

# The Nuclear Symmetry Energy: constraints from Giant Resonances

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# **INTRODUCTION**

# The Nuclear Many-Body Problem:

- ▶ **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- ▶ **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- ▶ **Complex systems:** **spin, isospin, pairing, deformation, ...**
- ▶ **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - ▶ **different nuclear interactions in the medium** are found **depending** on the **approach**
  - ▶ **EoS** and (only very recently) **few groups** in the world are able to perform extensive calculations for **light and medium mass nuclei**
- ▶ Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

# Approximate realization of an exact Nuclear Energy Density Functionals:

## Kohn-Sham iterative scheme (static approximation)

- ▶ Determine a good  $E[\rho]$
- ▶ Initial guess  $\rho_0$
- ▶ Calculate potential  $V_{\text{eff}}$  from  $\rho_0$
- ▶ Solve single particle (Schrödinger) equation and find single particle wave functions  $\phi_i$
- ▶ Use  $\phi_i$  for calculating new  $\rho_1 = \sum_i^A |\phi_i|^2$
- ▶ Repeat until convergence

**Runge-Gross Theorem:** dynamic generalization of the static EDFs.

$$\int dt \{ \langle \Phi(t) | i\partial_t | \Phi(t) \rangle - E[\rho(t), t] \} = 0$$

**Giant Resonances well described within the small amplitude limit (known as RPA approach)**

# Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

**Relativistic mean-field models**, based on Lagrangians where effective mesons carry the interaction:

$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\Gamma_{\sigma}(\bar{\Psi}, \Psi)\Psi\Phi_{\sigma} & + \bar{\Psi}\Gamma_{\delta}(\bar{\Psi}, \Psi)\tau\Psi\Phi_{\delta} \\ &- \bar{\Psi}\Gamma_{\omega}(\bar{\Psi}, \Psi)\gamma_{\mu}\Psi A^{(\omega)\mu} & - \bar{\Psi}\Gamma_{\rho}(\bar{\Psi}, \Psi)\gamma_{\mu}\tau\Psi A^{(\rho)\mu} \\ &- e\bar{\Psi}\hat{Q}\gamma_{\mu}\Psi A^{(\gamma)\mu}\end{aligned}$$

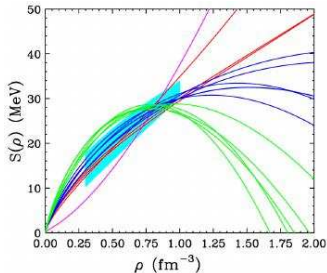
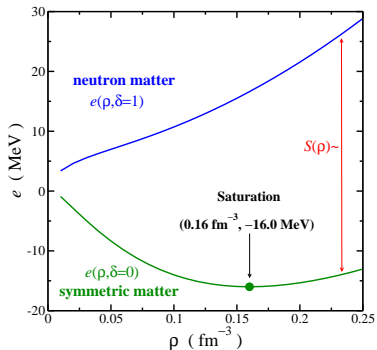
**Non-relativistic mean-field models**, based on Hamiltonians where effective interactions are proposed and tested:

$$V_{\text{Nucl}}^{\text{eff}} = V_{\text{attractive}}^{\text{long-range}} + V_{\text{repulsive}}^{\text{short-range}} + V_{\text{SO}}$$

- ▶ Fitted **parameters contain** (important) **correlations beyond the mean-field**
- ▶ Nuclear energy functionals are **phenomenological** → **not directly connected to any NN (or NNN) interaction**

# The Nuclear Equation of State: Infinite System

(Independent particle picture: nucleons move in an effective mean field potential generated by all of them)



Physical Review C **86** 015803 (2012)

$$e(\rho, \delta) = e(\rho, \delta = 0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4] \quad \text{with } \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$S(\rho) = J + L \frac{\rho - \rho_0}{3\rho_0} + K_{\text{sym}} \left( \frac{\rho - \rho_0}{3\rho_0} \right)^2 + \mathcal{O}[(\rho - \rho_0)^3]$$

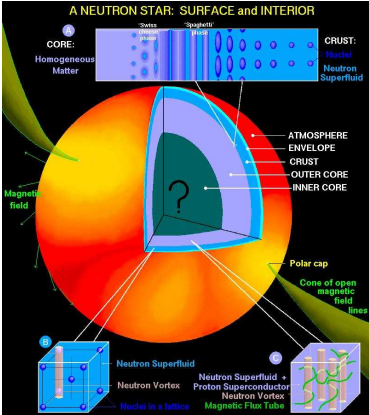
The uncertainties in  $S(\rho)$  impacts on many observables and, therefore, on our understanding on the nuclear effective interaction.

# MOTIVATION

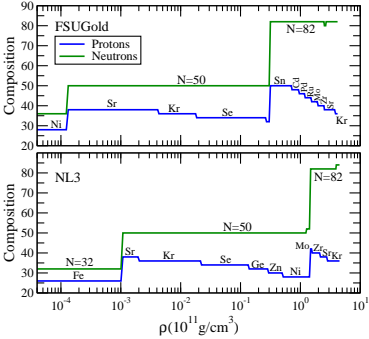


# Motivation:

## The symmetry energy and the structure and composition of a neutron star crust



Courtesy of Dany Page



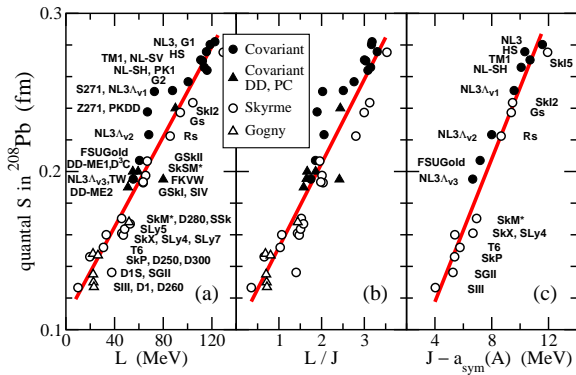
Physical Review C 78, 025807 (2008)

The faster the symmetry energy increases with density, the more exotic the composition of the outer crust.

# Motivation:

## The symmetry energy and the neutron distribution in nuclei

(quantal S: difference between the neutron and proton root mean square radius)



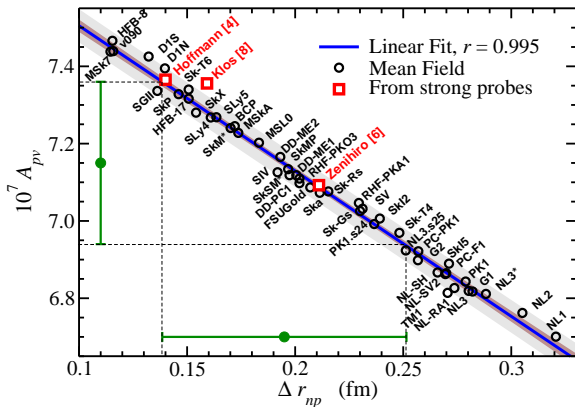
*Physical Review Letters* **102**, 122502 (2009)

The faster the symmetry energy increases with density, the largest the size of the neutron distribution in nuclei. [Exp. from strongly interacting probes:  $0.18 \pm 0.03$  fm (*Physical Review C* **86** 015803 (2012))].

# Motivation:

## The symmetry energy and parity violating electron scattering

( $A_{pv}$ : relative difference between the elastic cross sections of right- and left-handed electrons)



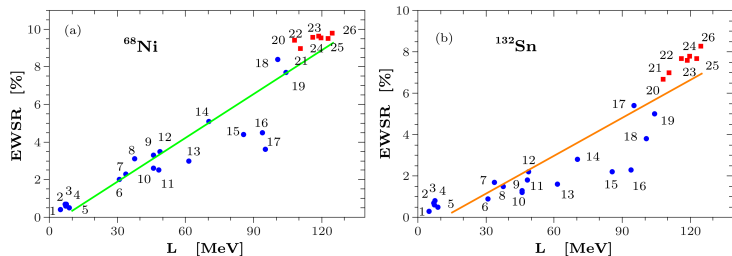
*Physical Review Letters* **106**, 252501 (2011)

The larger the size of the neutron distribution in nuclei, the smaller the elastic electron parity violating asymmetry. [Exp. from  
ew probes:  $0.302 \pm 0.175$  fm (*Physical Review C* **85**, 032501 (2012))].

# Motivation:

## The symmetry energy and the Pygmy Dipole Resonance

(Pygmy: low-energy excited state appearing in the dipole response of  $N \neq Z$  nuclei)



*Physical Review C* **81**, 041301 (2010)

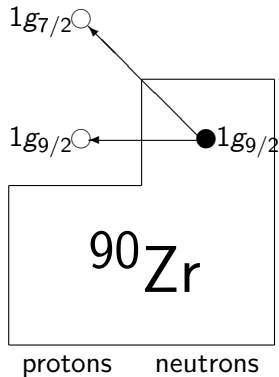
The faster the symmetry energy increases with density, the larger is the energy ( $E$ ) and the larger is the probability ( $P$ ) of exciting the Pygmy state  $\Rightarrow$  larger the Energy Weighted Sum Rule (EWSR)  $\propto E \times P$ .

# Motivation:

## The symmetry energy and the Gamow-Teller Resonance

(GT transitions change spin and isospin of the initial quantum state and favors  $\Delta n \rightarrow 0$ )

- ▶ Spin orbit splittings around the Fermi surface
- ▶ Proton and neutron single particle potentials  $\Rightarrow V_{\text{sym}}$





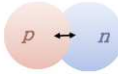

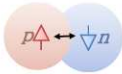


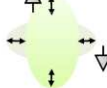
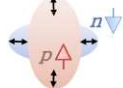


Gamow-Teller transitions determine:

- ▶ weak interaction rates essential in core-collapse dynamics of massive stars
- ▶ in neutron-rich environment, neutrino-induced nucleosynthesis
- ▶ studies of double- $\beta$  decay
- ▶ useful in the calibration of detectors used to measure neutrinos

# **GIANT RESONANCES**

# Giant Resonances

$\Delta L=0$	 ISGMR	 IVGMR	 ISSMR	 IVSMR
$\Delta L=1$		 IVGDR	 ISSDR	 IVSDR
$\Delta L=2$	 ISGQR	 IVGQR	 ISSQR	 IVSQR
	$\Delta S=0$	$\Delta S=0$	$\Delta S=1$	$\Delta S=1$
	$\Delta T=0$	$\Delta T=1$	$\Delta T=0$	$\Delta T=1$

<http://www.majimak.com/wordpress/>

$\Delta T = 1$ ,  $\Delta S = 0$  and  $\Delta L = 1 \rightarrow$  IsoVector **GDR**

$\Delta T = 1$ ,  $\Delta S = 0$  and  $\Delta L = 2 \rightarrow$  IsoVector **GQR**

# Isvector Giant Resonances

- ▶ In **isovector** giant resonances **neutrons and protons** “oscillate” **out of phase**  
**e.g.** within a classical picture: “**e-m interacting probes basically excite protons, protons drag neutrons thanks to the nuclear strong interaction, when neutrons approach too much to protons, they are pushed out**”
- ▶ **Isvector** resonances will depend on oscillations of the density  $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow \mathbf{S}(\rho)$  will drive such “oscillations”
- ▶ The **excitation energy ( $E_x$ )** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m} \frac{d^2 U}{dx^2}} \propto \sqrt{k} \rightarrow E_x \sim \sqrt{\frac{\delta^2 e}{\delta \beta^2}} \propto \sqrt{S(\rho)}$$

where  $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$



# Polarizability, Strength distribution and its moments

- ▶ The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s.,  $|0\rangle$ , to an excited state,  $|\nu\rangle$ , due to the **action of an external isovector oscillating field** (dipolar/quadrupolar in our case) of the form  $(Fe^{i\omega t} + F^\dagger e^{-i\omega t})$ :

$$F_{JM} = \sum_i^A r^J Y_{JM}(\hat{r}) \tau_z(i) \quad (\Delta L = 1, 2 \rightarrow \text{Dipole, Quadrupole})$$

- ▶ is proportional to the **static polarizability** for small oscillations

$$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_\nu |\langle \nu | F | 0 \rangle|^2 / E \quad \text{where } m_{-1} \text{ is the inverse energy weighted moment of the strength function, defined as, } S(E) = \sum_\nu |\langle \nu | F | 0 \rangle|^2 \delta(E - E_\nu)$$

- ▶ **Isvector energy weighted sum rules (EWSR)** are:

$$m_1 = \frac{\hbar^2}{2m} \frac{NZ}{A} (1 + \kappa_D) \quad m_1 = \frac{\hbar^2}{2m} \frac{50}{4\pi} A \langle r^2 \rangle (1 + \kappa_Q)$$

equal to one half of the HF expectation value of  $[\hat{F}, [H, \hat{F}]]$  (Thouless theorem) and where  $\kappa$  is the dipole/quadrupole enhancement factor

# Isvector Giant Dipole Resonance:

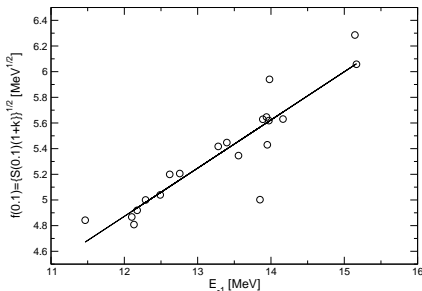


## Excitation energy

Guided by an hydrodynamical model<sup>†</sup> and the predictions of successful microscopic models (Energy Density Functionals), it was found:<sup>Physical Review C 77 061304 (2008)</sup>

$$E_{-1} = \sqrt{\frac{m_1}{m_{-1}}} \approx \sqrt{\frac{6\hbar^2}{m\langle r^2 \rangle}} S(\rho = 0.1)(1 + \kappa_D)$$

$S(\rho = 0.1 \text{ fm}^{-3})$  is **correlated** with the value of the excitation energy ( $E_{-1}$ ) of the IVGDR in spherical nuclei  $\Rightarrow$  experimental data on  $E_{-1}$  ( $^{208}\text{Pb}$ ) leads to



$$23.3 \text{ MeV} < S(\rho = 0.1 \text{ fm}^{-3}) < 24.9 \text{ MeV}$$

<sup>†</sup> *Physics Reports* 175 103-261 (1989)

# Isvector Giant Dipole Resonance:



Dipole polarizability: macroscopic approach

Guided by a constrained calculation ( $E(\lambda) = \langle \mathcal{H} - \lambda \mathcal{O} \rangle$ ) using the Droplet Model<sup>†</sup> and the dipole operator one may find:

$$m_{-1} \approx \frac{A \langle r^2 \rangle^{1/2}}{48J} \left( 1 + \frac{15}{4} \frac{J}{Q} A^{-1/3} \right)$$

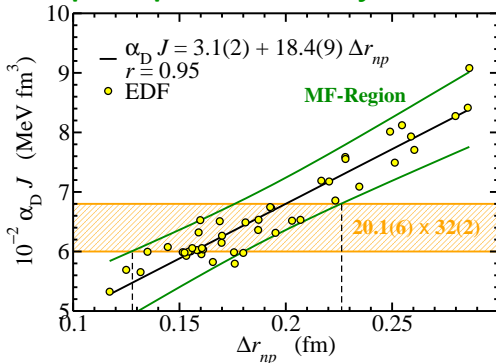
within the same model:

$$\Delta r_{np} = \sqrt{\frac{3}{5}} \left[ \mathbf{t} - \frac{e^2 Z}{70J} \right] + \Delta r_{np}^{\text{surf}}$$

$$\mathbf{t} \equiv \frac{3r_0}{2} \frac{J/Q}{1 + \frac{9}{4} \frac{J}{Q} A^{-1/3}} (I - I_C)$$

using these expressions:

$$\alpha_D \approx \frac{A \langle r^2 \rangle}{12J} \left[ 1 + \frac{5}{2} \frac{\Delta r_{np} + \sqrt{\frac{3}{5} \frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surf}}}}{\langle r^2 \rangle^{1/2} (I - I_C)} \right]$$



<sup>†</sup> Nuclear Physics A **385**, 269 (1982).

# Isvector Giant Quadrupole Resonance:



## IV (and IS) Excitation Energies in $^{208}\text{Pb}$

Guided by the Quantum Harmonic Oscillator approach, the relation  $S(\rho_A) \approx a_{\text{sym}}(A)$  within modern EDFs<sup>†</sup> and using experimental data, it was found: *Physical Review C* **87** 034301 (2013)

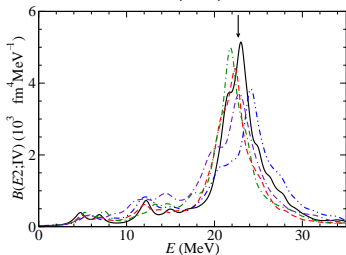
$$E_x^{\text{IS}} = \sqrt{\frac{2m}{m^*}} \hbar\omega_0$$

$$E_x^{\text{IV}} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4} \frac{\hbar^2}{2m} \frac{V_{\text{sym}} \langle r^2 \rangle}{(\hbar\omega_0)^2 \langle r^4 \rangle}}$$

where

$$V_{\text{sym}} \approx 8(S(\rho_A) - \varepsilon_{\text{F}0}/3)$$

$$S(\rho_A) = \frac{\varepsilon_{\text{F}0}}{3} \left\{ \frac{A^{2/3}}{8\varepsilon_{\text{F}0}^2} \left[ (E_x^{\text{IV}})^2 - 2(E_x^{\text{IS}})^2 \right] + 1 \right\}$$



$$S(\rho = 0.1 \text{ fm}^{-3} \text{ for } ^{208}\text{Pb}) = 23.3 \pm 0.6 \text{ MeV}$$

<sup>†</sup> *Phys. Rev. Lett.* **102**, 122502 (2009)

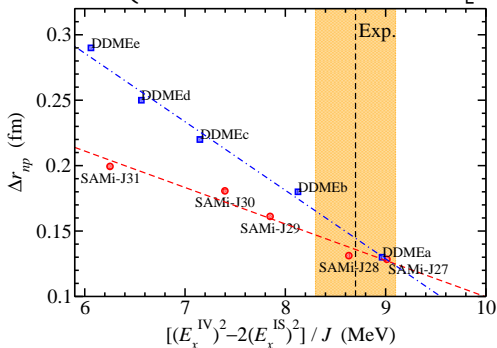


# Isovector Giant Quadrupole Resonance:

## IV (and IS) Excitation Energies in $^{208}\text{Pb}$

Consistently within the DM, one can also connect the symmetry potential proportional to  $a_{\text{sym}}(A) = J/(1 + \frac{9}{4} \frac{J}{Q} A^{-1/3})$  with the neutron skin and find,

$$\frac{\Delta r_{np} - \Delta r_{np}^{\text{surf}}}{\langle r^2 \rangle^{1/2}} = \frac{2}{3} (I - I_C) \left\{ 1 - \frac{\epsilon_{F\infty}}{3J} - \frac{3}{7} \frac{I_C}{I - I_C} - \frac{A^{2/3}}{24\epsilon_{F\infty}} \left[ \frac{(E_x^{\text{IV}})^2 - 2(E_x^{\text{IS}})^2}{J} \right] \right\}$$



$$\Delta r_{np} \approx 0.14 \pm 0.03 \text{ fm}$$

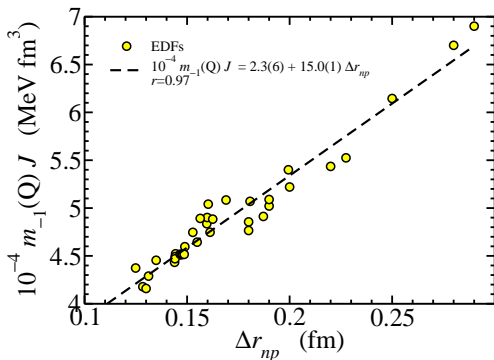
# Isvector Giant Quadrupole Resonance:



## Quadrupole polarizability

Guided by the Droplet Model (analogous to the dipole) and the predictions of successful EDFs, it can be found:

$$\alpha_Q \approx \frac{A \langle r^4 \rangle}{16\pi J} \left[ 1 + \frac{7}{2} \frac{\Delta r_{np} + \sqrt{\frac{3}{5}} \frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2 \rangle^{1/2} (I - I_C)} \right]$$



# CONCLUSIONS

# Conclusions:

## For medium-heavy and heavy mass nuclei:

- ▶ The **macroscopic model** presented here contains **relevant physics** for the description of the different **observables**.
- ▶ The **excitation energy** of the **IVGDR** depends on  $S(0.1 \text{ fm}^{-3})$  in EDFs.
- ▶  $S(0.1 \text{ fm}^{-3})$  or the  $\Delta r_{np}$  can be **determined by empirical data** on the excitation energies of the **IS-** and **IV-GQRs**.
- ▶ The **dipole and quadrupole polarizabilities** depend in an analogous way on the  $\Delta r_{np}$ .



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