New Skyrme energy density functional for a better description of spin-isospin resonances

Xavier Roca-Maza

Dipartimento di Fisica, Università degli Studi di Milano and INFN,
via Celoria 16, I-20133 Milano, Italy

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Motivation

- **Density functional theory** is a successful approach to address the description of Quantum Many-Body systems, extensively used in physics, chemistry and material sciences.
- The **H(F)+RPA** method based on nuclear effective interactions of the **Skyrme, Gogny or Relativistic** types enables an effective description of the nuclear many-body problem and can be understood as an **approximate realization of an EDF**
- **One of the open problems that need to be better understood and solved in current EDFs is the ...**

accurate determination of **spin-isospin properties**

This **implies** (for example) an **accurate description** in charge-exchange excitations such as the **GTR**

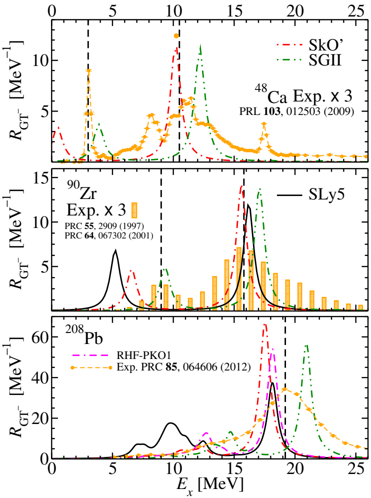
[GT transitions ...
govern electron capture during the core-colapse of supernovæ,
matrix el. are necessary for the study of double-β decay,
calibration of neutrino detectors ...]

Motivation: Gamow Teller Resonance I

The E_x is not properly described in H(F)+RPA

(Neither the strength → Beyond 1p-1h RPA effects)

- **SGII**[a] ⇒ earliest attempt to give a quantitative description of the GTR
- **SkO′**[b] ⇒ accurate in ground state finite nuclear properties and improves the GTR
- **PKO1**[c] ⇒ relativistic HF, reasonable GTR still not perfect.
- Relativistic H[d]: residual interaction modified *ad-hoc*



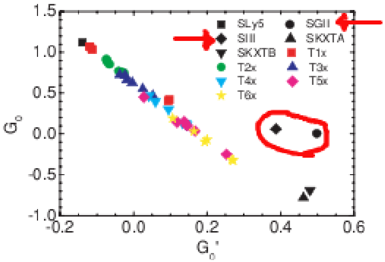
[a] N. Giai and H. Sagawa, Phys. Lett. B **106**, 379 (1981), [b] P.-G. Reinhard et al., Phys. Rev. C **60**, 014316 (1999), [c] H. Liang, N. Van Giai, and J. Meng, Phys. Rev. Lett. **101**, 122502 (2008), [d] N. Paar, T. Nikšić, D. Vretenar, and P. Ring, Phys. Rev. C 69, 054303

Motivation: which gs properties are important for describing the E_x^{GTR}?

The study[a] of the GTR and the spin-isospin Landau-Migdal parameter G_0' using several Skyrme sets,

- concluded that G_0' **is not the only important quantity** in determining the **excitation energy of the GTR**
- **spin-orbit splittings** also influences the GTR

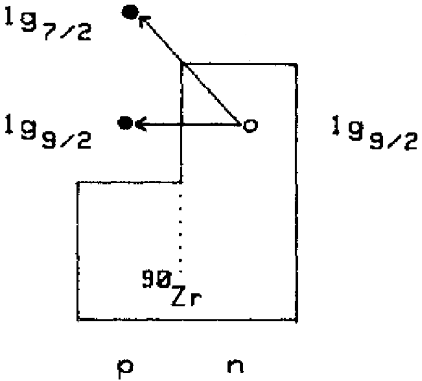
- Empirical indications[b] suggest that $G_0' > G_0 > 0$
- **Not a very common feature** within available Skyrme forces[c]



[a] M. Bender, J. Dobaczewski, J. Engel, and W. Nazarewicz, Phys. Rev. C **65**, 054322 (2002); [b] T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005); T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999), [c] Li-Gang Cao, G. Colo, and H. Sagawa, Phys. Rev. C **81**, 044302 (2010)

Why spin-orbit splittings are important in E_x^{GTR}?

Schematic picture of single-particle transitions involved in the Gamow Teller Resonance of ^{90}Zr. Transitions excited by $\sigma\tau_-$ operator.



$$E_x^1 \approx \epsilon_{\pi 1 g_{7/2}} - \epsilon_{\nu 1 g_{9/2}} + \epsilon_{ph}^1 \qquad E_x^2 \approx \epsilon_{\pi 1 g_{9/2}} - \epsilon_{\nu 1 g_{9/2}} + \epsilon_{ph}^2$$

$$\Delta E_x \approx \Delta\epsilon_{\pi 1 g} + \Delta\epsilon_{ph}$$

F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992)

New Skyrme interaction with improved spin-isospin properties

(Standard) Skyrme Model

[... have a quick look!]

Includes **central tensor terms (J^2 terms)** due to the coupling of tensor and spin and gradients terms and **two spin-orbit parameters** (same as SkO and some SkI forces)

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}} + H_{SO} + H_{sg} + \mathcal{H}_{\text{Coul}}$$

$$\mathcal{K} = \hbar^2\tau/2m$$

$$\mathcal{H}_0 = (1/4)t_0[(2+x_0)\rho^2 - (2x_0+1)(\rho_n^2+\rho_p^2)] \text{ (CENTRAL)}$$

$$\mathcal{H}_3 = (1/24)t_3\rho^\alpha[(2+x_3)\rho^2 - (2x_3+1)(\rho_n^2+\rho_p^2)] \text{ (DENSITY DEP.)}$$

$$\mathcal{H}_{\text{eff}} = (1/8)[t_1(2+x_1)+t_2(2+x_2)]\tau\rho + (1/8)[t_2(2x_2+1)-t_1(2x_1+1)](\tau_n\rho_n+\tau_p\rho_p) \text{ (EFF. MASS)}$$

$$\mathcal{H}_{\text{fin}} = (1/32)[3t_1(2+x_1)-t_2(2+x_2)](\nabla\rho)^2 - (1/32)[3t_1(2x_1+1)+t_2(2x_2+1)][(\nabla\rho_n)^2+(\nabla\rho_p)^2] \text{ (FIN RANGE)}$$

$$H_{SO} = (1/2)W_0\mathbf{J}\cdot\nabla\rho + (1/2)W_0'(\mathbf{J}\cdot_{\mathbf{n}}\nabla\rho_n + \mathbf{J_p}\cdot\nabla\rho_p)$$

$$H_{sg} = -(1/16)(t_1x_1+t_2x_2)\mathbf{J}^2 + (1/16)(t_1-t_2)(\mathbf{J_n}^2+\mathbf{J_p}^2)$$

Fitting Protocol: Inspired on SLy5

χ^2 definition: $\chi^2 = \frac{1}{N_{data}} \sum_i^{N_{data}} \frac{(\mathcal{O}_i^{theo.} - \mathcal{O}_i^{data})^2}{(\Delta \mathcal{O}_i^{data})^2}$

Landau-Migdal parameters in infinite nuclear matter G_0 and G_0' fixed to **0.15** and **0.35**, respectively, at ρ_0.

Table: Data and *pseudo*-data $\mathcal{O}_i$, adopted errors for the fit $\Delta\mathcal{O}_i$ and selected finite nuclei and EoS.

$\mathcal{O}_i$	$\Delta\mathcal{O}_i$	
B	1.00 MeV	^{40,48}Ca, ^{90}Zr, ^{132}Sn and ^{208}Pb
r_c	0.01 fm	^{40,48}Ca, ^{90}Zr and ^{208}Pb
ΔE_{SO}	$0.04\times\mathcal{O}_i$	$\pi 1g$ in ^{90}Zr and $\pi 2f$ in ^{208}Pb
$e_n(\rho)$	$0.20\times\mathcal{O}_i$	R. B. Wiringa *et al.*, PRC **38**, 1010 (1988)

Skyrme Aizu Milano interaction: SAMi

Parameter set and nuclear matter properties:

Table: SAMi parameter set and saturation properties with the estimated standard deviations inside parenthesis

	value(σ)			value(σ)	
t_0	−1877.75(75)	MeV fm^3	ρ_∞	0.159(1)	fm^{-3}
t_1	475.6(1.4)	MeV fm^5	e_∞	−15.93(9)	MeV
t_2	−85.2(1.0)	MeV fm^5	m^*_{IS}	0.6752(3)	
t_3	10219.6(7.6)	MeV fm$^{3+3\alpha}$	m^*_{IV}	0.664(13)	
x_0	0.320(16)		J	28(1)	MeV
x_1	−0.532(70)		L	44(7)	MeV
x_2	−0.014(15)		K_∞	245(1)	MeV
x_3	0.688(30)		G_0	0.15	(fixed)
W_0	137(11)		G'_0	0.35	(fixed)
W'_0	42(22)				
α	0.25614(37)				

Results

Equation of State: SAMi vs *ab*–initio calculations

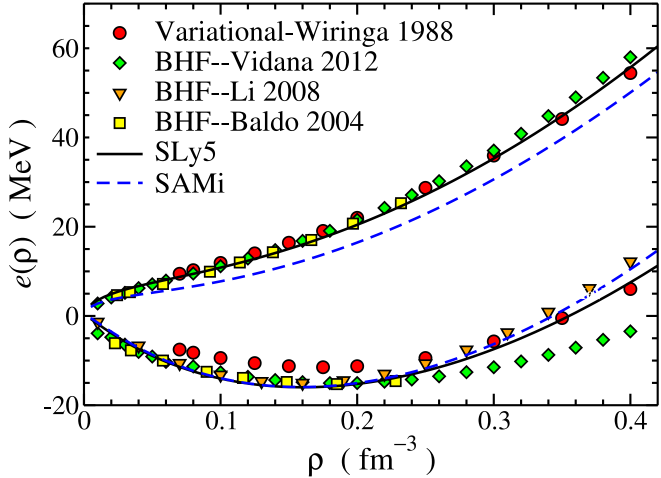


Figure: Neutron and symmetric matter EoS as predicted by the HF SAMi (dashed line) and SLy5 (solid line) interactions and by the benchmark microscopic calculations of R. B. Wiringa *et al.*, PRC **38**, 1010 (1988) (circles). State-of-the-art BHF calculations are shown by diamonds I. Vidaña, private communication, triangles Z. H. Li *et al.*, Phys. Rev. C **77**, 034316 (2008) and squares M. Baldo *et al.*, Nucl. Phys. A **736**, 241 (2004).

Results

Giant Monopole and Dipole Resonances in ^{208}Pb

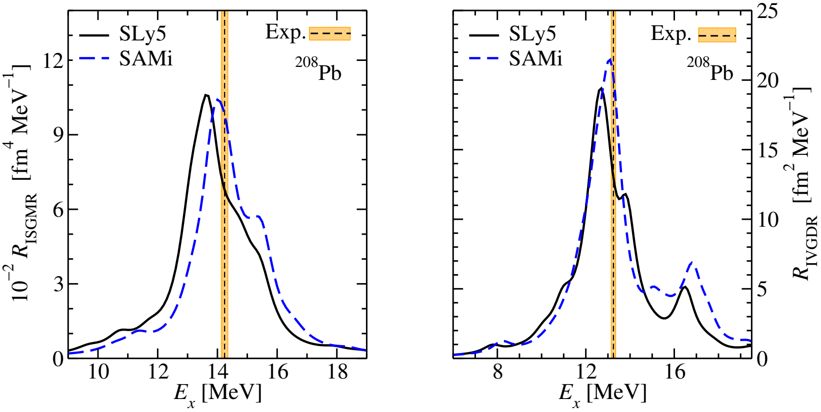


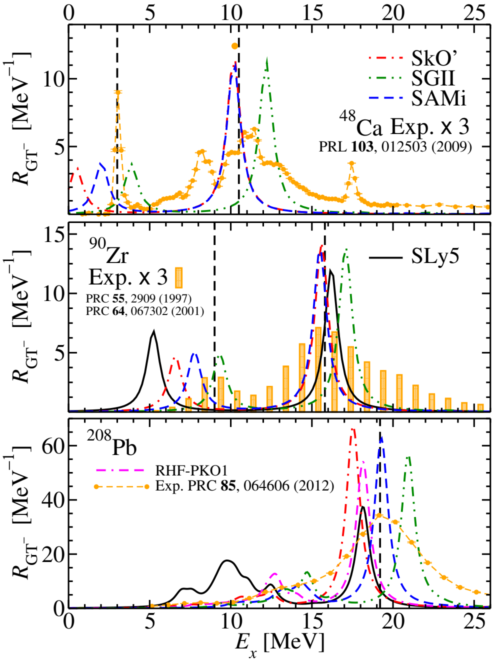
Figure: Strength function at the relevant excitation energies in ^{208}Pb as predicted by SLy5 and the SAMi interaction for GMR and GDR. A Lorentzian smearing parameter equal to 1 MeV is used. Experimental data for the centroid energies are also shown: $E_c(GMR) = 14.24 \pm 0.11$ MeV [D. H. Youngblood, et al., Phys. Rev. Lett. **82**, 691 (1999)] and $E_c(GDR) = 13.25 \pm 0.10$ MeV [N. Ryezayeva et al., Phys. Rev. Lett. **89**, 272502 (2002)].

Results

Gamow Teller Resonance in ^{48}Ca, ^{90}Zr and ^{208}Pb

$\sum_{i=1}^{A} \sigma(i)\tau_{\pm}(i)$

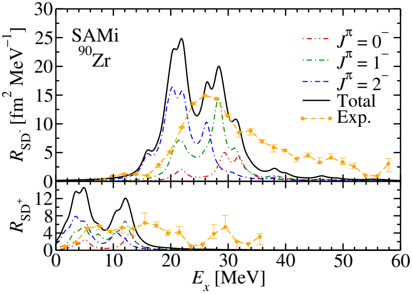
Figure: Gamow Teller strength distributions in ^{48}Ca (upper panel), ^{90}Zr (middle panel) and ^{208}Pb (lower panel) as measured in the experiment [T. Wakasa *et al.*, Phys. Rev. C **55**, 2909 (1997), K. Yako *et al.*, Phys. Rev. Lett. **103**, 012503 (2009), A. Krasznaborkay *et al.*, Phys. Rev. C **64**, 067302 (2001), H. Akimune *et al.*, Phys. Rev. C **52**, 604 (1995) and T. Wakasa et al., Phys. Rev. C **85**, 064606 (2012)] and predicted by SLy5, SkO′, SGII and SAMi forces.



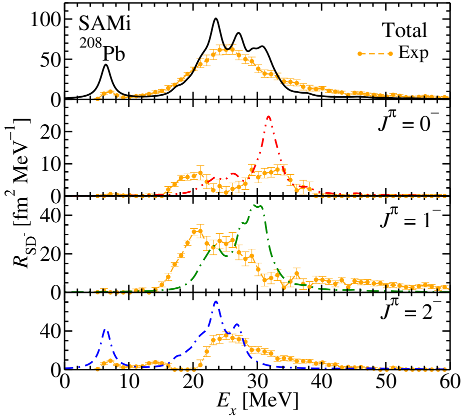
Results

Spin Dipole Resonances in ^{90}Zr and ^{208}Pb

Operator:
$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_i^L [Y_L(\hat{r}_i) \otimes \sigma(i)]_{JM}$
Sum Rule:
$\int [R_{SD-}(E) - R_{SD+}(E)] dE = \frac{9}{4\pi}(N\langle r_n^2\rangle - Z\langle r_p^2\rangle)$



Experiment: K. Yako *et al.*, Phys. Rev. C **74**, 051303(R) (2006). A Lorentzian smearing parameter 2 MeV is used.



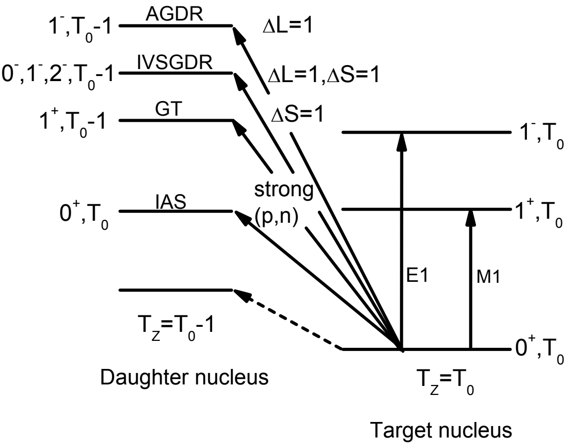
Experiment: T. Wakasa *et al.*, Phys. Rev. C **85**, 064606 (2012). A Lorentzian smearing parameter 2 MeV is used.

Analysis of the AGDR in ^{208}Pb

What we understand by AGDR

$T_0 - 1$ component of the charge-exchange of $\Delta J = 1$ $\Delta L = 1$ and $\Delta S = 0$

$$\hat{O}_{\pm} = \sum_i r_i Y_{1m}(\hat{r}_i) t_{\pm}(i)$$



$E_{AGDR} - E_{IAS}$ **versus** Δr_{np}: a simple macroscopic model

- Based on the simple **harmonic oscillator** approach in which the AGDR is exhausted in a single peak containing the full EWSR and $E_{IAS} \approx \Delta E_C$:

$$E_{AGDR} - E_{IAS} \approx \frac{5}{8}\sqrt{\frac{5}{3}}\frac{V_1(1+\gamma)}{\alpha Z}\frac{\hbar c}{mc^2\langle r^2\rangle^{1/2}}$$

where $2(1+\gamma) \equiv m_1^{AGDR}/m_1^{IVGDR} \approx$ constant (for SAMi-J), $V_1 \approx 8\left[a_{sym}(A) - \varepsilon_{F_\infty}/3\right]$ and $E^{unp}_{AGDR} \equiv \varepsilon - U + \Delta E_C \approx \Delta E_C$

- **Droplet Model** relation between $a_{sym}(A)$ and Δr_{np}:

$$E_{AGDR} - E_{IAS} \approx 5\sqrt{\frac{5}{3}}\frac{J}{I}\frac{1+\gamma}{\alpha Z}\frac{\hbar c}{m\langle r^2\rangle^{1/2}}\left[\left(1 - \frac{\varepsilon_{F_\infty}}{3J}\right)I - \frac{3}{2}\left(\frac{\Delta R_{np} - \Delta R_{np}^{surf}}{\langle r^2\rangle^{1/2}}\right) - \frac{3}{7}I_C\right]$$

- **Alternatively**, one may also write within the HO approach:

$$E_{AGDR} - E_{IAS} \approx \frac{\varepsilon}{\Delta E_C}\left(E_{IVGDR} - \varepsilon\right)\frac{m_1^{AGDR}}{m_1^{IVGDR}}$$

SAMi-J[a]: systematically varied interactions

For the analysis of isospin sensitive observables/quantities:
we have considered **families of functionals with systematically varied properties in the isovector channel**.

Allow to explore the isovector channel ensuring a good description of fitted (dominantly isoscalar) properties around the minimum of a given optimal model

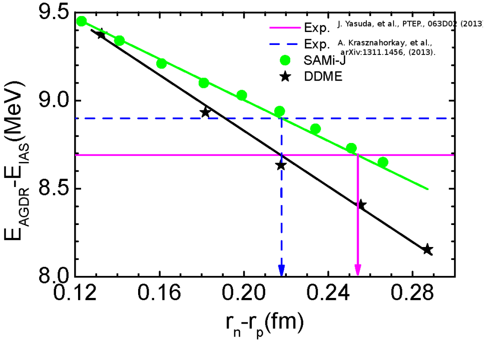
- Specificaly, using the **SAMi fitting protocol**, we fix K_∞ and m^* to the optimal values of SAMi and also fix J at different values for each interaction of the family: **from 27 MeV to 35 MeV** in steps of 1 MeV

9 interactions in the family SAMi-J

[a] X. Roca-Maza, M. Brenna, B. K. Agrawal, P. F. Bortignon, G. Colò, Li-Gang Cao, N. Paar, and D. Vretenar Phys. Rev. C 87, 034301 and L.G. Cao, X. Roca-Maza, G. Colò, H. Sagawa, arXiv:1504.07166

$E_{AGDR} - E_{IAS}$ versus Δr_{np}: micrsocopic predictions SAMi-J[a] and DDME[b]

- As suggested by the macroscopic model: **linear relation**
- **Clear model dependence:** non-accurate det. of Δr_{np}; better understanding is needed.



Note: Exps. are compatible albeit the central value of exp. in **pink** (8.69 ± 0.36 MeV) is clearly out from one sigma of exp. **blue** (8.90 ± 0.09 MeV)

[a] L.G. Cao, X. Roca-Maza, G. Colò, H. Sagawa, arXiv:1504.07166; [b] A. Krasznahorkay, N. Paar, D. Vretenar, and M. N. Harakeh, Phys. Scr. T 154, 014018 (2013) and A. Krasznahorkay, et al., arXiv:1311.1456.

Conclusions:

- we have **successfully determined a new Skyrme** energy density functional which **accounts** for the most relevant quantities in order to improve the description of **charge-exchange nuclear resonances**:
  - the **hierarchy** and **positive values** of the spin and spin-isospin Landau-Migdal parameters G_0 **and** G_0'
  - the **proton spin-orbit splittings** of different **high angular momenta** single-particle levels
- the **GTR** in ^{48}Ca and the **GTR**, **IAR**, and **SDR** in ^{90}**Zr** and ^{208}**Pb**, and the $E_{AGDR} - E_{IAS}$ in ^{208}Pb are predicted with **reasonable accuracy by SAMi**.
- **SAMi** does **not deteriorate** the description of other **nuclear observables**.
- **applicability in nuclear physics and astrophysics**

Thank you!

Work in collaboration with:

G. Colò, H. Sagawa and Li-Gang Cao

Extra Material

Motivation: Gamow Teller Resonance II

Quenching of the strength

- **Experimentally**, the **GTR** exhausts **60–70%** of the **Ikeda sum rule**: $\int[R_{GT^-}(E) - R_{GT^+}(E)]dE = 3(N - Z)$
- To **explain** the problem, two possibilities that go beyond $(1p - 1h)$ RPA correlations have been drawn:
  - the effects of the second-order configuration mixing: **2p-2h correlations**
  - within the quark model, a **n(p)** can become a **p(n)** or a $\Delta^+(\Delta^{++})$ under the action of the GT^- operator and since there is **no Pauli blocking for Δ−h excitations** ⇒ it may **contribute to the GTR**.
- The **experimental analysis of ^{90}Zr** ⇒ **quenching** (2/3) has to be **mainly attributed to 2p-2h** coupling and not to Δ−isobar effects much smaller [T. Wakasa *et. al.*, Phys. Rev. C **55**, 2909 (1997)].
- E_x **GTR in nuclei** mainly in the region of several **tens of MeV** and the Δ−h states are hundreds of MeV above the GT ⇒ **hard to excite the Δ** in the nuclear medium.

Empirical constraints on G_0 and G_0'

- **Gamow-Teller Resonance** using RPA based on the Woods-Saxon potential have been studied and the **Landau-Migdal parameters estimated by comparing experiment** with theoretical calculations in Refs. [T. Wakasa, M. Ichimura, and H. Sakai, Phys. Rev. C **72**, 067303 (2005) and T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999)].
- In our fit, **we do not use the obtained values as pseudodata** because **our theoretical framework is different and the results are associated to different** m^* (our sp energies are based on HF calculations instead of a Wood-Saxon potential).
- **We use** the empirical result in which an **hierarchy** between spin and spin-isospin parameters is suggested:

$$G_0' > G_0 > 0$$

Results

Finite Nulcei: spherical double-magic nuclei

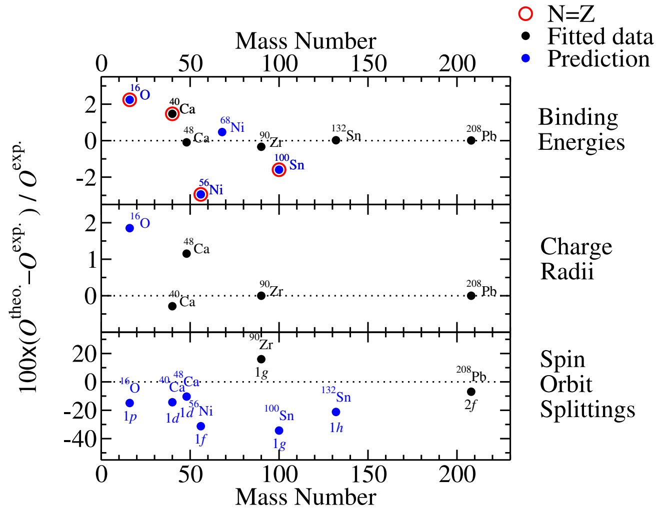
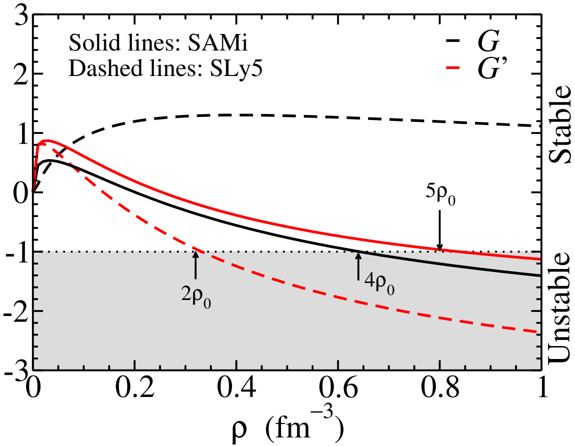


Figure: Finite nuclei properties as predicted by the HF SAMi (black circles) and some predictions (blue circles) for spherical double-magic nuclei. Experimental data taken from Refs. G. Audi *et al.*, NPA **729**, 337 (2003), I. Angeli, ADNDT **87**, 185 (2004), M. Zalewski *et al.*, PRC **77**, 024316 (2008)

SAMi: spin and spin-isospin instabilities

Imposing that spin and isospin d.o.f. at the Fermi surface are stable under generalized deformations [Bäckman *et al.*, Nucl. Phys. A **321**, 10 (1979)]

$$1 + G_0 > 0 \qquad 1 + G_0' > 0$$



Model dependece: possible sources to be investigated

- **Coulomb-exhange?** DD-ME does not contain this contribution and SAMi-J does it, it might be important in high Z nuclei. [The Coulomb shift $\Delta E_C \sim 2\left(\frac{3}{5}\right)^{3/2} \frac{e^2 Z}{\langle r^2 \rangle^{1/2}}$]
- **Macroscopic** model we use for guidance is **non-relativistic**, we cannot assess if model dependendence arise form the covariance (or not) of the functionals.
- **Differences on the effective mass?** The excitation energy is modified by m^*, in our case $\varepsilon = 41 A^{-1/3} \sqrt{m/m^*}$ MeV [non-relativistic estimation[a] of the effective mass associated to DD-ME is different than the one of SAMi-J].
- ...
- **suggestions?**

[a] M. Jaminon and C. Mahaux, Phys. Rev. C 40, 354 (1989)