Parity violating asymmetry, dipole polarizability, and the neutron skin thickness in ^{48}Ca and ^{208}Pb

Xavier Roca-Maza
Università degli Studi di Milano and INFN
Via Celoria 16, I-20133, Milano (Italy)

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The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different nuclear interactions in the medium** are found **depending** on the **approach**
  - **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

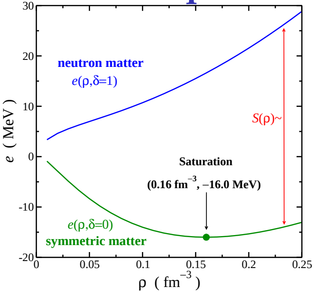
$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma && +\bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ &-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} && -\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\boldsymbol{\tau}\Psi A^{(\rho)\mu} \\ &-e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}\end{aligned}$$

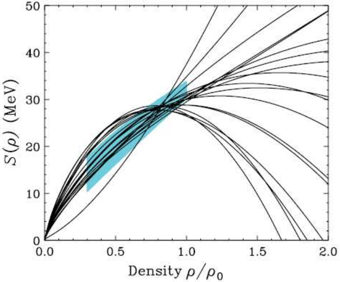
Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\mathrm{eff}}_{\mathrm{Nucl}} = V^{\mathrm{long-range}}_{\mathrm{attractive}} + V^{\mathrm{short-range}}_{\mathrm{repulsive}} + V_{\mathrm{SO}}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** → **not directly connected to any NN** (or NNN) **interaction**

The Nuclear Equation of State: Infinite System



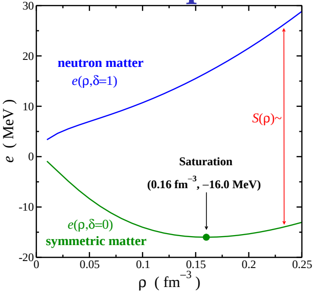


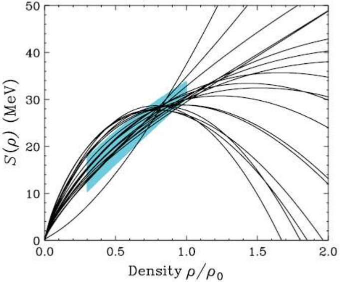
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



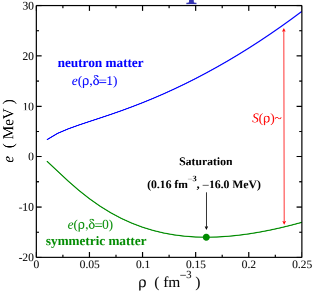


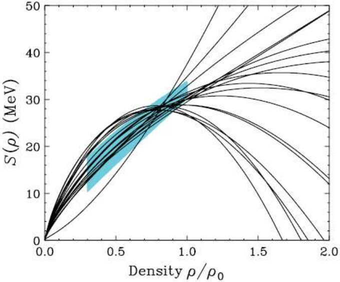
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



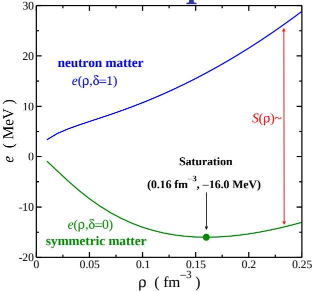


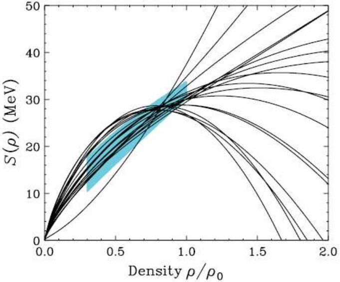
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter
- Symmetry energy

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



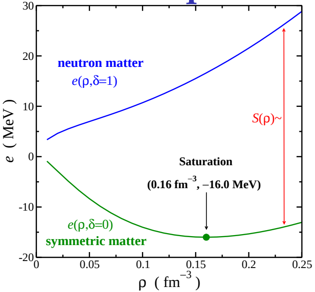


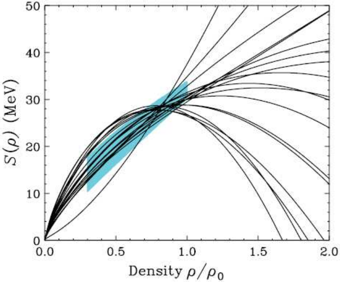
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho,\beta=0) + \beta^2\left(J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3)\right)$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho-\rho_0}{3\rho_0}\right]$$

The Nuclear Equation of State: Infinite System





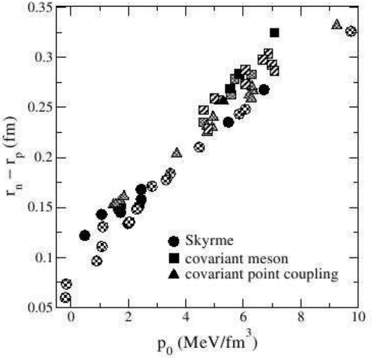
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + \beta^2 \left(\boxed{J} + \boxed{L}\, x + \frac{1}{2} \boxed{K_{sym}}\, x^2 + \mathcal{O}(x^3) \right)$$

- $S(\rho_0) = J$
- $\frac{d}{d\rho} S(\rho) \Big|_{\rho_0} = \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2}$
- $\frac{d^2}{d\rho^2} S(\rho) \Big|_{\rho_0} = \frac{K_{sym}}{9\rho_0^2}$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0} \right]$$

It is possible to clearly connect observables with general isovector properties of the nuclear effective interaction?

Example:
Modern nuclear models of current use show a clear **correlation** between Δr_{np} of a medium and heavy nucleus and the density slope of the symmetry energy ($L = 3\rho_0 \partial_\rho S(\rho)|_{\rho_0} = 3\rho_0 p_0$).



R.J. Furnstahl, NPA, **706**, 85 (2002)

Motivation: importance of determining isovector properties in nuclei

- **In the past** (and also in the present), **neutron properties** in **stable** medium and heavy nuclei have been mainly measured by using **strongly interacting probes**.

  $\Downarrow$

  Limited knowledge of isovector properties

- **At present**,
  - the use of **rare ion beams** has opened the possibility of measuring properties of **exotic nuclei**
  - **parity violating elastic electron scattering** (PVES), a **model independent technique**, has allowed to estimate the **neutron radius** of a stable heavy nucleus like ^{208}Pb

  $\Downarrow$

  Promising perspectives for the near future

Parity violating elastic electron scattering in ^{48}Ca and ^{208}Pb

From theory: calculation of $e - A$ elastic scattering

- **Electrons** interact by exchanging a γ or a Z_0 boson.
- While **protons** couple basically to γ, **neutrons** do it to Z_0.
- **Ultra-relativistic electrons**, depending on their helicity, interact with the nucleons $V_{\pm} = V_{\mathrm{Coulomb}} \pm V_{\mathrm{Weak}}$.
- Input for the calculation of $V_{\pm}$ are the ρ_n **and** ρ_p **(from nuclear models main uncertainties)** and **nucleon form factors** for the e-m and the weak neutral current.
- Electron scattering from a heavy nucleus is modified substantially by Coulomb distortions ($\sim \alpha Z$).
- **Ultra-relativistic electrons** moving under the effect of $V_{\pm}$ where **Coulomb distortions** are important $\Rightarrow$ solution of the Dirac equation via the Distorted Wave Born Approximation (**DWBA**).

Refs: Phys. Rev. C **57** 3430 (1998); Phys. Rev. C **63**, 025501 (2001); Phys. Rev. C **78**, 044332 (2008); Phys. Rev. C **82**, 054314 (2010); Phys. Rev. Lett. **106** 252501 (2011)

From Experiment:

The **P**b and **C**a **R**adius **Ex**periments (**PREx** and **CREx**) @JLab measure, model-independently, the **parity violating asymmetry**,

$$A_{pv} = \frac{\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}}{\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}}$$

at 1.06 GeV and for a single angle (~ 5 deg.) in ^{208}Pb and at 2.20 GeV and for a single angle (~ 4 deg.) in ^{48}Ca

- ρ_p is accurately known for these nuclei via parity conserving elastic electron scattering
- **ρ_n of ^{208}Pb and ^{48}Ca are the quantities to be determined**
- A precise determination of Δr_{np} would constrain the density dependence of the symmetry energy around saturation.

Qualitatively,

- A_{pv} within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)}\right]$$

- ... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$\begin{aligned} -\frac{q^2}{6}\left(\langle r_n^2\rangle - \langle r_p^2\rangle\right) &= -\frac{q^2}{6}\left[\Delta r_{np}(\langle r_n^2\rangle^{1/2} + \langle r_p^2\rangle^{1/2})\right] \\ &= -\frac{q^2}{6}\left(2\langle r_p^2\rangle^{1/2}\Delta r_{np} + \Delta r_{np}^2\right) \end{aligned}$$

- variation of A_{pv} at a fixed q dominated by the variation of Δr_{np}. $\langle r_p^2\rangle^{1/2}$ well fixed by experiment

^{208}Pb: direct correlations

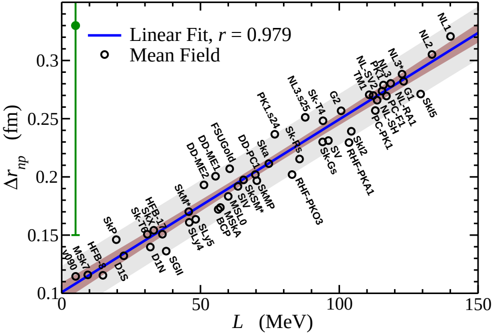
X. Roca-Maza, M. Centelles, X. Viñas, and M.

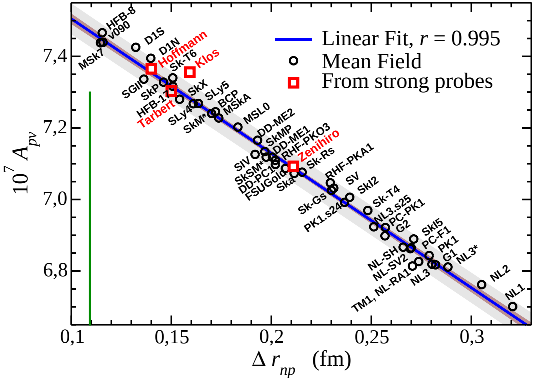
Warda, Phys. Rev. Lett. **106** 252501 (2011)

$\delta A_{pv} \sim$ **1%**

$\delta \Delta r_{np} \sim 0.02$ **fm**

or $\delta L \sim$ **10 MeV**

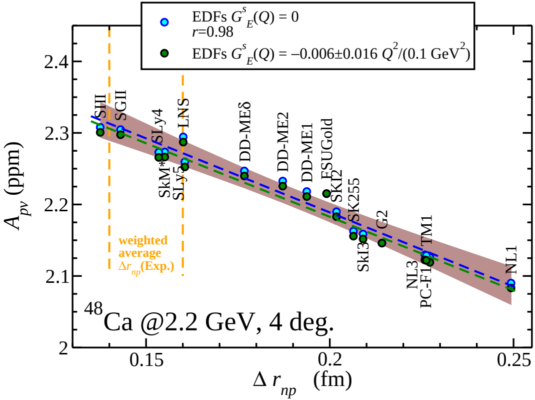




EDF correlations allows to determine Δr_{np} and L without direct assumpt. on ρ, new run of PREx [and Mainz] expected accuracy $\rightarrow$ constrain L

Different experiments using strongly interacting probes agree with the correlation

^{48}Ca: direct correlations within EDF



- A_{pv} decreases by around 5 ppb with an error of about 10 - 20 ppb when $G^s_E(Q^2)$ is included.
- Spin-orbit effects shifts to lower values the A_{pv} by about several tens of ppb. This predicts a reduction of Δr_{np} of about 0.05 fm.

- Shell Model calculations based on χEFT with NN to N3LO (fixed to scattering data) and 3N to N2LO (fixed to B tritium and R of $\alpha-$particle) **[J. Menendez (TU Darmstadt)]** $\rightarrow$ 3N-forces used shifts downwards the A_{pv} by about **0.05 ppm**

Used $G^s_E(Q^2)$ from PRC 76, 025202 (2007) by Liu, McKeown, and Ramsey-Musolf **Average Δr_{np} from hadronic probes: PRC12, 778 1978; PRL87, 08250113, 343 (2004); Phys. Rev. 174, 1380 (1968); Physics Letters 57B 47 (1975); PRC 67, 054605 (2003) and PRC33 1624 (1986).**

Electric dipole polarizability as a complementary observable

- **Macroscopic picture:** the electric polarizability measures the tendency of the nuclear charge distribution to be distorted $\rightarrow \alpha = \dfrac{\text{electric dipole moment}}{\text{external electric field}}$
- **Microscopic definition:** The linear response or dynamic polarizability of a nuclear system excited from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the action of an external oscillating dipolar field of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_D = \frac{Z}{A}\sum_i^N r_n Y_{1M}(\hat{r}_n) - \frac{N}{A}\sum_i^Z r_p Y_{1M}(\hat{r}_p)$$

- is proportional to the **static dipole polarizability, α_D,** for small oscillations

$$\alpha_D = \frac{8\pi}{9}e^2 \sum_\nu \frac{|\langle\nu|F_D|0\rangle|^2}{E}$$

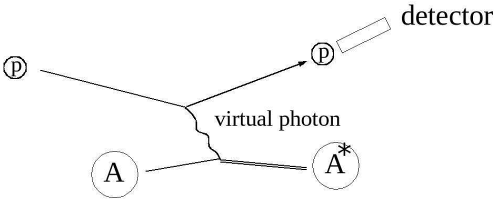
- DM: $\alpha_D \approx \dfrac{A\langle r^2\rangle}{12J}\left[1 + \dfrac{5}{2}\dfrac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$

Recent measurement in ^{208}Pb at RCNP

At the Research Center for Nuclear Physics (RCNP), Osaka ...

- using **polarized protons**
- measuring protons **scattered inelatically**
- excitations via virtual photons (**Coulomb excitation**)
- able to cover a **broad range of excitation energies**
- set up with **high-resolution and efficiency**

Very good agreement with previous measurements is found



A. Tamii et al., PRL107 (2011) 062502

Recent measurement in ^{208}Pb at RCNP

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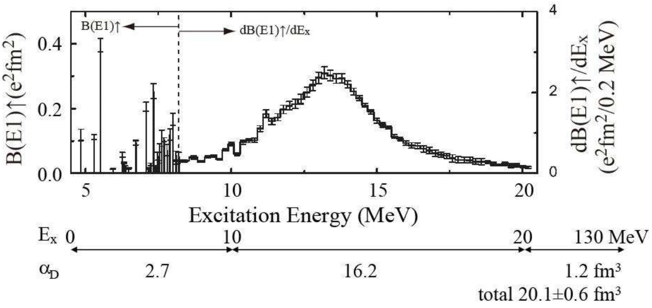
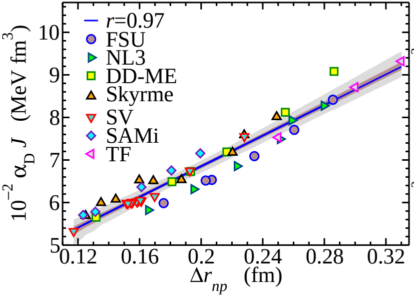


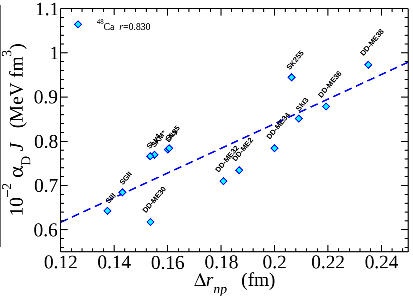
Figure taken from A. Tamii's talk at INPC 2013

Dipole polarizability is determined with high accuracy by taking the average of the RCNP data plus available data in ^{208}Pb† covering a wide range of excitation energies: $\alpha_D = 20.1 \pm 0.6$ **fm^3**

†K. Schelhaas et al., Nucl. Phys. A 489, 189 (1988); A. Veyssiere, H. Beil, R. Bergere, P. Carlos, and A. Lepretre, Nuclear Physics A 159, 561 (1970)

Electric dipole polarizability in ^{48}Ca and ^{208}Pb:





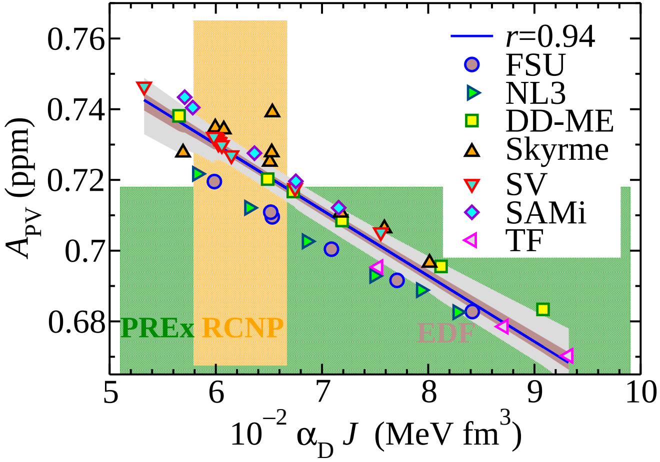
X. Roca-Maza, *et al.*, Phys. Rev. C 88, 024316 (2013)

- Using exp. $\alpha_D = 20.1 \pm 0.6\ \mathbf{fm}^3$ † in ^{208}Pb one finds the relation

$$\Delta r_{np} = -0.156 \pm (0.014)_{\text{theo.}} + \left[1.04 \pm (0.03)_{\text{exp.}} \pm (0.04)_{\text{theo.}}\right] \times 10^{-2} J$$

† A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011)

A_{pv} and α_D in ^{208}Pb



Conclusions:

- A precise and **model-independent** determination of Δr_{np} in ^{48}Ca and ^{208}Pb via PVES experiments would **probe** the nuclear **symmetry energy**.
- We demonstrate a close **linear correlation** between $A_{p\nu}$ and Δr_{np} within the same framework in which the Δr_{np} is correlated with L (expected to be better as heavier the nucleus).
- Other **experiments** fairly **agree** with the **correlation** between $A_{p\nu}$ and Δr_{np} in ^{208}Pb.

Conclusions:

- Modern nuclear energy density functionals show a linear correlation between $\alpha_D J$ and Δr_{np} and the measurement of α_D in ^{208}Pb has allowed us to find $\Delta r_{np} = -0.156 \pm (0.014)_{\rm theo.} + \left[1.04 \pm (0.03)_{\rm exp.} \pm (0.04)_{\rm theo.}\right] \times 10^{-2} J$
- $A_{p\nu}$ and α_D are complementary **observables** (though only $A_{p\nu}$ is basically model independent) that may set tight **constraints** on the **density dependence of the symmetry energy.**

Collaborators:

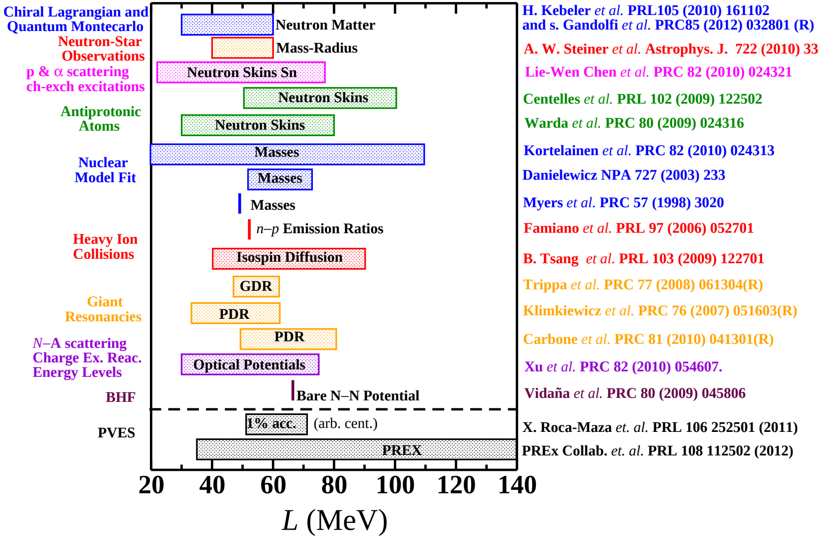
B. K. Agrawal[1]
M. Brenna
G. Colò[2,3]
W. Nazarewicz[4,5,6]
N. Paar[7]
J. Piekarewicz[8]

P.-G. Reinhard[9]
D. Vretenar[7] Michal Warda[10]
Mario Centelles[11]
Xavier Viñas[11]

[1] Saha Institute of Nuclear Physics, Kolkata 700064, India
[2] Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy
[3] INFN, Sezione di Milano, via Celoria 16, I-20133 Milano, Italy
[4] Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA
[5] Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA
[6] Institute of Theoretical Physics, University of Warsaw, ulitsa HoÅija 69, PL-00-681 Warsaw, Poland
[7] Physics Department, Faculty of Science, University of Zagreb, Zagreb, Croatia
[8] Department of Physics, Florida State University, Tallahassee, Florida 32306, USA
[9] Institut für Theoretische Physik II, UniversitÃd't Erlangen-Nürnberg, Staudtstrasse 7, D-91058 Erlangen, Germany
[10] Katedra Fizyki Teoretycznej, Uniwersytet Marii Curie-Sklodowskiej, ul. Radziszewskiego 10, PL-20-031 Lublin, Poland
[11] Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Facultat de Física, Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain

Extra Material

Available constraints on L



AIP Conference Proceedings **1491**, *101 (2012)*

Macroscopic approach: α_D

- Being E the energy of a nucleus within the Liquid Drop Model, Droplet Model,... and F_{ext} an external field (dipole operator) $\rightarrow$ constrained calculation keeping N and Z fixed:

$$\delta \left\{ E(\rho,\delta) + F_{ext}(\Lambda,\rho,\delta) - \lambda_n \int_0^\infty \rho_n(r)dr - \lambda_p \int_0^\infty \rho_p(r)dr \right\} = 0$$

- **LDM**[1]:

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}$$

- **DM**[2]:

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

[1] A.B . Migdal, J. Phys. USSR **8** (1944) 331; J.S . Levinger, Nuclear photodisintegration (Oxford Univ . Press, London, 1960) sect. 3-1.

[2] J. Meyer, P. Quentin, and B. K. Jennings, Nucl. Phys. A **386** (1982) 269.

Droplet model approach: symmetry energy and neutron skin

- The symmetry energy with surface effects is written in the DM:

$$a_{sym}(A) = \frac{J}{1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}},$$

- while the neutron skin thickness (also including surface effects),

$$\Delta r_{np} = \sqrt{\frac{3}{5}}\left[t - \frac{e^2 Z}{70J}\right] + \Delta r_{np}^{surface}$$

$$t \equiv \frac{3r_0}{2}\frac{J/Q}{1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}}(I - I_C),$$

$I \equiv (N - Z)/A$
$I_C \equiv (e^2 Z)/(20JR)$
$R = r_0 A^{1/3}$

$\Delta r_{np}^{surface} = \sqrt{(3/5)}[5(b_n^2 - b_p^2)/(2R)]$ is a correction caused by the difference in the surface widths b_n and b_p of the neutron and proton density profiles

Droplet model approach: connection between α_D and the neutron skin

- Combining these formulas,

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1+\frac{5}{2}\frac{\Delta r_{np}+\sqrt{\frac{3}{5}}\frac{e^2Z}{70J}-\Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$

- For a heavy nucleus and assuming small variations† of $\langle r^2\rangle$, $e^2Z/70J$ and $\Delta r_{np}^{\text{surface}}$ as compared to that of J and Δr_{np},

$$\alpha_D J \approx p_1 + p_2\Delta r_{np}$$

†Some numbers: ^{208}Pb and assuming $J = 32 \pm 2$ MeV, $\rho_0 = 0.160 \pm 0.05$ fm^{-3} and $\Delta r_{np}^{\text{surface}} \approx 0.09 \pm 0.01$ (EDF) $\Rightarrow (e^2Z)/(70J) - \Delta r_{np}^{\text{surface}} \approx -0.04 \pm 0.01$ fm $\langle r^2\rangle^{1/2} \approx 5.23 \pm 0.55$ fm, $I_c \approx 0.027 \pm 0.003$

Covariance analysis: χ^2 test

Observables $\mathcal{O}$ are used to calibrate the parameters $\mathbf{p}$ of a given model. The optimum parametrization $\mathbf{p}_0$ is determined by a least-squares fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the χ^2 is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned} \chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j}) \\ &\equiv \sum_{i,j}^{n} (p_i - p_{0i}) \mathcal{M}_{ij} (p_j - p_{0j}) \end{aligned}$$

where $\mathcal{M}$ is the curvature matrix.

Covariance analysis: χ^2 test

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(\mathbf{p})$),

$$\Delta\mathcal{A} = \sqrt{\sum_{\imath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\imath} \partial_{p_\imath} A} \tag{1}$$

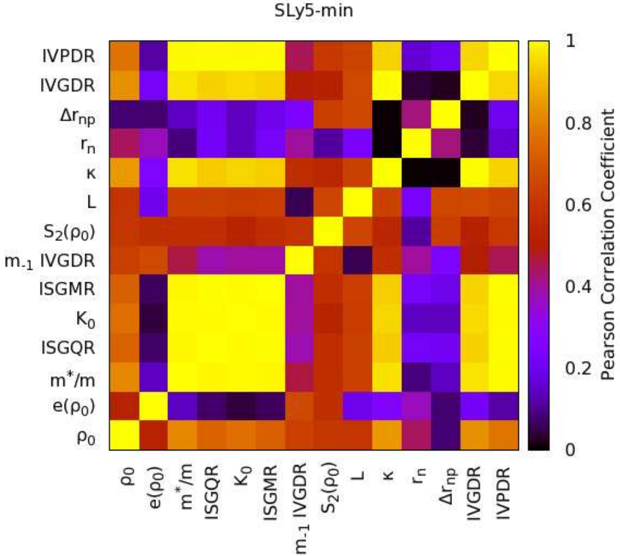
$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}} \tag{2}$$

where,

$$C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{\imath\jmath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\jmath} \partial_{p_\jmath} B$$

Covariance analysis: SLy5-min as an example



Covariance analysis: SLy5-min as an example

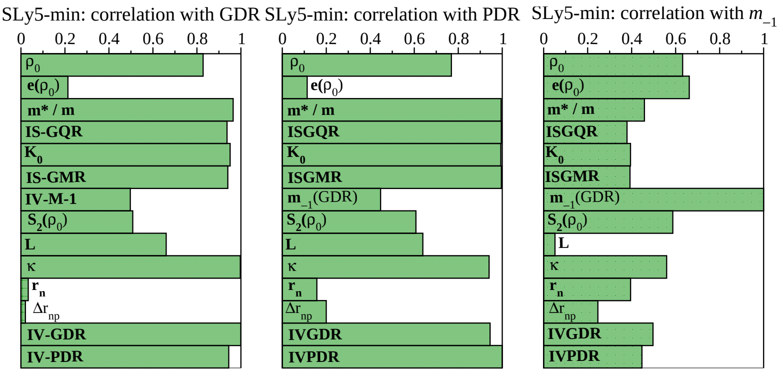


Figure: Pearson product-moment correlation coefficient for the IVGDR (left panel), IVPDR (middle panel) and m_{-1}(IVGDR) (right panel) with all other studied properties as predicted by the covariance analysis of SLy5.