The Nuclear Symmetry Energy: constraints from Giant Resonances

Xavier Roca-Maza

Dipartimento di Fisica, Università degli Studi di Milano
and
INFN, Sezione di Milano

3rd International Symposium on Nuclear Symmetry Energy,
NSCL/FRIB, East Lansing, Michigan — July 22 - 26, 2013

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RECENT EXPERIMENTS

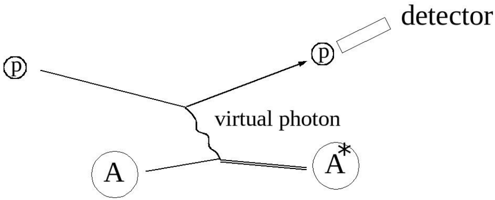
on the Dipole and Quadrupole responses

Isovector Giant Dipole Resonance in ^{208}Pb

At the Research Center for Nuclear Physics (RCNP), Osaka ...

- using **polarized protons**
- measuring protons **scattered inelatically**
- excitations via virtual photons (**Coulomb excitation**)
- able to cover a **broad range of excitation energies**
- set up with **high-resolution and efficiency**

Very good agreement with previous measurements is found



A. Tamii et al., PRL107 (2011) 062502

Isovector Giant Dipole Resonance in ^{208}Pb

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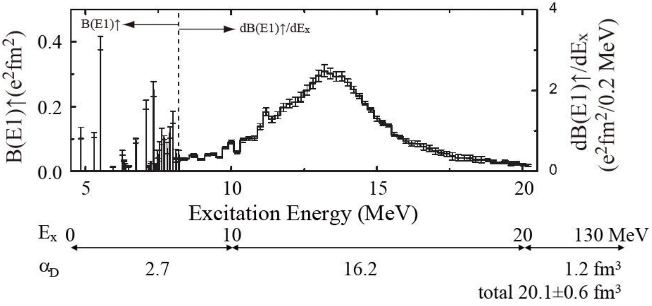


Figure taken from A. Tamii's talk at INPC 2013

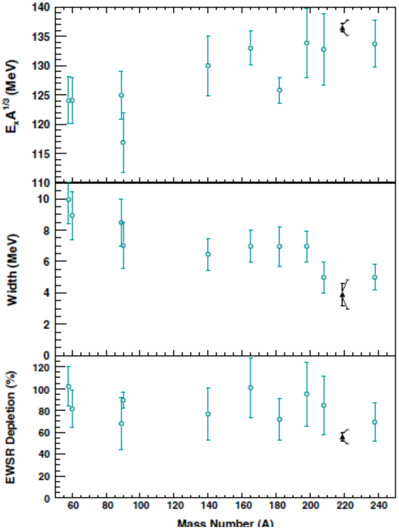
Dipole polarizability is determined with high accuracy by taking the average of the RCNP data plus available data in ^{208}Pb[†] covering a wide range of excitation energies: $\alpha_D = 20.1 \pm 0.6$ **fm^3**

A. Tamii e...

[†] K. Schelhaas et al., Nucl. Phys. A 489, 189 (1988); A. Veyssiere, H. Beil, R. Bergere, P. Carlos, and A. Lepretre, Nuclear Physics A 159, 561 (1970)

Isovector Giant Quadrupole Resonance in ^{208}Pb

It was experimentally known since the 80's **[R. Pitthan, proceedings of Giant Multiple Resonance conference, Oak Ridge 1980]** but the use of the **High Intensity γ–ray source** facility at **Duke U.** (oparated by **TUNL**) in combination with a new experimental technique has allowed to improve the accuracy **[S.S. Henshaw, M.W. Ahmed, G. Feldman, A.M. Nathan, and H.R. Weller PRL107 (2011)]**



$E_{excitation}$, **width and EWSR**

Isovector Giant Quadrupole Resonance in ^{208}Pb

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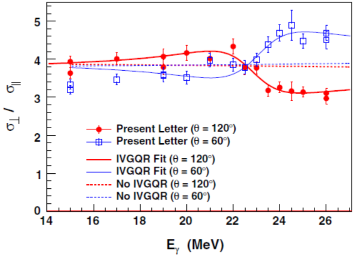
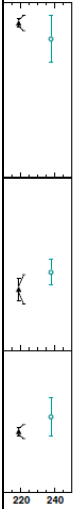
$$\frac{\sigma_{\|}}{\sigma_{\perp}} = \cos^2\theta + \frac{2|f_{E2}|\cos(\phi_{E2} - \phi_{E1})[\cos^3\theta - \cos\theta]}{|f_{E1} + D(E_\gamma, \theta)|}$$





Key features in the new polarized Compton scattering experiment:

- almost **monoenergetic and polarized $\gamma-$ray beam**
- E1 – E2 **interference** term has **opposite signs** in the forward and backward angles

[**S.S. Henshaw, M.W. Ahmed, G. Feldman, A.M. Nathan, and H.R. Weller** PRL107 (2011)]

THEORY

IN VERY SHORT

Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

$$\begin{aligned}\mathcal{L}_{int} &= \bar{\Psi}\Gamma_\sigma(\bar{\Psi},\Psi)\Psi\Phi_\sigma \quad + \bar{\Psi}\Gamma_\delta(\bar{\Psi},\Psi)\boldsymbol{\tau}\Psi\Phi_\delta \\ &-\bar{\Psi}\Gamma_\omega(\bar{\Psi},\Psi)\gamma_\mu\Psi A^{(\omega)\mu} \quad -\bar{\Psi}\Gamma_\rho(\bar{\Psi},\Psi)\gamma_\mu\tau\Psi A^{(\rho)\mu} \\ &-e\bar{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu}\end{aligned}$$

Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

$$V^{\mathrm{eff}}_{\mathrm{Nucl}} = V^{\mathrm{long-range}}_{\mathrm{attractive}} + V^{\mathrm{short-range}}_{\mathrm{repulsive}} + V_{\mathrm{SO}} + V_{\mathrm{pair}}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** → **not directly connected to any NN** (or NNN) **interaction** (opposite to "ab-initio" calculations)

Approximate realization of an exact Nuclear Energy Density Functional:

Kohn-Sham iterative scheme (static approximation)

- Determine a good $E[\rho]$
- Initial guess ρ_0
- Calculate potential V_{eff} from ρ_0
- Solve single particle (Schrödinger) equation and find single particle wave functions ϕ_i
- Use ϕ_i for calculating new $\rho_1 = \sum_i^A |\phi_i|^2$
- Repeat until convergence

Runge-Gross Theorem: dynamic generalization of the static EDFs.

$$\int dt \{\langle \Phi(t)|i\partial_t|\Phi(t)\rangle - E[\rho(t), t]\} = 0$$

Giant Resonances well described within the small amplitude limit (known as RPA approach)

Isovector Giant Resonances

- In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**
  e.g. within a classical picture: **"e-m interacting probes basically excite protons, protons drag neutrons thanks to the nuclear strong interaction, when neutrons approach too much to protons, they are pushed out"**
- **Isovector** resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such "oscillations"
- The **excitation energy (E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \rightarrow E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{S(\rho)}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

Polarizability, Strength distribution and its moments

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \; (\Delta L = 1,\, 2 \rightarrow \text{Dipole, Quadrupole})$$

- is proportional to the **static polarizability** for small oscillations

$$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$$

  where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_{\nu} |\langle\nu|F|0\rangle|^2 \delta(E - E_{\nu})$

- **Isovector energy weighted sum rules (EWSR)** are:

$$m_1 = \frac{\hbar^2}{2m}\frac{NZ}{A}(1 + \kappa_D) \qquad m_1 = \frac{\hbar^2}{2m}\frac{50}{4\pi}A\langle r^2\rangle\left(1 + \kappa_Q\right)$$

equal to one half of the HF expectation value of $[\hat{F}, [H, \hat{F}]]$ (Thouless theorem) and where κ is the

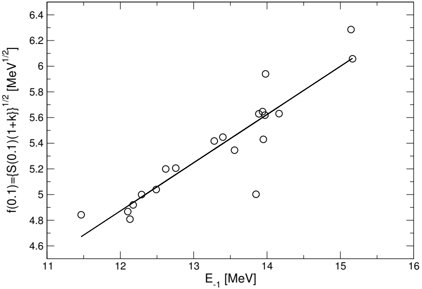
RESULTS

Excitation energy of the IVGDR in ^{208}Pb

Excitation Energy of the IV Giant Dipole Resonance in ^{208}Pb

Guided by an **hydrodynamical model**† one may derive the approximate formula:

$$\mathbf{E_{-1}} = \sqrt{\frac{m_1}{m_{-1}}} \approx \sqrt{\frac{6\hbar^2}{m\langle r^2\rangle} S(\rho = 0.1)(1+\kappa_D)}$$



Luca Trippa, Gianluca Colò, and Enrico Vigezzi, PRC **77** *061304 (2008)*

and found that **EDFs where also obeying such a correlation**

The **larger** $S(\rho = 0.1\ \mathbf{fm}^{-3})$, the **larger the** E_x^{GDR}

† Lipparini and Stringari, Physics Reports **175** 103-261 (1989)

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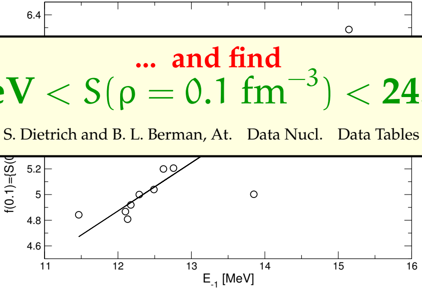

... and find

23.3 MeV $< S(\rho = 0.1\ \text{fm}^{-3}) <$ **24.9 MeV**

Using $E_{-1}^{\text{exp.}\,\dagger\dagger}$ from S. S. Dietrich and B. L. Berman, At. Data Nucl. Data Tables 38, 199 (1988)

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Dipole polarizability

Insights from a macroscopic approach

Within the DM model:

$$\Delta r_{np} = \sqrt{\frac{3}{5}}\left[t - \frac{e^2 Z}{70J}\right] + \Delta r_{np}^{\text{surface}}$$

$$t \equiv \frac{3r_0}{2}\frac{J/Q}{1+\frac{9}{4}\frac{J}{Q}A^{-1/3}}(I - I_C)$$

Q is the so-called surface stiffness coefficient, $I \equiv (N-Z)/A$ is the relative neutron excess, $\rho_0 = 3A/4\pi r_0^3$, $I_C = (e^2 Z)/(20JR)$, $R \equiv \sqrt{3/5}\, r_0 A^{1/3}$, and $\Delta r_{np}^{\text{surf}} = \sqrt{3/5}[5(b_n^2 - b_p^2)/(2R)]$ is a correction caused by the difference in the surface width b_n (b_p) of the neutron (proton) density profile

using these expressions:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Adopting a value of $J = 31 \pm 2$ MeV† one finds for ^{208}Pb that $I_c \approx 0.028 \pm 0.002$ $\sqrt{3/5}(e^2 Z)/(70J)$ is around 0.042 ± 0.003 fm. $\Delta r_{np}^{\text{surf}}$ for ^{208}Pb is almost constant (0.09 ± 0.01 fm) in EDFs††

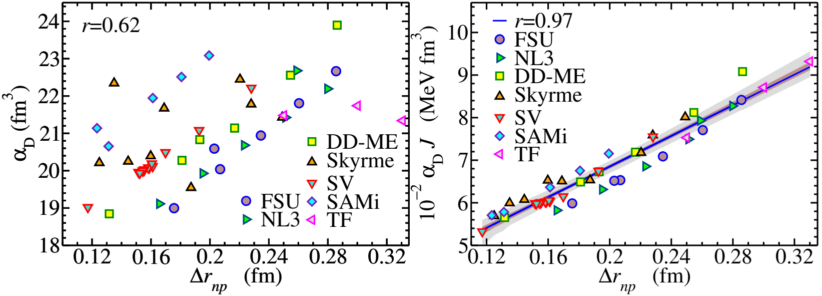
In the DM Δr_{np} (and L) are better correlated with $\alpha_D J$ than with α_D alone in a heavy nucleus such as ^{208}Pb

† James M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

†† M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C 82, 054314 (2010).

Dipole polarizability in ^{208}Pb

Insights from a macroscopic approach



X. Roca-Maza et al., arXiv:1307.4806 (2013)

Using exp. $\alpha_D = 20.1 \pm 0.6$ **fm**^{3} † in ^{208}Pb on finds the relation

$\Delta r_{np} = -0.156 \pm (0.014)_{\text{theo.}} + \left[1.04 \pm (0.03)_{\text{exp.}} \pm (0.04)_{\text{theo.}}\right] \times 10^{-2} J$

Adopting $J = 31 \pm (2)_{\text{est.}}$ **MeV** †† one obtains

$\Delta r_{np} = 0.168 \pm (0.009)_{\text{exp.}} \pm (0.019)_{\text{theo.}} \pm (0.021)_{\text{est.}}$ **fm**

† A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011) †† James M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

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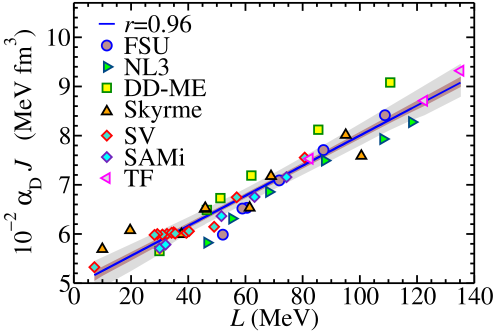

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Using exp. $\alpha_D = 20.1 \pm 0.6$ **fm**$^{3\,\dagger}$ in ^{208}Pb on finds the relation

$$L = -145 \pm (9)_{theo.} + \left[6.07 \pm (0.18)_{exp.} \pm (0.26)_{theo.}\right] J$$

Adopting $J = 31 \pm (2)_{est.}$ **MeV** †† one obtains

$$L = 43 \pm (6)_{exp.} \pm (12)_{theo.} \pm (12)_{est.}\ \textbf{MeV}$$

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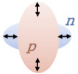
Excitation energy of the IVGQR in ^{208}Pb



IV and IS Excitation Energies in ^{208}Pb

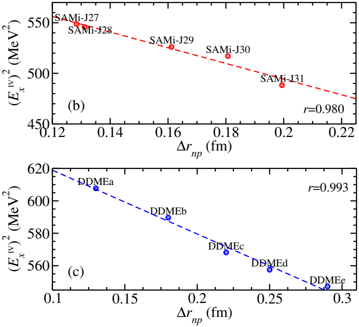
Guided by the Quantum Harmonic Oscillator approach (Bohr and Mottelson)

$$E_x^{IS} = \sqrt{\frac{2m}{m^*}}\hbar\omega_0 \Rightarrow \hbar\omega_0 \approx 41A^{-1/3} \text{ shell gap}$$

$$E_x^{IV} = 2\sqrt{\frac{\left(E_x^{IS}\right)^2}{2} + 10\frac{\hbar^2}{2m}\frac{a_{sym}^{pot}\langle r^2\rangle}{\langle r^4\rangle}}$$

$S(\rho_A) \approx a_{sym}(A)$ within modern EDFs†

$$S(\rho_A) = \frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$

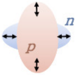


The **larger** $S(\rho = 0.1\ \mathbf{fm}^{-3})$, the **larger** $\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2$

X. Roca-Maza et al. PRC 87 034301 (2013)

† *M. Centelles, X. Roca-Maza, M. Warda, and X. Viñas, PRL 102, 122502 (2009)*

Excitation energy of the IVGQR in ^{208}Pb



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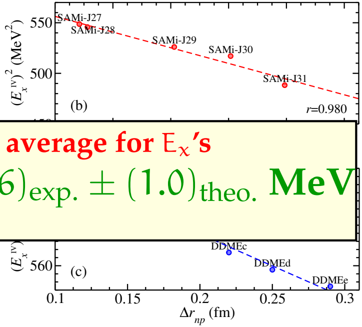
$$E_x^{IS} = \sqrt{\frac{2m}{m^*}}\hbar\omega_0 \Rightarrow \hbar\omega_0 \approx 41A^{-1/3} \text{ shell gap}$$

$$E_x^{IV} = 2\sqrt{\frac{(E_x^{IS})^2}{\ldots} + \ldots \hbar^2 \, a_{sym}^{pot}\langle r^2\rangle}$$

... and using experimental average for E_x's

$$S(\rho_{208} = 0.1\ \mathbf{fm}^{-3}) = 23.3 \pm (0.6)_{exp.} \pm (1.0)_{theo.}\ \mathbf{MeV}$$

$$\frac{\varepsilon_{F_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{F_0}^2}\left[\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2\right] + 1\right\}$$



The **larger** $S(\rho = 0.1\ \mathbf{fm}^{-3})$, the **larger** $\left(E_x^{IV}\right)^2 - 2\left(E_x^{IS}\right)^2$

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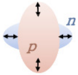
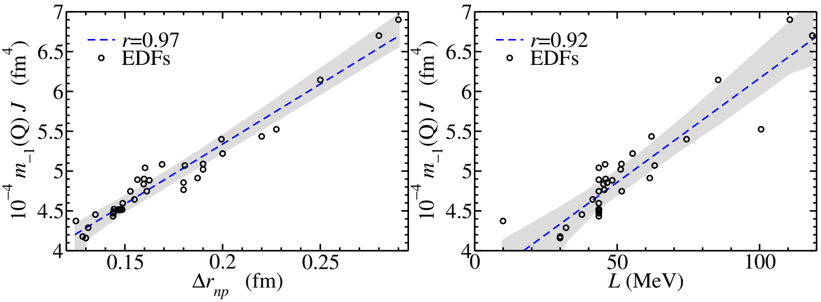
† *M. Centelles, X. Roca-Maza, M. Warda, and X. Viñas, PRL 102, 122502 (2009)*

Quadrupole polarizability in ^{208}Pb



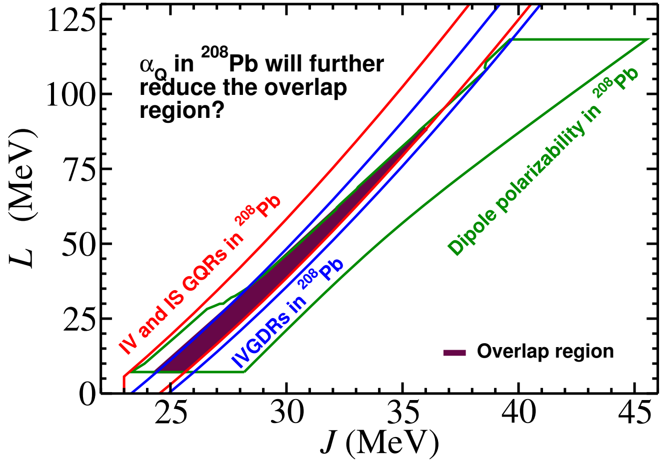
$$\alpha_Q \approx \frac{A\langle r^4\rangle}{16\pi J}\left[1+\frac{7}{2}\,\frac{\Delta r_{np}+\sqrt{\frac{3}{5}\frac{e^2 Z}{70J}}-\Delta r_{np}^{\mathrm{surface}}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$



HI$\vec{\gamma}$S measurment of IVGQR with high accuracy but no information on the α_Q was published [S.S. Henshaw, M.W. Ahmed, G. Feldman, A.M. Nathan, and H.R. Weller PRL107 (2011)].

J versus L

From our analysis of the experimental E_x of the IVGDR, ISGQR, IVGQR and α_D in ^{208}Pb based on EDFs



DIPOLE RESPONSE IN EXOTIC NUCLEI

Dipole polarizability in exotic nuclei

Rare Isotope facilities constitute a unique experimental tool for the study of fundamental properties of exotic nuclei

As near future example:

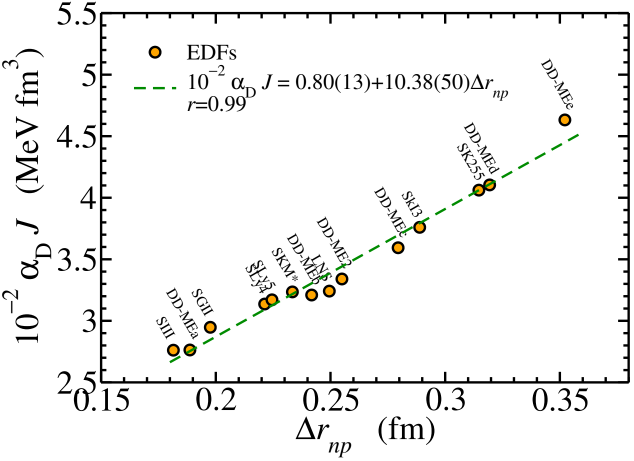
- The **e-m charge distribution of unstable Sn (Z=50) isotopes** will be measured at the **SCRIT† (RIKEN) facility next year via electron elastic scattering**.

  (arXiv:1307.2315: "Shell and isospin effects in nuclear charge radii" the authors estimated $L = 54 \pm 19$ MeV using the isotope shift between ^{30}S and ^{30}Si)

- If measuring the **dipole** response from inelastic electrons at forward angles **becomes feasible using SCRIT††**, the **proton and neutron rms of exotic nuclei** (and L) might be **extracted experimentally using the same facility** via the correlation between $\alpha_D J$ **and** Δr_{np} in EDFs **presented here**.
- Might be **proton inelastic scattering by exotic nuclei at forward angles** may also become **feasible in existent and/or future RI facilites**.

† http://www.riken.jp/en/research/labs/rnc/instrum_dev/scrit/ †† T. Suda et al. Prog. Theor. Exp. Phys. 2012, 03C008

Dipole polarizability in the exotic ^{132}Sn nucleus



X. Roca-Maza in preparation (DD-ME calculations provided by Nils Paar)

CONCLUSIONS

Conclusions:

For medium-heavy and heavy mass nuclei we expect:

- the **macroscopic models** presented here contain **relevant physics** for the description of the
  - E_x**'s** of the **IVGDR, ISGQR and IVGQR**
  - **dipole and quadrupole polarizabilities**
- $\alpha_{D(Q)} J$ are strongly correlated with the Δr_{np} and L in EDFs.

For the case of ^{208}Pb using exp. values on

- E_x^{IVGDR}: 23.3 MeV $< S(\rho = 0.1 \text{ fm}^{-3}) <$ 24.9 MeV
- E_x^{ISGQR} and E_x^{IVGQR}:
  $S(\rho = 0.1 \text{ fm}^{-3}) = 23.3 \pm (0.6)_{\text{exp.}} \pm (1.0)_{\text{theo.}}$ MeV
- α_D:
  $\Delta r_{np} = -0.156 \pm (0.014)_{\text{th.}} + \left[1.04 \pm (0.03)_{\text{exp.}} \pm (0.04)_{\text{th.}}\right] 10^{-2} J$
  $L = -145 \pm (9)_{\text{theo.}} + \left[6.07 \pm (0.18)_{\text{exp.}} \pm (0.26)_{\text{theo.}}\right] J$
  ... and assuming $J = 31 \pm (2)_{\text{est}}$ MeV:
  $\Delta r_{np} = 0.168 \pm (0.009)_{\text{exp.}} \pm (0.019)_{\text{theo.}} \pm (0.021)_{\text{est.}}$ fm
  $L = 43 \pm (6)_{\text{exp.}} \pm (12)_{\text{theo.}} \pm (12)_{\text{est.}}$ MeV

Co-workers:

G. Colò, P. F. Bortignon and M. Brenna (U. Milan, Italy)

M. Centelles and X. Viñas (U. Barcelona, Spain)

N. Paar and D. Vretenar (U. Zagreb, Croatia)

B. K. Agrawal (SINP, Kolkata, India)

W. Nazarewicz (U. Tennessee & ORNL, USA)

J. Piekarewicz (Florida State University, USA)

P.-G. Reinhard (Universität Erlangen-Nürnberg, Germany)

L. Cao (IMP-CAS, Lanzhou, P.R. China)

EXTRA MATERIAL

Dipole polarizability

Insights from a macroscopic approach

Given that **only the m_{-1}** moment is **required** for the calculation of the **dipole polarizability**, one may perform a constrained calculation

$$\delta\{\langle\mathcal{H}\rangle - \lambda\langle\mathcal{D}\rangle\} = 0$$

This defines the *constrained energy* $E(\lambda)$. **The dielectric theorem** establishes that the m_{-1} moment may be computed as

$$m_{-1}(E1) = \frac{1}{2}\left.\frac{\partial^2 E(\lambda)}{\partial\lambda^2}\right|_{\lambda=0}$$

Applying **this procedure** in combination with the **droplet model** approach of Myers and Swiatecki[†] yields the following result[††]:

$$\alpha_D = \frac{8\pi}{9}e^2\frac{A\langle r^2\rangle}{48J}\left(1 + \frac{5}{3}\frac{9J}{4Q}A^{-1/3}\right)$$

† W. Myers and W. Swiatecki, Annals of Physics 84, 186 (1974)

†† J. Meyer, P. Quentin, and B. Jennings, Nuclear Physics A 385, 269 (1982)

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$$t \equiv \frac{3r_0}{2}\,\frac{J/Q}{1+\frac{9}{4}\frac{J}{Q}A^{-1/3}}(I - I_C)$$

Q is the so-called surface stiffness coefficient, $I \equiv (N-Z)/A$ is the relative neutron excess, $\rho_0 = 3A/4\pi r_0^3$, $I_C = (e^2 Z)/(20JR)$, $R \equiv \sqrt{3/5}\,r_0 A^{1/3}$, and $\Delta r_{np}^{\text{surf}} = \sqrt{3/5}[5(b_n^2 - b_p^2)/(2R)]$ is a correction caused by the difference in the surface width b_n (b_p) of the neutron (proton) density profile

using these expressions:

$$\alpha_D \approx \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J}\left[1 + \frac{5}{2}\,\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Adopting a value of $J = 31 \pm 2$ MeV† one finds for ^{208}Pb that $I_c \approx 0.028 \pm 0.002$ $\sqrt{3/5}(e^2 Z)/(70J)$ is around 0.042 ± 0.003 fm. $\Delta r_{np}^{\text{surf}}$ for ^{208}Pb is almost constant (0.09 ± 0.01 fm) in EDFs††

In the DM Δr_{np} and L are better correlated with $\alpha_D J$ than with α_D alone in a heavy nucleus such as ^{208}Pb

† James M. Lattimer, Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

†† M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C 82, 054314 (2010).

Dipole polarizability: Correlations in EDFs

Insights from a macroscopic approach

Starting from the DM experessions

$$\alpha_D = \frac{8\pi}{9} e^2 \frac{A\langle r^2\rangle}{48J}\left(1 + \frac{5}{3}\frac{9J}{4Q}A^{-1/3}\right) \;\&\; a_{sym}(A) = \frac{J}{1+\frac{9J}{4Q}A^{-1/3}}$$

one can write

$$\alpha_D \approx \frac{8\pi}{9} e^2 \frac{A\langle r^2\rangle}{48J}\left(1 + \frac{5}{3}\frac{J - a_{sym}(A)}{J}\right)$$

and assuming that the **symmetry energy coefficient of a finite nucleus is very close to that of the infinite system**[†] at an **appropriate** sub-saturation density ρ_A: $a_{sym}(A) \approx S(\rho_A)$:

$$\alpha_D^{DM} \approx \frac{\pi e^2}{54}\frac{A\langle r^2\rangle}{J}\left[1 + \frac{5}{3}\frac{L}{J}\epsilon_A\right]$$

where $\epsilon_A \equiv \dfrac{\rho_0 - \rho_A}{3\rho_0}$ and $\epsilon_{208} = 1/8$ for $\rho_0 = 0.16$ fm-3 for the case of ^{208}Pb

[†] M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. 102, 122502 (2009)

Dipole polarizability: Correlations in EDFs

Covariance analysis within a model: theory

Given as set of observables $\mathcal{O}$ used to calibrate the parameters $\mathbf{p}$ of a given model, the optimum parametrization $\mathbf{p}_0$ is determined by a fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{m} \left(\frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the χ^2 is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned} \chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2 (p_j - p_{0j}) \\ &\equiv \sum_{i,j}^{n} (p_i - p_{0i}) \mathcal{M}_{ij} (p_j - p_{0j}) \end{aligned}$$

where $\mathcal{M}$ is the curvature matrix.

Dipole polarizability: Correlations in EDFs

Covariance analysis within a model: theory

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(\mathbf{p})$),

$$\Delta\mathcal{A} = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A} \tag{1}$$

$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

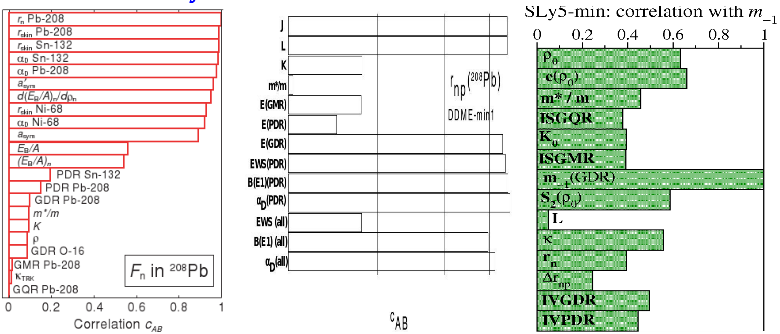
$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}} \tag{2}$$

where,

$$C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$$

Dipole polarizability: correlations in EDFs

Covariance analysis within a model: results



From left to right: SV: P.-G. Reinhard and W. Nazarewicz, Phys. Rev. C 81, 051303(R) (2010); DD-ME1: Nils talk at the INPC 2013; SLy5: X. Roca-Maza

Using the **experimental value** $\alpha_D = 20.1 \pm 0.6$ **fm**^{3} † in ^{208}Pb the **covariance analisis of SV model**, a value $\Delta r_{np} = 0.156^{+0.025}_{-0.021}$ **fm** was found†.

† A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011)