have **remainded** some of the **problems** in the **spin-isospin channels** in Skyrme and RH models using as an example the **GTR**
- We have **briefly presented**
  - the benefits of the **new proposed fitting protocol** that cure part of the previous problems
  - test the new protocol and show **some results when applied with a Skyrme interaction**
  - the benefits of using a **RHF model**
  - proposed a **new method** to determine a **RH** model keeping the benefits of a RHF

Thank you!

Work in collaboration with:

G. Colò, H. Sagawa, Li-Gang Cao, H. Liang, J. Meng, P. Ring and P. Zhao