The Symmetry Energy: Giant Quadrupole Resonances and Parity Violating Electron Scattering

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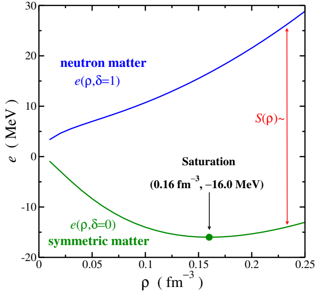
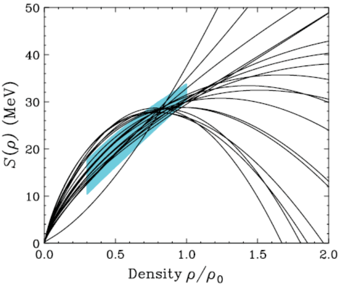
Brief Introduction:

- The **nucleus** is a **complex and self-bound system** of fermions: **spin, isospin, pairing, deformation,** ...
- *ab-initio* many-body calculations based on **bare nucleon-nucleon interaction** cannot be **extensively** applied to **ground state and excited state properties** of medium and heavy mass nuclei yet.
- **Nuclear Energy Density Functionals**, based on effective interactions, are **successful** in the description of overall properties such as **masses, nuclear sizes, deformation, Giant Resonances,...**

The Nuclear Equation of State: Infinite System



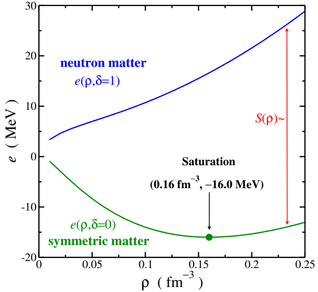


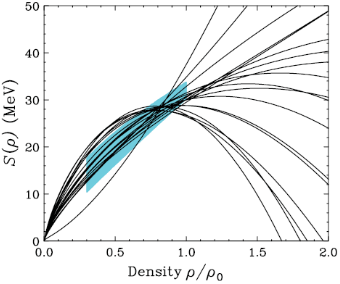
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System





$$\boxed{\frac{E}{A}(\rho, \beta)} = \boxed{\frac{E}{A}(\rho, \beta = 0)} + \boxed{S(\rho)\beta^2} + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter

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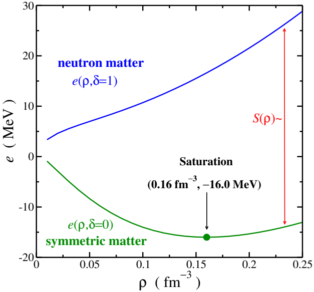
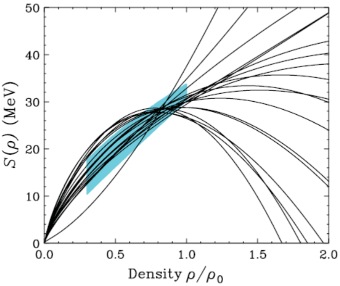
The Nuclear Equation of State: Infinite System



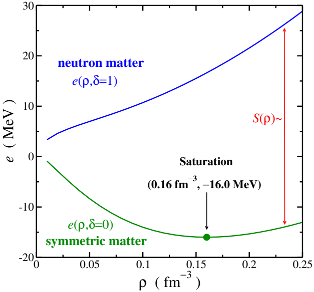


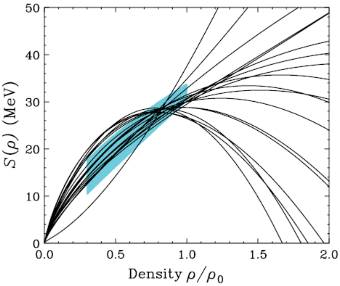
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

- Nuclear Matter
- Symmetric Matter
- Symmetry energy

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}\right]$$

The Nuclear Equation of State: Infinite System



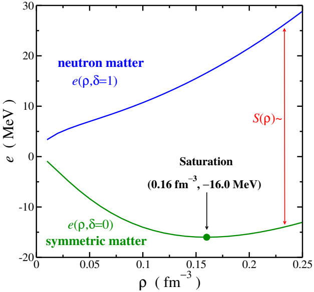


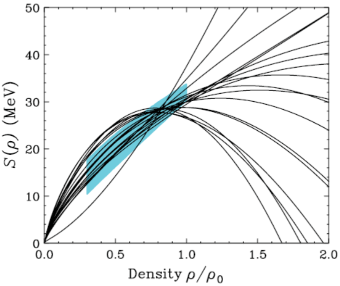
$$\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4)$$

$$= \frac{E}{A}(\rho, \beta = 0) + \beta^2 \left(J + Lx + \frac{1}{2}K_{sym}x^2 + \mathcal{O}(x^3) \right)$$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0} \right]$$

The Nuclear Equation of State: Infinite System





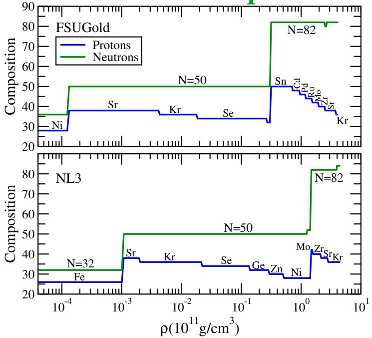
$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta=0) + \beta^2\left(\boxed{J} + \boxed{L}x + \frac{1}{2}\boxed{K_{sym}}x^2 + \mathcal{O}(x^3)\right)$$

- $S(\rho_0) = J$
- $\left.\dfrac{d}{d\rho}S(\rho)\right|_{\rho_0} = \dfrac{L}{3\rho_0} = \dfrac{P_0}{\rho_0^2}$
- $\left.\dfrac{d^2}{d\rho^2}S(\rho)\right|_{\rho_0} = \dfrac{K_{sym}}{9\rho_0^2}$

$$\left[\beta = \frac{\rho_n - \rho_p}{\rho}; \quad x = \frac{\rho - \rho_0}{3\rho_0}\right]$$

Motivation: Examples where $S(\rho)$ is relevant

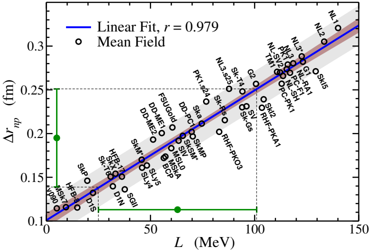
Neutron star composition



X. Roca-Maza and J. Piekarewicz, PRC **78***, 025807 (2008)*

The **faster $S(\rho)$ increases** with density, the **more exotic** the composition of the **outer crust**.

Size of neutron distribution



B.A. Brown, PRL 85, 5296 (2000); M. Centelles, X. Roca-Maza, M. Warda, and X. Viñas, PRL **102***, 122502 (2009); X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, PRL* **106***, 252501 (2011)*

The **faster $S(\rho)$ increases** with density, the **larger** the **size of the neutron distribution** in nuclei.

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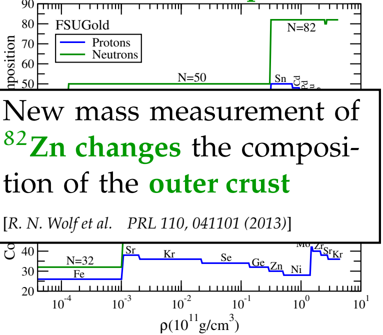

New mass measurement of ^{82}**Zn changes** the composition of the **outer crust**

[*R. N. Wolf et al.* *PRL 110, 041101 (2013)*]

X. Roca-Maza and J. Piekarewicz, PRC **78***, 025807 (2008)*

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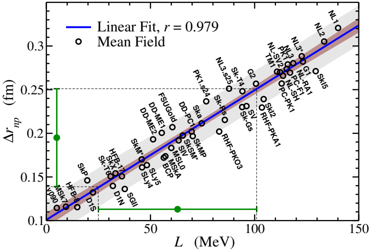

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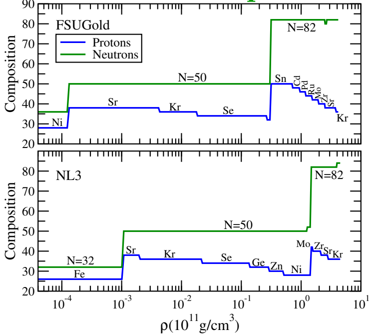

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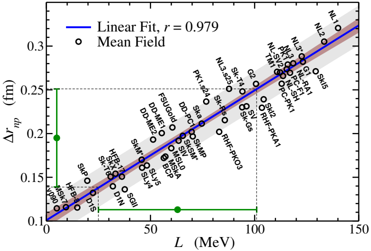

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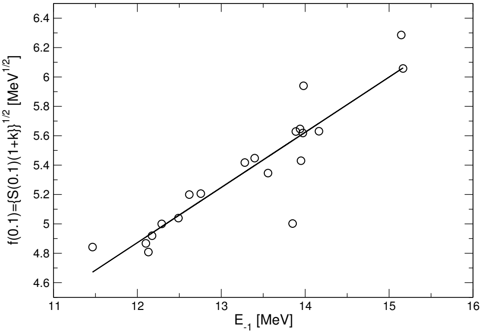
Motivation: Examples where $S(\rho)$ is relevant

Excitation Energy of the Isovector Giant Dipole Resonance



Luca Trippa, Gianluca Colò, and Enrico Vigezzi, PRC **77** *061304 (2008)*

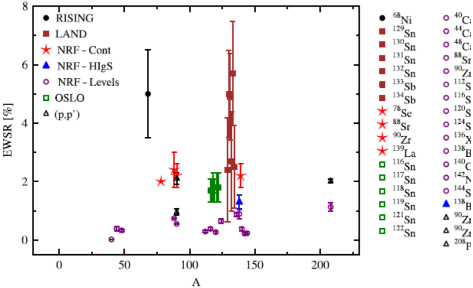
$23.3\ \mathrm{MeV} < S(\rho = 0.1\ \mathrm{fm}^{-3}) < 24.9\ \mathrm{MeV}$

The **larger $S(\rho)$** at subsaturation, the larger the E_x^{GDR}

Motivation: Examples where $S(\rho)$ is relevant

Experimental evidencies of low-lying electric dipole strength using real photons, Coulomb excitation and hadronic probes

D. Savran, T. Aumann and A. Zilges PPNP 70 (2013) 210



K. Govaert *et al.*, **PRC 57 (1998) 2229**; P. Adrich *et al.* **PRL 95 (2005) 132501**; D. Savran *et al.*, **PRC 84 (2011) 024326**; T. Hartmann *et al.*, **PRL 93 (2004) 192501**; A. Klimkiewicz *et al.*, **PRC 76 (2007) 051603**; R. Schwengner *et al.*, **PRC 76 (2007) 034321**; R. Schwengner *et al.*, **PRC 78 (2008) 064314**; B. Ozel *et al.*, **NPA 788 (2007) 385**; S. Volz *et al.*, **NPA 779 (2006) 1**; A. Makinaga *et al.*, **PRC 82 (2010) 024314**; R. Schwengner *et al.*, **PRC 81 (2010) 054315**; G. Schramm *et al.*, **PRC 85 (2012) 014311**; O. Wieland *et al.*, **PRL 102 (2009) 092502**; I. Poltoratska *et al.*, **PRC 85 (2012) 041304**; C. Iwamoto *et al.*, **PRL 108 (2012) 262501**; H. K. Toft *et al.*, **PRC 83 (2011) 044320**.

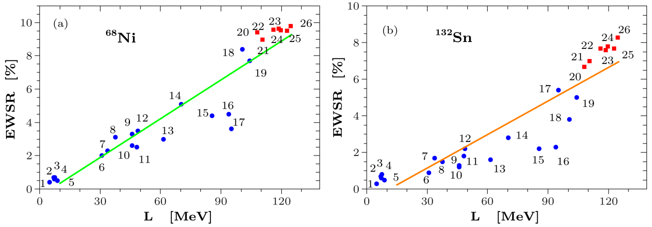
Motivation: Examples where $S(\rho)$ is relevant

Exper ... **ength**

using ... **obes**

D. Savran ...

Theory (RPA)



Andrea Carbone et al. *PRC* **81**, *041301 (2010)*

X. Roca-Maza, et al. *PRC85, 024601 (2012)*

The **faster $S(\rho)$ increases** with density, the **larger** is the **energy** (E) times the **probability** (P) of **exciting the Pygmy state** ⇒ the larger the Energy Weighted Sum Rule (EWSR) $\propto E \times P$.

K. Govaer ... **) 024326**;

T. Hartma ... ., **PRC 76**

(2007) 034 ... **NPA 779**

(2006) 1; A. Makinaga *et al.*, **PRC 82 (2010) 024314**; R. Schwengner *et al.*, **PRC 81 (2010) 054315**; G. Schramm *et al.*, **PRC 85 (2012) 014311**; O. Wieland *et al.*, **PRL 102 (2009) 092502**; I. Poltoratska *et al.*, **PRC 85 (2012) 041304**; C. Iwamoto *et al.*, **PRL 108 (2012) 262501**; H. K. Toft *et al.*, **PRC 83 (2011) 044320**.

Isovector Giant Resonances

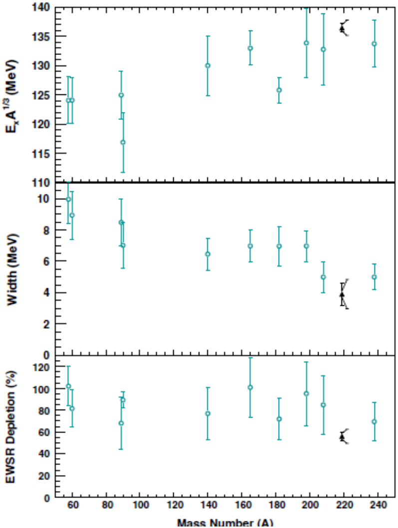
- In **isovector** giant resonances **neutrons and protons oscillate out of phase**
  e.g. within a classical picture: **"e-m interacting probes basically excite protons, protons drag neutrons thanks to the long range atraction of the nuclear strong interaction, when neutrons approach too much to protons, due to the short range repulsion they are pushed out"**
- **Isovector** resonances will depend on oscillations of the isovector density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such oscillations.
- The **excitation energy** (E_x) within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \to E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{S(\rho)}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

Giant Quadrupole Resonances

IVGQR: was experimentally known **[R. Pitthan, proceedings of Giant Multiple Resonance conference, Oak Ridge 1980]** but via a recent experimental technique the accuracy has been improved **[S.S. Henshaw, M.W. Ahmed, G. Feldman, A.M. Nathan, and H.R. Weller PRL107 (2011)]**



$E_{excitation}$, width and EWSR

Giant Quadrupole Resonances

IVGQR
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Giant M
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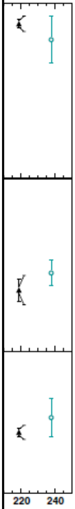
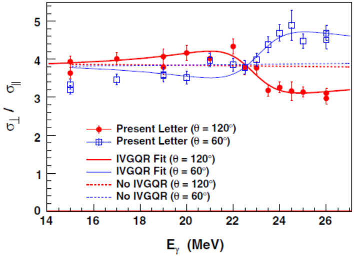
M.W. A
A.M. N
Weller



$$\frac{\sigma_{\parallel}}{\sigma_{\perp}} = \cos^2\theta + \frac{2|f_{E2}|\cos(\phi_{E2} - \phi_{E1})[\cos^3\theta - \cos\theta]}{|f_{E1} + D(E_\gamma, \theta)|}$$

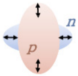


Key features in the new polarized Compton scattering experiment:

- almost **monoenergetic and polarized $\gamma-$ray beam**
- **E1 — E2 interference** term has **opposite signs** in the forward and backward angles

[**S.S. Henshaw, M.W. Ahmed, G. Feldman, A.M. Nathan, and H.R. Weller** PRL107 (2011)]

Giant Quadrupole Resonances:



IV and IS Excitation Energies in ^{208}Pb

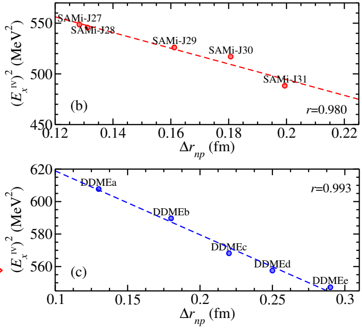
Guided by the Quantum Harmonic Oscillator approach (Bohr and Mottelson)

$$E_x^{IS} = \sqrt{\frac{2m}{m^*}}\hbar\omega_0 \Rightarrow \hbar\omega_0 \approx 41 A^{-1/3} \text{ shell gap}$$

$$E_x^{IV} = 2\sqrt{\frac{\left(E_x^{IS}\right)^2}{2} + 10\frac{\hbar^2}{2m}\frac{a_{sym}^{pot}\langle r^2\rangle}{\langle r^4\rangle}}$$

$S(\rho_A) \approx a_{sym}(A)$ within modern EDFs†

$$S(\rho_A) = \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$



$$S(\rho_{208} = 0.1 \text{ fm}^{-3}) = 23.3 \pm (0.6)_{exp.} \pm (1.0)_{teo.} \text{ MeV}$$

X. Roca-Maza et al. PRC 87 034301 (2013)

† *M. Centelles, X. Roca-Maza, M. Warda, and X. Viñas, PRL 102, 122502 (2009)*

PARITY VIOLATING ELECTRON SCATTERING

T.W. Donnelly, J. Dubach, and Ingo Sick, Nucl. Phys. A503, 589 (1989); **C. J. Horowitz**, Phys. Rev. C **57** 3430 (1998); **C. J. Horowitz, S. J. Pollock, P. A. Souder, and R. Michaels**, Phys. Rev. C **63**, 025501 (2001); **M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda**, Phys. Rev. C **82**, 054314 (2010); **X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda**, Phys. Rev. Lett. **106** 252501 (2011); **PREx collaboration**, Phys. Rev. Lett. 108, 112502 (2012).

Parity Violating Elastic Electron Scattering

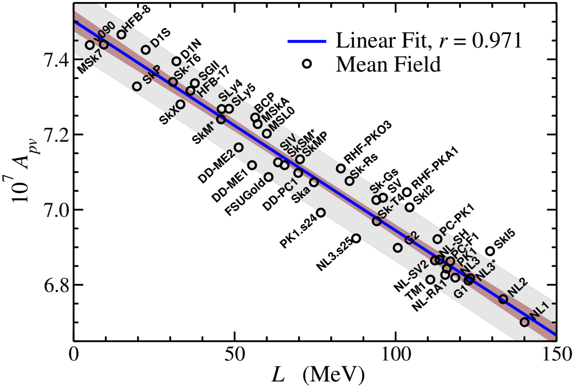
- The **electron interacts** with a **nucleus** by exchanging either a γ or a Z_0 boson.
- **γ couples basically to protons** and **Z_0 couples basically to neutrons**.
- **Ultra-relativistic electons** interact with the **Coulomb** + or − the **Weak potential** depending on their **helicity**.
  $V_{tot} = V_C \pm V_W$ where $V_W = G_F \rho_W(r)/2^{2/3}$
- The effect of the **parity-violating part** due to the weak interaction may be **isolated** by measuring the **parity-violating asymmetry**,

$$A_{PV} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega}$$

  where +/− indicates positive or negative helicity of e.
- **Coulomb distortions** are **improtant** ⇒ **D**istorted **W**ave **B**orn **A**pproximation

Parity Violating Elastic Electron Scattering

The largest the size of the neutron distribution in nuclei, the largest the L and the smaller the parity violating asymmetry



X. Roca-Maza, M. Centelles, X. Vinas, and M. Warda, PRL 106, 252501 (2011)

Exp. $[6.56 \pm 0.60_{(\mathrm{stat})} \pm 0.14_{(\mathrm{syst})}] \times 10^7$

PREx collaboration, PRL 108, 112502 (2012)

Conclusions:

For medium-heavy and heavy mass nuclei:

- **macroscopic models** may contain **relevant physics** and be useful for understanding properties such as nuclear sizes and excitation energies of Giant Resonances.
- $S(\rho = 0.1 \text{ fm}^{-3})$ (and the Δr_{np}) can be **determined** by a combination of **empirical data** on the excitation energies of the **ISGQR** and **IVGQRs** in EDFs.
- The estimated value for the symmetry energy
  $$S(\rho = 0.1 \text{ fm}^{-3}) = 23.3 \pm (0.6)_{\text{exp.}} \pm (1.0)_{\text{teo.}} \text{ MeV}$$
  is compatible with other available estimates

  M.B. Tsang, et al. Phys. Rev. C 86, 015803 (2012).

- The **parity violating asymmetry** is linearly **correlated** with the **slope of the symmetry energy** around saturation (and the neutron skin thickness of ^{208}Pb) in EDFs.

Collaborators:

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G. Colò[2,3]
W. Nazarewicz[6,7,8]

N. Paar[9]
J. Piekarewicz[10]
P.-G. Reinhard[11]
D. Vretenar[9]
X. Viñas[5]
M. Warda[12]

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EXTRA MATERIAL

Polarizability, Strength distribution and its moments

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar/quadrupolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \; (\Delta L = 1, 2 \rightarrow \mathrm{Dipole, Quadrupole})$$

- is proportional to the **static polarizability** for small oscillations

$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_{\nu} |\langle\nu|F|0\rangle|^2 \delta(E - E_{\nu})$

- **Isovector energy weighted sum rules (EWSR)** are:

$$m_1 = \frac{\hbar^2}{2m}\frac{NZ}{A}(1 + \kappa_D) \qquad m_1 = \frac{\hbar^2}{2m}\frac{50}{4\pi}A\langle r^2\rangle\left(1 + \kappa_Q\right)$$

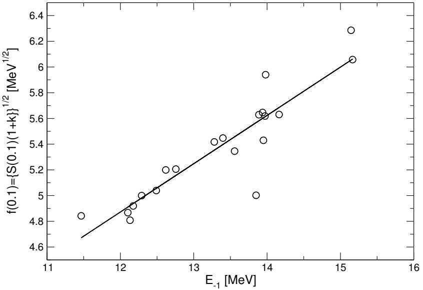
Isovector Giant Dipole Resonance:

Excitation energy in ^{208}Pb

Guided by an hydrodynamical model† and the predictions of successful microscopic models (Energy Density Functionals), it was found: *Physical Review C* **77** *061304 (2008)*

$$E_{-1} = \sqrt{\frac{m_1}{m_{-1}}} \approx \sqrt{\frac{6\hbar^2}{m\langle r^2 \rangle} S(\rho = 0.1)(1 + \kappa_D)}$$

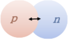
$S(\rho = 0.1$ **fm**$^{-1})$ is **correlated** with the value of the excitation energy (E_{-1}) of the IVGDR in spherical nuclei ⇒ experimental data on $E_{-1}(^{208}\mathrm{Pb})$ leads to



$$\mathbf{23.3\ MeV} < S(\rho = 0.1\ \mathrm{fm}^{-3}) < \mathbf{24.9\ MeV}$$

† *Physics Reports* **175** *103-261 (1989)*

Isovector Giant Dipole Resonance:



Dipole polarizability in ^{208}Pb

Guided by a constrained calculation ($E(\lambda) = \langle \mathcal{H} - \lambda \mathcal{O} \rangle$) using the Droplet Model[†] and the dipole operator one may find:

$$\alpha_D \approx \frac{A\langle r^2 \rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2 \rangle^{1/2}(I - I_C)}\right]$$

$$\alpha_D = \frac{8\pi}{9}e^2\frac{A\langle r^2 \rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

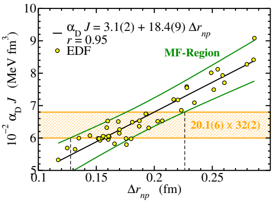
Within the DM model:

$$\Delta r_{np} = \sqrt{\frac{3}{5}}\left[t - \frac{e^2 Z}{70J}\right] + \Delta r_{np}^{\text{surface}}$$

$$t \equiv \frac{3r_0}{2}\frac{J/Q}{1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}}(I - I_C)$$

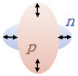
and the symmetry energy:

$$a_{sym}(A) = J/\left(1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}\right)$$



† *Nuclear Physics A* **385**, 269 (1982).

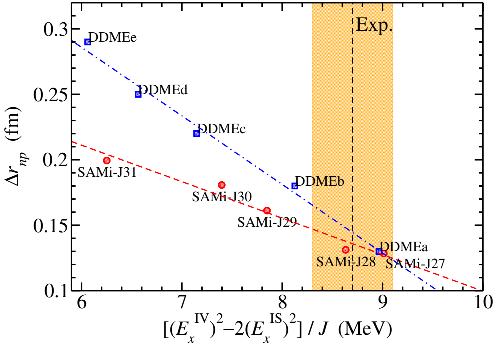
Isovector Giant Quadrupole Resonance:



Neutron skin and excitation energies in ^{208}Pb

Correlation within families of interactions

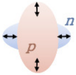
Physical Review C **87** *034301 (2013)*



$$\frac{\Delta r_{np} - \Delta r_{np}^{surf}}{\langle r^2\rangle^{1/2}} = \frac{2}{3}(I - I_C)\left\{1 - \frac{\varepsilon_{F_\infty}}{3J} - \frac{3}{7}\frac{I_C}{I - I_C} - \frac{A^{2/3}}{24\varepsilon_{F_\infty}}\left[\frac{\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2}{J}\right]\right\}$$

$$\Delta r_{np} \approx 0.14 \pm 0.03 \ \textbf{fm}$$

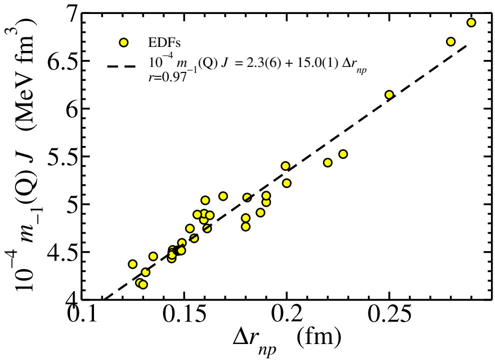
Isovector Giant Quadrupole Resonance:



Quadrupole polarizability in ^{208}Pb

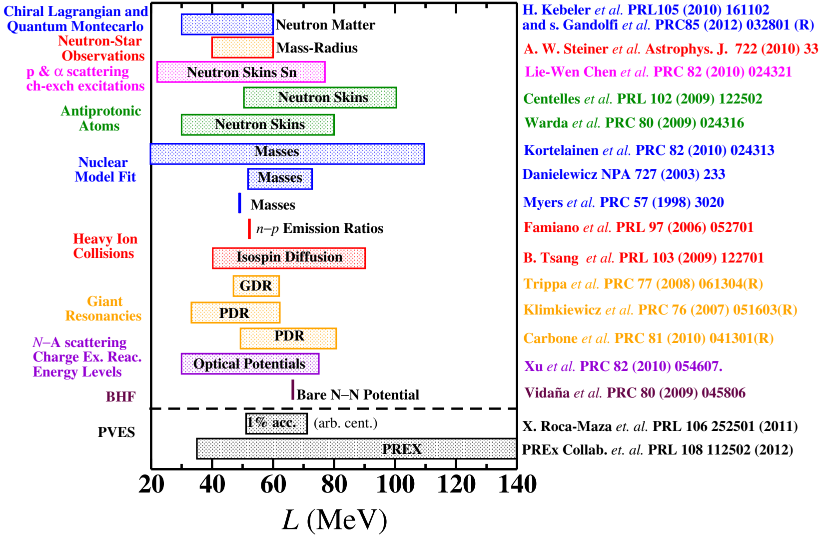
Guided by a constrained calculation ($E(\lambda) = \langle \mathcal{H} - \lambda \mathcal{O} \rangle$) using a macroscopic approach†:

$$\alpha_Q \approx \frac{A\langle r^4 \rangle}{16\pi J}\left[1 + \frac{7}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2 \rangle^{1/2}(I - I_C)}\right]$$



† *Physics Reports* **175** 103-261 (1989) and *Nuclear Physics A* **385**, 269 (1982)

Available constraints on L



AIP Conference Proceedings **1491**, *101 (2012)*