Elastic electron scattering as a revitalized experimental tool in modern nuclear physics: a theoretical point of view

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Theoretical study of elastic electron scattering off stable and exotic nuclei **X. Roca-Maza, M. Centelles, F. Salvat, and X. Viñas Phys. Rev. C 78, 044332 (2008).**

Electron scattering in isotonic chains as a probe of the proton shell structure of unstable nuclei **X. Roca-Maza, M. Centelles, F. Salvat, and X. Viñas Phys. Rev. C 87, 014304 (2013).**

Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment **X. Roca-Maza, M. Centelles, X. Vias, and M. Warda Phys. Rev. Lett. 106, 252501.**

Motivation

- **In-medium nuclear (effective) interaction is not well understood for extreme values of isospin asymmetry, that is, far from the stability valley**
- **Experimental studies of elastic electron scattering by unstable nuclei:**
  - will be **feasible** in rare ion beam facilities such as RIKEN (Japan) and GSI (Germany)
  - determine e-m charge distribution **model independently**
  - better understanding of nuclei under more **extreme conditions**
  - data on **large isospin asymmetries**
- **Theoretical studies of elastic electron scattering by unstable nuclei:**
  - Physical process **well understood** since many years ago.
  - **Exact calculations available** once the exact electromagnetic charge distribution is known
  - **Theoretical guidance for future experiments**

Motivation

- **In addition ...**
- **Experimental studies of ineslastic electron scattering by unstable nuclei at forward angles that prominently measure the $E1$ response:**
  - will be also **feasible** in facilities such as SCRIT (Japan)
  - determine the GDR in unstable nuclei (some mixing with other resonances will reduce the accuracy)
  - better understanding of the $E1$ response of unstable nuclei
- **Theoretical studies on the GDR in unstable nuclei:**
  - Physical process **well understood**
  - **Calculations available**
  - **Theoretical guidance for experiments**

Motivation

- **In-medium nuclear (effective) interaction for moderate values of isospin asymmetry, that is, close/within the stability valley is not precisely determined (neither)**
- **Experimental studies of parity violating elastic electron scattering by stable medium and heavy nuclei where isospin asymmetries are larger:**
  - are **feasible** in facilities such as JLab (USA) and MAMI (Germany)
  - determine the weak charge distribution **model independently**
  - better understanding of neutron distribution in nuclei
- **Theoretical studies of parity violating elastic electron scattering by stable nuclei:**
  - Physical process **well understood**.
  - **Exact calculations available** once the exact electromagnetic and weak charge distributions are known
  - **Theoretical guidance for experiments**

Table of contents

Elastic Scattering of Electrons by Nuclei

Exact solution: Dirac partial-wave (also known as DWBA) calculation of elastic scattering of electrons by nuclei. X. Roca-Maza, M. Centelles, F. Salvat, and X. Viñas & Phys. Rev. C 78, 044332 (2008). F. Salvat *et al.* Comp. Phys. Comm. 165 157-190 (2005).

Theory: study of the nuclear charge distribution

- **$\mathbf{E}_{\rm beam}$** $\sim 2\pi \frac{\hbar c}{\lambda_{\rm Nucl.size}}$ where $\lambda_{\rm nucl.size} \sim 2\langle r^2\rangle^{1/2} \sim 2 - 10$ fm $\Rightarrow$ **$\mathbf{100 - 600}$ MeV**.
- **Relativistic treatment** is needed $m_e c^2/E_{\rm beam} \lesssim 0.005$.
- At these energies, effect of **screening by the orbiting atomic electrons** is limited to scattering angles **smaller than 1 degree** (we will not calculate them here).
- The interaction potential is $V_{\rm nucl.elec.}$ calculated from $\rho_{\rm ch}$ (parametrized, model, ...)

$$V_{\rm nucl.elec.} = 4\pi Z_0 e^2 \left\{ \frac{1}{r}\int_0^r \rho_{\rm ch}(u) u^2 du + \int_r^\infty \rho_{\rm ch}(u) u du \right\}$$

- spherical symmetry assumed

Theory: direct and spin-flip amplitudes

- The **scattering of relativistic electrons by a central field $V(r)$ is completely described** by the **direct** scattering amplitude, $f(\theta)$, and the **spin-flip** scattering amplitude, $g(\theta)$.
- $f(\theta)$ and $g(\theta)$ are complex functions solutions of the Dirac equation for $V(r)$ that behave asymptotically as a plane wave plus an outgoing spherical wave.
- $f(\theta)$ and $g(\theta)$ admit the so called partial-wave expansion,

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} \left\{ (l+1) \left[e^{2i\delta_{\kappa=-l-1}} - 1 \right] + l \left[e^{2i\delta_{\kappa=l}} - 1 \right] \right\} P_l(\cos(\theta))$$

and,

$$g(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} \left[e^{2i\delta_{\kappa=l}} - e^{2i\delta_{\kappa=-l-1}} \right] P_l^1(\cos(\theta))$$

where k is the projectile wave number ($\hbar k = p$), P_l and P_l^1 are Legendre polynomials and δ_κ **are the phase shifts induced by the central potential**

Theory: phase shifts δ_κ

- The phase shifts δ_κ **represent the large-r behavior of the Dirac spherical waves**, solution of the Dirac equation,

  $$\psi_{E\kappa m}(\mathbf{r}) = \frac{1}{r}\begin{pmatrix} P_{E\kappa}(r)\Omega_{\kappa,m}(\hat{\mathbf{r}}) \\ iQ_{E\kappa}(r)\Omega_{-\kappa,m}(\hat{\mathbf{r}}) \end{pmatrix},$$

  where $\Omega_{\kappa,m}(\hat{\mathbf{r}})$ are the spherical spinors and $P_{E\kappa}(r)$ and $Q_{E\kappa}(r)$ satisfy,

  $$\begin{aligned} \frac{\mathrm{d}P_{E\kappa}(r)}{\mathrm{d}r} &= -\frac{\kappa}{r}P_{E\kappa} + \frac{E - V + 2m_e c^2}{c\hbar}Q_{E\kappa} \\ \frac{\mathrm{d}Q_{E\kappa}(r)}{\mathrm{d}r} &= -\frac{\kappa}{r}Q_{E\kappa} - \frac{E - V}{c\hbar}P_{E\kappa} \end{aligned}$$

  where $\kappa = (l - j)(2j + 1)$ is the relativistic quantum number.
- $P_{E\kappa}(r \to \infty) \approx \sin(kr - l\pi/2 + \delta_\kappa)$ for finite range fields.
- **Attractive (repulsive) potentials give positive (negative) phase shifts**.

Theory: Basic quantities

- Elastic DCS per unit solid angle for spin unpolarized electrons

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |g(\theta)|^2$$

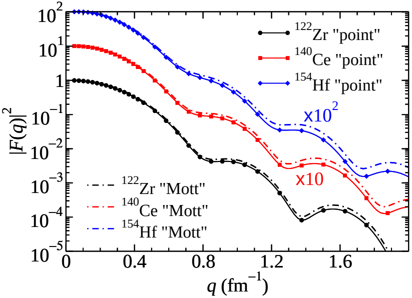
- Spin polarization function of the electrons from an initially unpolarized beam (Sherman function)

$$S(\theta) \equiv i\frac{f(\theta)g^*(\theta) - f^*(\theta)g(\theta)}{|f(\theta)|^2 + |g(\theta)|^2}$$

Theory: the Form Factor

$$|F_{DWBA}(q)|^2 \equiv \frac{d\sigma/d\Omega}{d\sigma_{\mathrm{point}}/d\Omega}$$

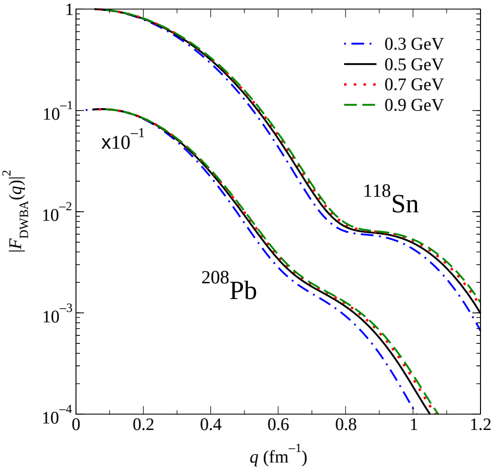
where $d\sigma_{\mathrm{point}}/d\Omega$ is the DWBA solution for a point nucleus and $c\hbar q = 2E\sin(\theta/2)$.



- This definition, as compared to $\frac{d\sigma/d\Omega}{d\sigma_{\mathrm{Mott}}/d\Omega}$, disentagles better the finite size effects of the nucleus.
- Nevertheless, it is found that the choice is not critical for the low momentum transfer regime.

Mott DCS: $\frac{d\sigma_{\mathrm{Mott}}}{d\Omega} = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\theta}{\sin^4\theta}$; for small angles diverges as θ^{-4}

Theory: Energy Dependence in the e-m Form Factor



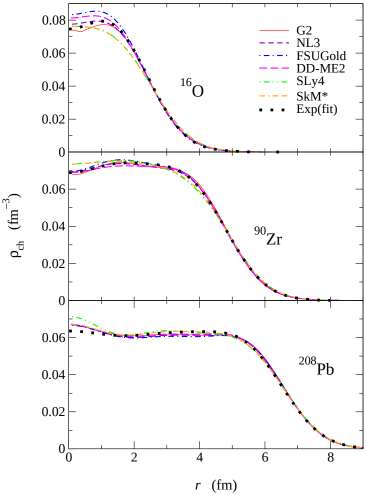
Test: The form factor in DWBA is almost energy-independent in the low q-regime

$F_{DWBA}(q)$ is a good quantity for the study of the electromagnetic structure of the nucleus

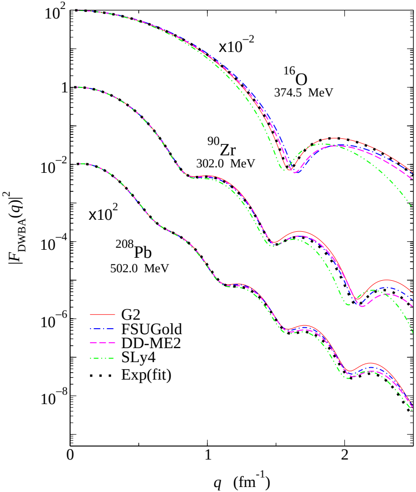
Stable Nuclei: Overview

Experiment versus Theory in stable nuclei

Nuclear Model (NM) provides:

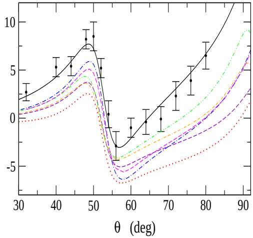


NM+DWBA provides:

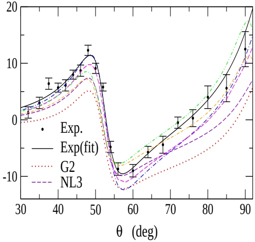


… and a more demanding test: $D(A-B) \equiv (A-B)/(A+B)$

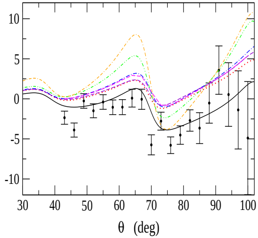
$D({}^{40}\text{Ca} - {}^{42}\text{Ca})$

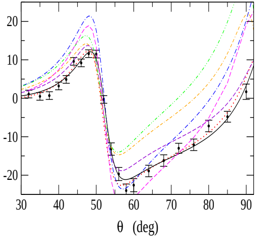


$D({}^{40}\text{Ca} - {}^{44}\text{Ca})$

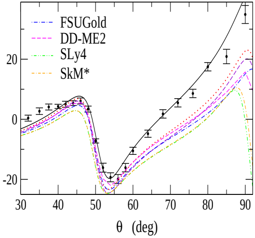


$D({}^{116}\text{Sn} - {}^{118}\text{Sn})$

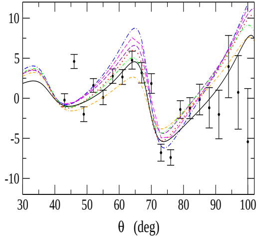




$D({}^{40}\text{Ca} - {}^{48}\text{Ca})$



$D({}^{48}\text{Ca} - {}^{48}\text{Ti})$



$D({}^{118}\text{Sn} - {}^{124}\text{Sn})$

Ability test for the models

Nucleus	E_e	d_w^2							"Best"
	MeV	**Exp. fit**	DD-ME2	G2	NL3	FSUGold	SLy4	SKM*	model
^{16}O	374.5	**11.1**[b]	88.7	13.1	38.6	206.	191.	194.	**G2**
^{40}Ca	250.0	**7.18**[b]	3.15	16.2	13.9	0.84	24.4	24.3	**FSUGold**
	500.0	**3.48**[b]	1.49	42.9	19.7	5.79	40.0	39.0	**DD-ME2**
^{48}Ca	250.0	**6.66**[b]	4.85	9.74	7.14	4.08	14.9	13.6	**FSUGold**
	500.0	**3.19**[b]	1.11	17.0	3.53	2.57	21.84	18.5	**DD-ME2**
^{90}Zr	209.6	**0.78**[b]	0.87	2.21	1.36	0.65	6.53	5.36	**FSUGold**
	302.0	**0.86**[b]	0.91	9.92	3.27	0.67	9.35	7.19	**FSUGold**
^{118}Sn	225.0	**5.43**[a]	18.4	34.8	25.5	31.8	2.75	4.20	**SLy4**
^{208}Pb	248.2	**30.6**[b]	44.4	154.	74.8	89.5	89.2	61.0	**DD-ME2**
	502.0	**21.2**[b]	14.1	186.	50.5	61.1	95.9	76.5	**DD-ME2**
$D(^{40}Ca - {}^{42}Ca)$	250.0	**0.56**[c]	9.1	28.3	16.0	11.1	9.1	12.9	**DD-ME2**/**SLy4**
$D(^{40}Ca - {}^{44}Ca)$	250.0	**1.14**[c]	4.5	29.6	12.2	3.88	7.08	9.13	**FSUGold**
$D(^{40}Ca - {}^{48}Ca)$	250.0	**1.06**[c]	16.4	4.89	7.74	38.5	94.1	49.3	**G2**
$D(^{48}Ca - {}^{48}Ti)$	250.0	**2.49**[c]	18.0	19.6	31.0	37.8	71.8	64.9	**DD-ME2**
$D(^{116}Sn - {}^{118}Sn)$	225.0	**2.05**[a]	8.05	7.80	9.00	10.1	13.2	18.5	**G2**
$D(^{118}Sn - {}^{124}Sn)$	225.0	**4.03**[a]	5.35	6.98	7.50	9.22	7.05	7.18	**DD-ME2**

[a] A.S. Litvinenko et al., Nucl.Phys. **A182**, 265 (1972). [b] B. Dreher, J. Friedrich, K. Merle, H. Rothhaas and G. Luhrs, Nucl.Phys.**A235**, 219 (1974). [c] R.F. Frosch et al. Phys. Rev. **174** (1968) 1380.

Conclusions

- Theory is well understood and calculations are feasible: **exact solution of the scattering process** once the nuclear e-m charge distribution has been provided.
- **disagreement** with the experiment **due exclusively** to the **nuclear model**
- The defined **Form Factor**
  - include **all finite size effects**
  - is **nearly energy-independent** at low momentum transfer
- Exist a quasi-model-independent q-regime: Up to $1 - 1.5$ fm^{-1} for the studied models and scattering processes.

New experimental landscape: e - Rare Isotope Beams

ELISe@FAIR and SCRIT@RIKEN projects

- **Self-Confinig Rare Isotope Target (e-RI scattering) - SCRIT Operative**
- **ELectron-Ion Scattering in a Storage Ring (eA collider) - ELISe Still under developement**

At the beginnig of next year, SCRIT collacoration will start measuring the e-m distribution of unstable Sn isotopes, from N=82 to N=62

How can theory help in the experimental analysis? Could we find simple and general trends for $F(q)$ in exotic nuclei?

Helm Model: **2** parameters fitted to theoretical predictions to mimic future experimental analysis

- Helm Charge Form Factor: R_0 & σ

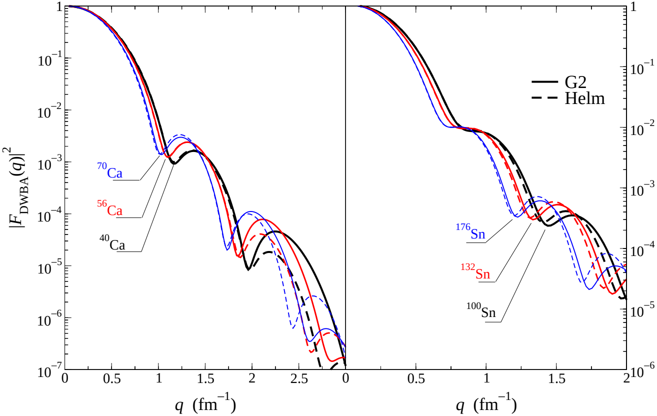
  $$F_H(q) = \int e^{i\vec{q}\vec{r}} \rho_H(\vec{r}) d\vec{r} = \frac{3}{\mathbf{R_0} q} j_1(q\mathbf{R_0}) e^{-\sigma^2 q^2/2}$$

  where σ measures the surface fall-off of the density distribution and R_0 measures its bulk extension.
- How we determine the parameters:
  - R_0**:** one requires that the first zero of F_H occurs at the same q of F_{PWBA} (fourier transform of the self-consistent density). Therefore, it coincides with the sharp radius.
  - σ**:** is chosen to reproduce the height of the second maximum of $|F_{PWBA}|$

Results: Z=50 and Z=20 isotopic chains

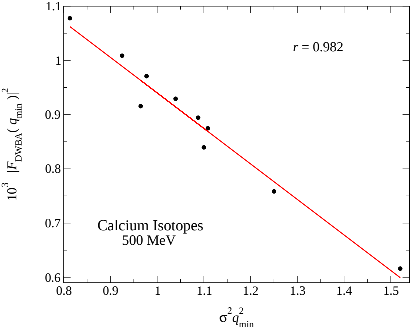
Charge Form Factor

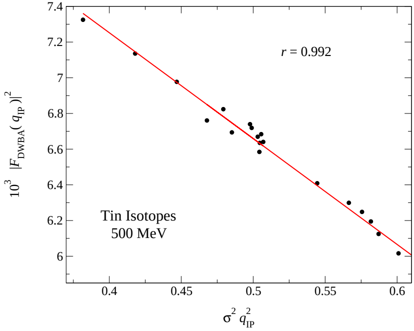
F_{DWBA} increases and shifts towards smaller q as the neutron number increases



Methodology accurate for low-momentum transfer

Correlations: the smaller the bulk part of the nuclear charge distribution and the compact the surface, the smaller the form factor



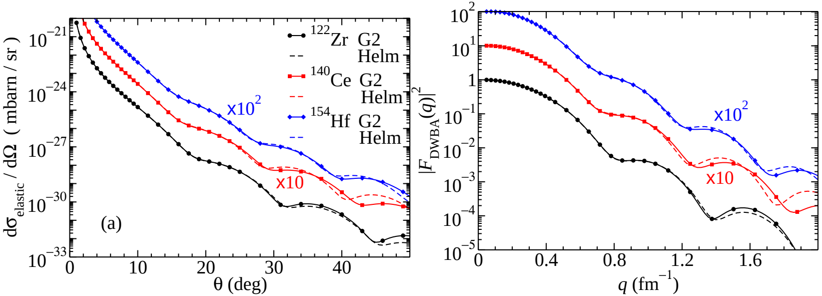


Therefore, if two or more isotopes have been measured ...

- linear correlations would provide, for an **unknown nucleus** of the chain, a **hint** on the value expected for the **square of the experimental electric charge form factor** at its first minimum
- if the value of the squared modulus of the **form factor is determined experimentally at its first minimum**, the **charge density in the Helm model can be sketched** from similar correlations
- use of **more elaborated versions of the Helm model** that take into account the central depression of the charge density should allow one to **extend the domain of validity of our method** up to larger values of the momentum transfer.

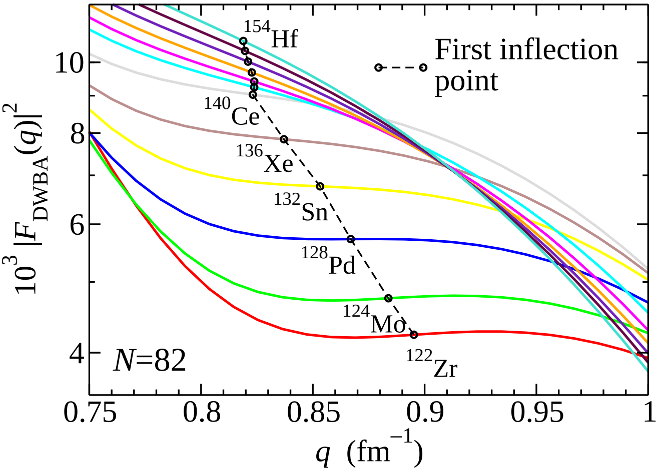
Results & Correlations: N=82, N=50 and N=14 isotonic chains

Differential cross sections and form factors

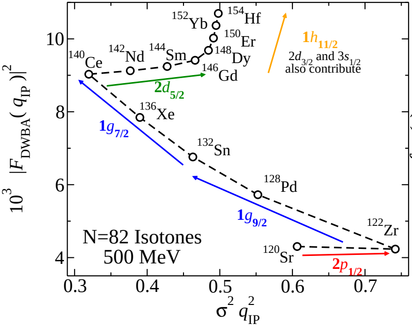


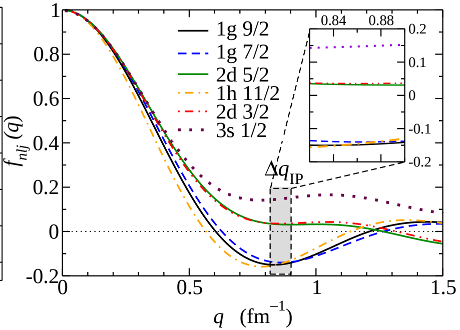
Methodology accurate for low-momentum transfer

Charge form factors F_{DWBA} increases and shifts towards smaller q as the neutron number increases

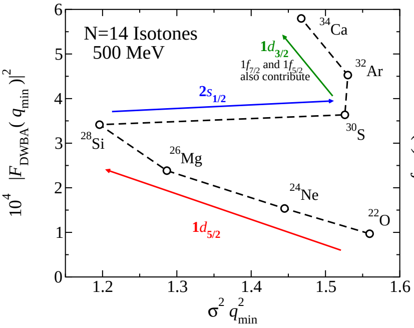


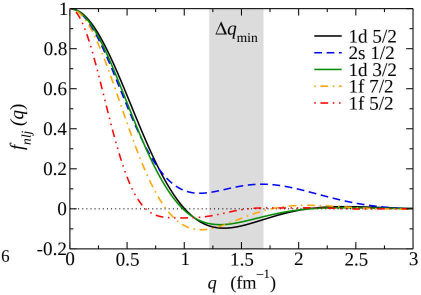
The increasing rate of the form factor basically depends on the proton level which is being filled!!





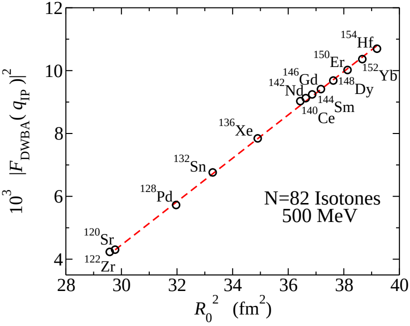
Also in lighter isotonic chains...

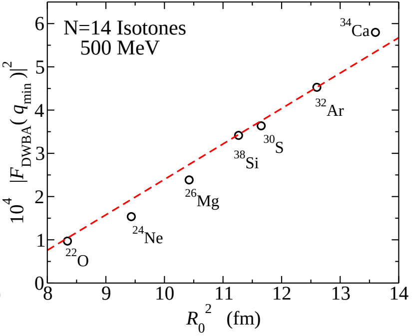




The larger the number of protons, the larger the formfactor

...this was clear, less clear was that it is almost linear along isotonic chains





Conclusions: isotopic chains

- The described analysis is **potentially useful** for future electron-nucleus elastic **scattering experiments**,
    - the **linear correlations** shown would provide, for an unknown nucleus of a chain, a hint on the value expected for $|F_{\rm exp}(q_{\rm min})|^2$.
- The **exact analysis of the Coulomb phase shifts** applied to a exotic nuclei and compared with future measurments could, potentially, elucidate some aspects related with the **isospin asymmetry** of the nuclear force.
- The use of more **elaborated versions** of the Helm model should allow one to **extend** the domain of **validity** of our method up to **larger** values of q

Conclusions: isotonic chains

- Rate of change of the **electric charge form factor** is **extremely sensitive** on the **proton level which is being filled**
  - levels with large *n* and small *l* contribute with opposite sign with respect to levels without radial nodes and large angular momenta.
  - plotting $|F(q)|^2$ against $\sigma^2 q^2_{\mathrm{min}}$ magnifies such effects
- Therefore, **electron scattering** in isotonic chains can be a useful tool to **probe the proton single-particle shell structure of exotic nuclei**: filling order and occupancy of the different valence proton orbitals.

Conclusions: warning...

Extensive experimental investigations more difficult because of the limitations arising from **small production rates, short half-lives, and small cross sections** when one deals with unstable nuclei

Parity violating electron scattering

Refs: C. J. Horowitz, Phys. Rev. C **57** 3430 (1998); C. J. Horowitz, S. J. Pollock, P. A. Souder, and R. Michaels, Phys. Rev. C **63**, 025501 (2001); M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C **82**, 054314 (2010); X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, Phys. Rev. Lett. **106** 252501 (2011) and (for the electric proton and neutron form factors) J. Friedrich and Th. Walcher, Eur. Phys. J. A **17**, 607623 (2003)

Theory:

- The **electron interacts** with a **nucleus** by exchanging either a γ or a Z_0 boson.
- γ couples basically to protons, $Q_{\mathrm{em}} = Z$, and Z_0 couples basically to neutrons,
  $$Q_{\mathrm{W}} = -N + (1 - 4\sin^2(\theta_W))Z \approx -N + 0.1Z.$$
- Ultra-relativistic electons interact with the Coulomb $+$ or $-$ the Weak potential depending on the helicity of the electrons,
  $$V_{\mathrm{tot}} = V_C \pm V_W \text{ where } V_W = G_F \rho_W(r)/2^{2/3},$$
  This produces a parity-violating amplitude in the scattering process.
- The effect of the parity-violating part of the weak interaction may be isolated by measuring the parity-violating asymmetry,
  $$A_{\mathrm{PV}} = \frac{\mathrm{d}\sigma_+/\mathrm{d}\Omega - \mathrm{d}\sigma_-/\mathrm{d}\Omega}{\mathrm{d}\sigma_+/\mathrm{d}\Omega + \mathrm{d}\sigma_-/\mathrm{d}\Omega}$$
  where $+/-$ indicates positive or negative helicity of e.

- **Parity violating elastic scattering determine the nuclear weak charge distribution in a similar way as the electromagnetic charge distribution is determined in parity conserving elastic electron scattering**
- **The determination of A_{PV} is model-independent.**
- **Experiments at different anlges are not planned for the near future ⇒ on cannot map the whole weak density in nuclei in a model independent way.**

Theory:

Qualitatively,

- A_{pv} within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{\mathbf{F_n(q) - F_p(q)}}{F_p(q)}\right]$$

- ... which depends on $\mathbf{F_n(q) - F_p(q)}$. For $q \to 0$, it is approximately,

$$\begin{aligned} -\frac{q^2}{6}\left(\langle r_n^2\rangle - \langle r_p^2\rangle\right) &= -\frac{q^2}{6}\left[\boldsymbol{\Delta}\mathbf{r_{np}}(\langle r_n^2\rangle^{1/2} + \langle r_p^2\rangle^{1/2})\right] \\ &= -\frac{q^2}{6}\left(2\langle r_p^2\rangle^{1/2}\boldsymbol{\Delta}\mathbf{r_{np}} + \Delta r_{np}^2\right) \end{aligned}$$

- variation of A_{pv} at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

Past and Future Experiments

Past

- PREx measured A_{PV} in ^{208}Pb @ 5deg and 1.063 GeV model-independently → first electro-weak probe of the excistence of a weak charge radius larger than the electromagnetic charge radius in a heavy nucleus.

Future

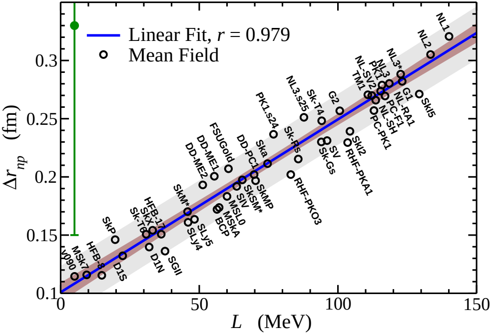
- PREx II: improve accuracy of PREx (JLab)
- CREx: measure A_{PV} in ^{48}Ca @ 4deg and 2.2 GeV (JLab)
- Super PREx or PV-RAPTOR: A_{PV} in ^{208}Pb with better accuracy than PREx and PREx II (MAMI)*

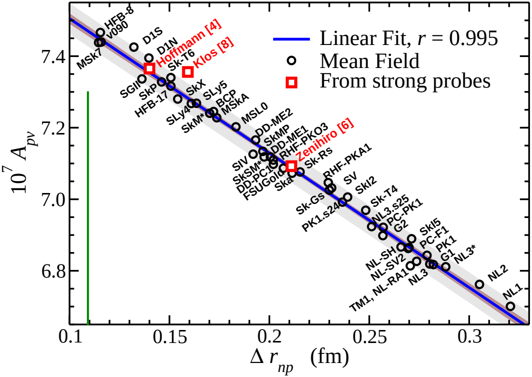
* Open also to measure other nuclei if well motivated from the theory.

^{208}Pb: direct correlations

DWBA; no radiative corrections or strange quark effects included

X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, Phys. Rev. Lett. **106** 252501 (2011)

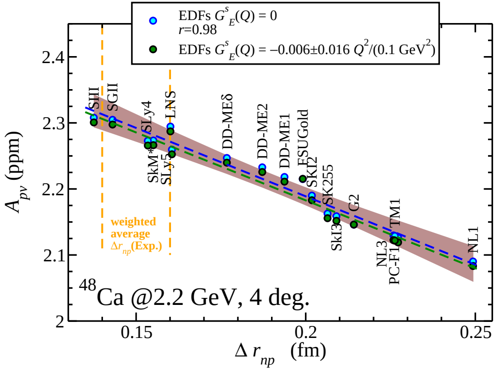




MF correlations allows to determine Δr_{np} and L without direct assumptions on ρ, PREx-II and PV-RAPTOR expected accuracy $\rightarrow$ constrain on L

Different experiments on proton elastic scattering and antirpotonic atoms agrees with the correlation

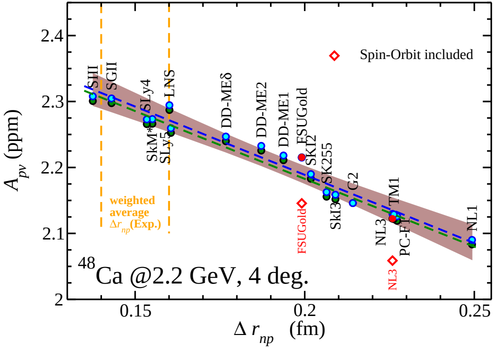
^{48}Ca: direct correlations within MF including radiative corrections and strange quark effects



A_{pv} decreases by around 0.005 ppm with an error of about 0.01 - 0.02 ppm when $G_E^s(Q^2)$ is included.

Used $G_E^s(Q^2)$ from PRC 76, 025202 (2007) by Liu, McKeown, and Ramsey-Musolf Average Δr_{np} from hadronic probes: PRC12, 778 1978; PRL87, 08250113, 343 (2004); Phys. Rev. 174, 1380 (1968); Physics Letters 57B 47 (1975); PRC 67, 054605 (2003) and PRC33 1624 (1986).

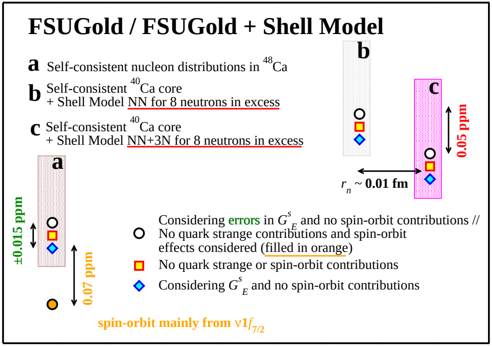
^{48}Ca: estimation of spin-orbit effects



In the two tested models, spin-orbit effects shifts to lower values the A_{pv} consistently by about 0.07 ppm. This predicts a reduction of Δr_{np} of about 0.05 fm.

Charge density distributions including spin orbit effects provided by J. Piekarewicz (FSU).

^{48}Ca: Estimation of three-neutron forces effects in comparison with other corrections



FSUGold / FSUGold + Shell Model

a Self-consistent nucleon distributions in ^{48}Ca

b Self-consistent ^{40}Ca core
+ Shell Model NN for 8 neutrons in excess

c Self-consistent ^{40}Ca core
+ Shell Model NN+3N for 8 neutrons in excess

○ Considering errors in G^s_E and no spin-orbit contributions // No quark strange contributions and spin-orbit effects considered (filled in orange)

🟥 No quark strange or spin-orbit contributions

◆ Considering G^s_E and no spin-orbit contributions

Shell Model calculations based on χEFT with NN to N3LO (fixed to scattering data) and 3N to N2LO (fixed to B tritium and R of alpha particle) **provided by J. Menendez (TU Darmstadt)**.

Three-neutron forces used here shifts downwards the A_{pv} by about **0.05 ppm (very similar to spin-orbit effect)**

Conclusions

- A precise and **model-independent** determination of Δr_{np} in ^{48}Ca and ^{208}Pb via PVES experiments would **probe** at the same time the density dependence of the nuclear **symmetry energy** and the relevance of **three neutron-forces** in ^{48}Ca. Eventually, it can also provide indirect indications on the impact of 3N in ^{208}Pb.
- We demonstrate a close **linear correlation** between A_{pv} and Δr_{np} within the same framework in which the Δr_{np} is correlated with L.
- Other **experiments** fairly **agree** with the **correlation** between A_{pv} and Δr_{np}.

Collaborators:

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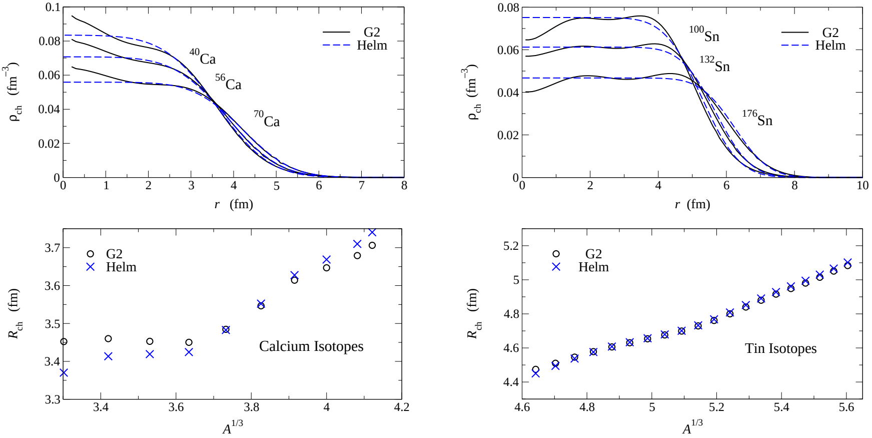
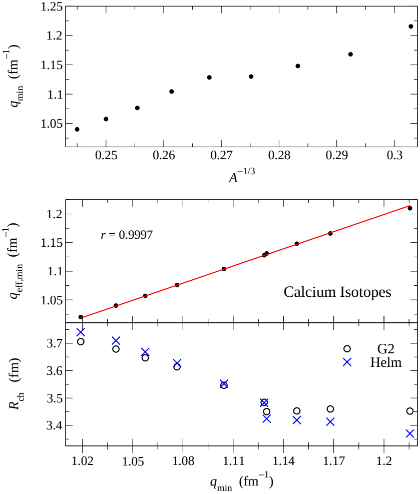
Extra material

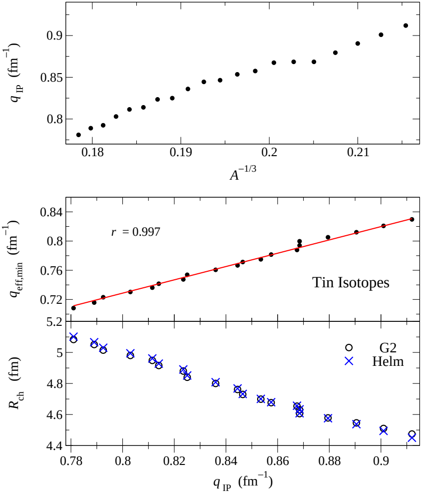
Isotopes

Helm and self-consistent charge densities and charge radii



Correlations: evolution of first minimum or inflection point





Isotones

Charge densities and proton single particle levels