Dipole Polarizability and Parity Violating Asymmetry in ^{208}Pb

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Motivation:

The importance of determining isovector properties in nuclei

- **In the past** (and also in the present), **neutron properties** in stable medium and heavy nuclei have been mainly measured by using **strongly interacting probes**.

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  Limited knowledge of isovector properties

- **At present**,
  - the use of **rare ion beams** has opened the possibility of measuring properties of **exotic nuclei**.
  - **parity violating elastic electron scattering** (PVES), a **model independent technique**, has allowed to estimate the **neutron radius** of a stable heavy nucleus like ^{208}Pb (PREx@JLab).

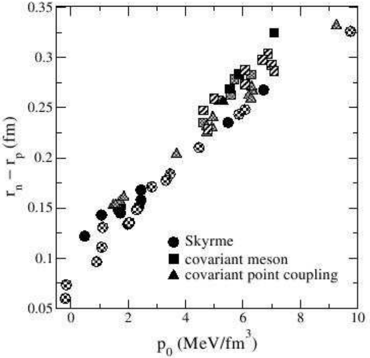
  $\Downarrow$

  Promising perspectives for the near future.

Motivation:

It is possible to connect observables with general isovector properties of the nuclear effective interaction?

Example:
Mean-Field predictions show a clear **correlation** between Δr_{np} of a medium and heavy nucleus and the density slope of the symmetry energy ($L = 3\rho_0 \partial_\rho S(\rho)|_{\rho_0} = 3\rho_0 p_0$).



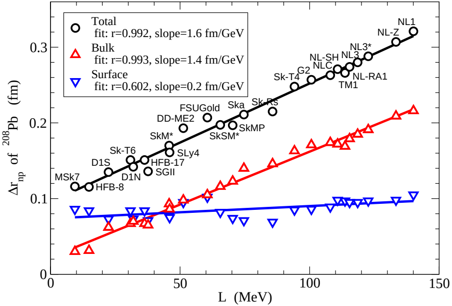
R.J. Furnstahl, NPA, **706**, 85 (2002)

Motivation:

More generally within **MF**, it has been found a semi-empirical law: $a_{\rm sym}(A) \approx S(\rho_A)$ with $\rho_A = \rho_0 - \rho_0/(1 + cA^{1/3}) \Rightarrow$

direct and clear connection of any ground state isospin sensitive **observable** with the parameters of the **EoS**.

Following the same example: $\Delta r_{np}^{\rm total}(A,I) = \Delta r_{np}^{\rm bulk}(A,I) + \Delta r_{np}^{\rm surface}(A,I)$



$$\Delta r_{np}^{\rm bulk}(A,I) \approx \frac{2r_0}{3J}\, L \left(1 - \epsilon_A \frac{K_{\rm sym}}{2L}\right) \epsilon_A A^{1/3} (I - I_{\rm C})$$

M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. **102**, 122502 (2009); Phys. Rev. C **80** 024316 (2009); Phys. Rev. C **81** 054309 (2010) and Phys. Rev. C **82**, 054314 (2010)

Motivation:

Observables, processes and observations known to be correlated with the isovector properties of the nuclear effective interaction

- **Binding energies**
- **Neutron distributions** (proton elastic scattering, antiprotonic atoms, parity violating asymmetry,...)
- **Giant Resonances**: Giant Dipole, Gamow-Teller, Isobaric Analog, Spin Dipole and Anti-analog of the Giant Dipole Resonances (inelastic hadron-nucleus, nucleus-nucleus and γ-nucleus scattering).
- **Heavy Ion Collisions** (EoS — transport models)
- **Neutron Star properties**: mass-radius relation, transition density crust-core, composition,... (observational data).
- Low-energy dipole response (?)
- Isovector Giant Quadrupole Resonance (?)
- Isoscalar Giant Resonances along isotopic chains (?)
- ...

Isovector static dipole polarizability

Definition: α_D

- The linear response or dynamic polarizability of a nuclear system excited from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the action of an external oscillating dipolar field of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_D = \frac{Z}{A}\sum_{i}^{N} r_n Y_{1M}(\hat{r}_n) - \frac{N}{A}\sum_{i}^{Z} r_p Y_{1M}(\hat{r}_p)$$

- is proportional to the **static dipole polarizability,** α_D**,** for small oscillations

$$\alpha_D = \frac{8\pi}{9}e^2 m_{-1} = \frac{8\pi}{9}e^2 \sum_{\nu} \frac{|\langle\nu|F_D|0\rangle|^2}{E}$$

where m_{-1} is the inverse energy weighted moment of the strength function,

$$S_D(E) = \sum_{\nu} |\langle\nu|F_D|0\rangle|^2 \delta(E - E_\nu)$$

Macroscopic approach: α_D

- Being E the energy of a nucleus within the Liquid Drop Model, Droplet Model,... and F_{ext} an external field (dipole operator) $\rightarrow$ constrained calculation keeping N and Z fixed:

$$\delta \left\{ E(\rho,\delta) + F_{ext}(\Lambda,\rho,\delta) - \lambda_n \int_0^\infty \rho_n(r) d\mathbf{r} - \lambda_p \int_0^\infty \rho_p(r) d\mathbf{r} \right\} = 0$$

- **LDM**[1]:

$$m_{-1} \approx \frac{\mathbf{A \langle r^2 \rangle^{1/2}}}{\mathbf{48J}}$$

- **DM**[2]:

$$m_{-1} \approx \frac{\mathbf{A \langle r^2 \rangle^{1/2}}}{\mathbf{48J}} \left(1 + \frac{\mathbf{15}}{\mathbf{4}} \frac{\mathbf{J}}{\mathbf{Q}} \mathbf{A}^{-1/3} \right)$$

[1] A.B . Migdal, J. Phys. USSR **8** (1944) 331; J.S . Levinger, Nuclear photodisintegration (Oxford Univ . Press, London, 1960) sect. 3-1.

[2] J. Meyer, P. Quentin, and B. K. Jennings, Nucl. Phys. A **386** (1982) 269.

Droplet model approach: symmetry energy and neutron skin

- The symmetry energy with surface effects is written in the DM:

$$a_{\rm sym}(A) = \frac{J}{1+\frac{9}{4}\frac{J}{Q}A^{-1/3}},$$

- while the neutron skin thickness (also including surface effects),

$$\begin{aligned} \Delta r_{np} &= \sqrt{\frac{3}{5}}\left[t - \frac{e^2 Z}{70J}\right] + \Delta r_{np}^{\rm surface} \\ t &\equiv \frac{3r_0}{2}\frac{J/Q}{1+\frac{9}{4}\frac{J}{Q}A^{-1/3}}(I - I_C), \end{aligned}$$

$I \equiv (N-Z)/A$
$I_C \equiv (e^2 Z)/(20JR)$
$R = r_0 A^{1/3}$

$\Delta r_{np}^{\rm surface} = \sqrt{(3/5)}[5(b_n^2 - b_p^2)/(2R)]$ is a correction caused by the difference in the surface widths b_n and b_p of the neutron and proton density profiles

Droplet model approach: connection between α_D and the neutron skin

- Combining these formulas,

$$\alpha_{\mathbf{D}} \approx \frac{A\langle r^2\rangle}{12\mathbf{J}}\left[1+\frac{5}{2}\frac{\mathbf{\Delta r_{np}}+\sqrt{\frac{3}{5}}\frac{e^2 Z}{70J}-\Delta r_{np}^{\rm surface}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$

- For a heavy nucleus and assuming small variations† of $\langle r^2\rangle$, $e^2Z/70J$ and $\Delta r_{np}^{\rm surface}$ as compared to that of J and Δr_{np},

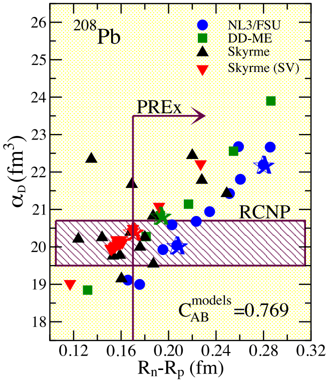
$$\alpha_D J \approx p_1 + p_2 \Delta r_{np}$$

†Some numbers: ^{208}Pb and assuming $J = 32 \pm 2$ MeV, $\rho_0 = 0.160 \pm 0.05$ fm^{-3} and

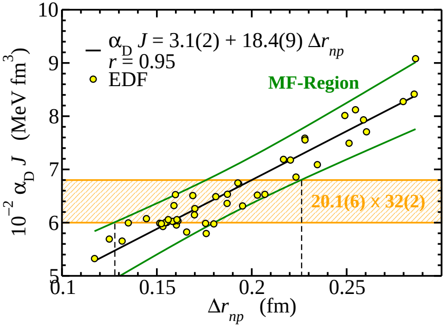
$\Delta r_{np}^{\rm surface} \approx 0.09 \pm 0.01$ (MF models) $\Rightarrow (e^2Z)/(70J) - \Delta r_{np}^{\rm surface} \approx -0.04 \pm 0.01$ fm

$\langle r^2\rangle^{1/2} \approx 5.23 \pm 0.55$ fm, $I_c \approx 0.027 \pm 0.003$

Mean-Field + RPA results for ^{208}Pb



J. Piekarewicz, B. K. Agrawal, G. Colò, W. Nazarewicz, N. Paar, P.-G. Reinhard, X. Roca-Maza and D. Vretenar, Phys. Rev. C **85** 041302 (2012) (R).



$$\Delta r_{np} \approx 0.18 \pm 0.05 \text{ fm}$$

assuming $J = 32 \pm 2$ MeV and α_D from A. Tamii *et al.* Phys. Rev. Lett. **107**, 062502 (2011)

Parity violating elastic electron scattering in ^{208}Pb

From previous talks, we have seen that,

- **Electrons** interact by exchanging a γ or a Z_0 boson.
- While **protons** couple basically to γ, **neutrons** do it to Z_0.
- **Ultra-relativistic electrons**, depending on their helicity, interact with the nucleons $V_{\pm} = V_{\mathrm{Coulomb}} \pm V_{\mathrm{Weak}}$.
- **Ultra-relativistic electrons** moving under the effect of $V_{\pm}$ where **Coulomb distortions** are important $\Rightarrow$ solution of the Dirac equation via the Distorted Wave Born Approximation (**DWBA**).
- **Input for the calculation: ρ_n and ρ_p**

Refs: C. J. Horowitz, Phys. Rev. C **57** 3430 (1998); C. J. Horowitz, S. J. Pollock, P. A. Souder, and R. Michaels, Phys. Rev. C **63**, 025501 (2001); M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C **82**, 054314 (2010); X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, Phys. Rev. Lett. **106** 252501 (2011) and (for the electric proton and neutron form factors) J. Friedrich and Th. Walcher, Eur. Phys. J. A **17**, 607623 (2003)

PREx data analysis:

- **PREx** measures, model-independently, the **parity violating asymmetry** at 1.06 GeV and for a single angle (~ 5 deg.) in ^{208}Pb,

$$\mathbf{A_{pv}} = \left(\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}\right) \Big/ \left(\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}\right)$$

- ρ_n of ^{208}Pb is the quantity to be determined, so...
- **Which is the optimal neutron distribution in reproducing $A_{pv}(E_{\mathrm{beam}} = 1.06$ GeV,$\theta = 5$ deg) from PREx?**

Analysis I:

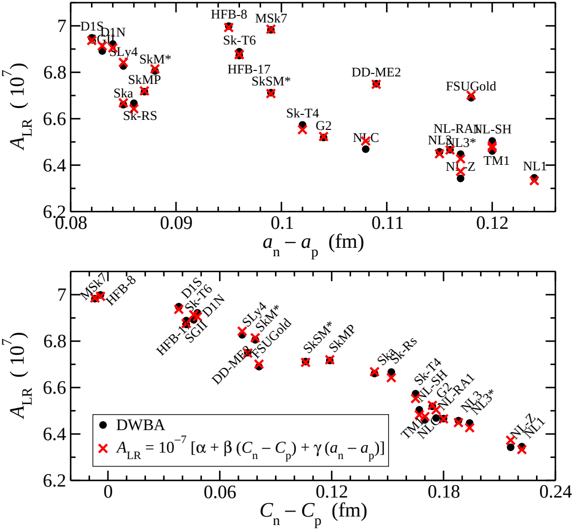
- Fix ρ_p with the experimental data on elastic electron scattering.
- Assume the electric proton and neutron form factors from other experiments.
- Assume a parametrized density for the neutron distribution such as a two parameter Fermi function, 2pF:

$$\rho_n(r) = \frac{\rho_{0n}}{1 + e^{(r - C_n)/a_n}} \qquad \int_0^\infty \rho_n(r) d\mathbf{r} = N$$

- Fit the parameters (e.g. C_n and a_n), keeping the number of neutrons fixed, to the data on A_{pv}

Problem: One can only fix one of the parameters since A_{pv} is known at only one scattering angle.
Solution: Fix a range for the other parameter based on theoretical calculations.
Problem: Model dependence is introduced.

Analysis II: 2pF densities fitted to MF ρ_n and ρ_p



M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C **82**, 054314 (2010)

A qualitative guide to understand the previous correlation:

- A_{pv} within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{\mathbf{F_n(q) - F_p(q)}}{F_p(q)}\right]$$

- ... which depends on $\mathbf{F_n(q) - F_p(q)}$ that for a 2pF and $q \to 0$, it is approximately,

$$-\frac{q^2}{6}\left[\frac{3}{5}(\mathbf{C_n - C_p})(C_n + C_p) + \frac{7\pi}{5}(\mathbf{a_n - a_p})(a_n + a_p)\right]$$

- variation of A_{pv} dominated by $(C_n - C_p)$ and $(a_n - a_p)$.

In this case: C_p and a_p fixed from the experiment; C_n from the value of A_{pv} and the rather good linear dependence shown in the lower panel of the last figure; a_n using the correlation shown (calibrated with MF).

Problem: Model dependence.

The solution to this problem: measurements of A_{pv} at **different angles**

$$\Downarrow$$

model-independent determination of Δr_{np}

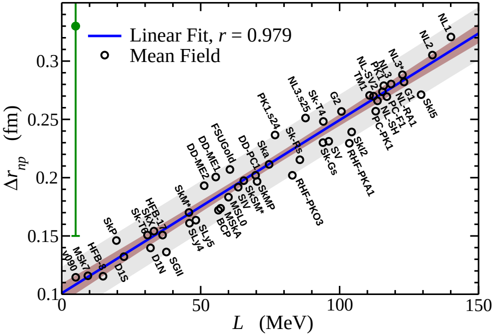
In case in which it is not possible, **we propose the following analysis...**

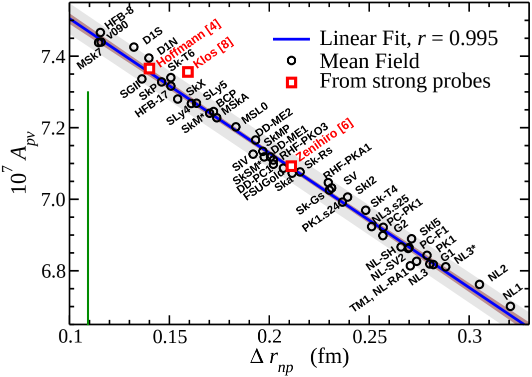
Analysis III: direct correlations within MF

X. Roca-Maza, M. Centelles, X. Viñas, and

M. Warda, Phys. Rev. Lett. **106** 252501

(2011)

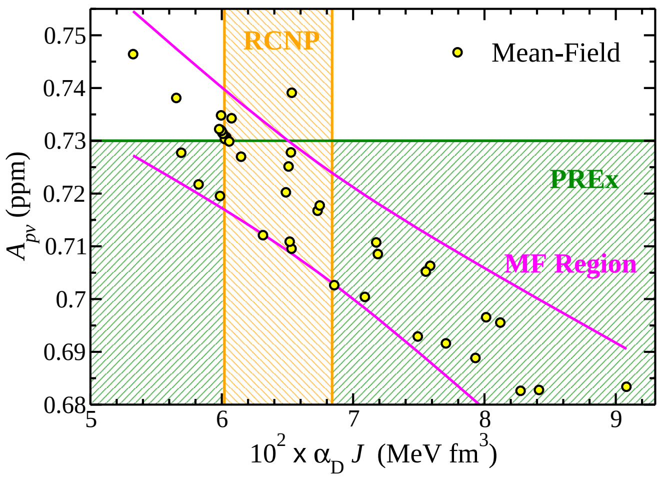




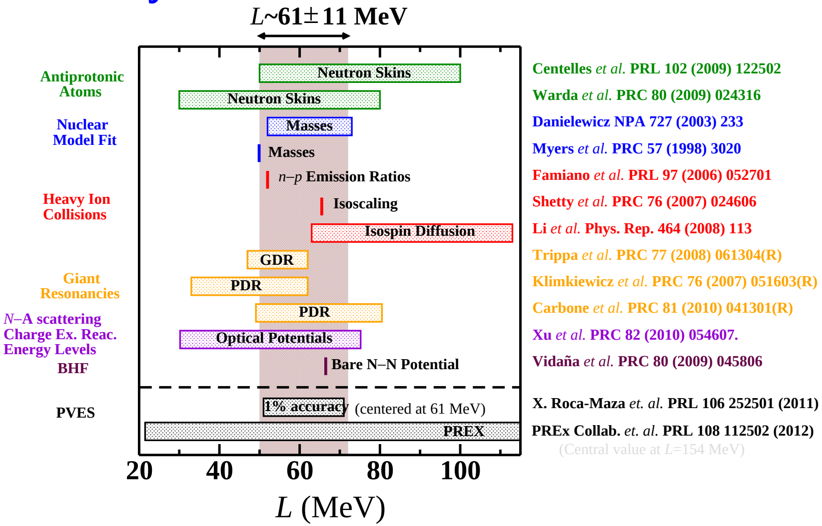
MF correlations allows to determine Δr_{np} and L without direct assumptions on ρ

Different experiments on proton elastic scattering and antirpotonic atoms agrees with the correlation

Constraints set by A_{pv} measured at JLab and α_D measured at RCNP on MF calculations.



Summary of different constraints on L



Conclusions:

- **Families of MF** models predict a high linear **correlation** between α_D and Δr_{np} in ^{208}Pb.
- The **Droplet Model** suggest a linear **correlation** between $\alpha_D J$ and Δr_{np}. **(This result should be analyzed more precisely on the basis of microscopic calculations)**

Conclusions:

- A **model-independent** determination of Δr_{np} in ^{208}Pb via PVES experiments would need a measurement of A_{pv} at **different scattering angles.**
- We demonstrate a close **linear correlation** between A_{pv} and Δr_{np} within the same framework in which the Δr_{np} is correlated with L. This allows to **determine unambiguously** the value of Δr_{np} and L within the **MF framework**.
- Other **experiments** fairly **agree** with the **correlation** between A_{pv} and Δr_{np}.
- A_{pv} measured by the PREx collaboration at JLab and α_D measured at RCNP are complementary **observables** that may set tight **constraints** on the **density dependence of the symmetry energy.**
- Most of the **empirical estimations of L** agrees in a range for its central value that lies within **50 and 70 MeV**.

Collaborators:

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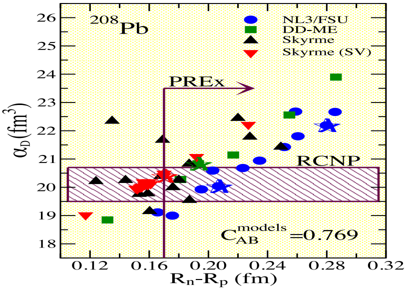
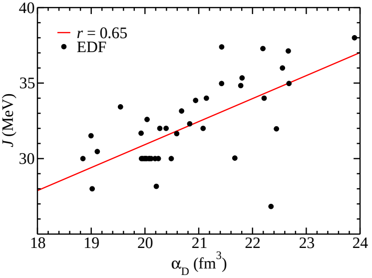
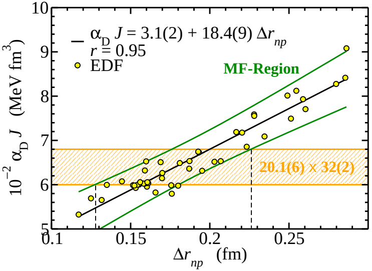
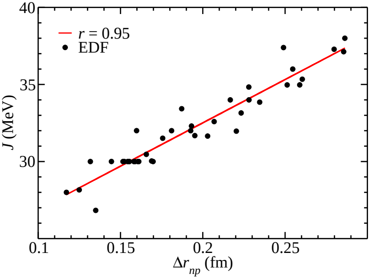
Extra Material

Correlations: J, α_D and Δr_{np}

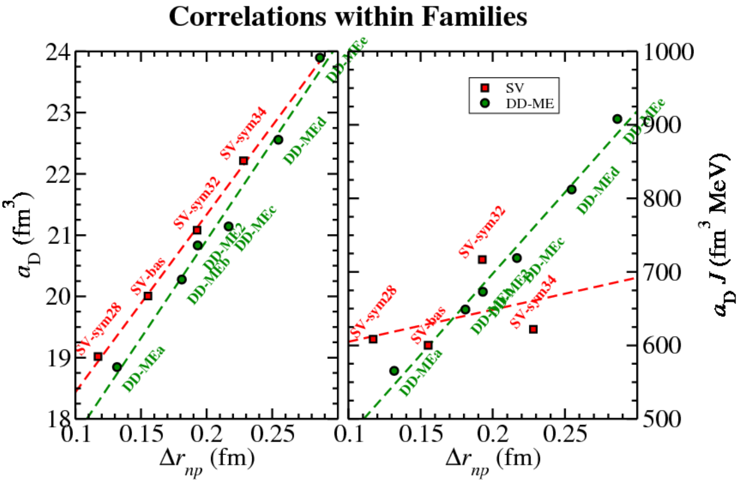








Correlations: J, α_D and Δr_{np}



DM versus other calculations and experiment

TABLE 2

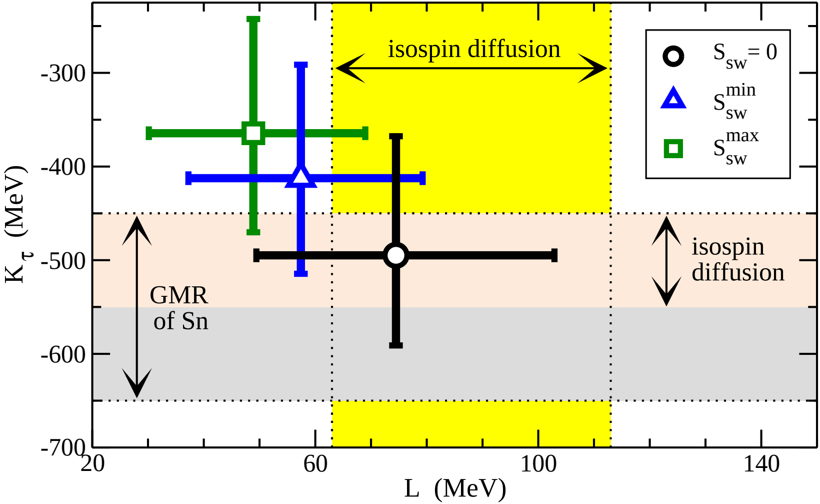
Total photoabsorption cross sections $\sigma_{-2}A^{-5/3}$ (in $\mu b \cdot MeV^{-1}$)

	GT	Migdal	Droplet	RPA	CHF	Exp
^{16}O	4.08	2.31	4.26	5.7	5.0	5.8 ± 0.1 [a])
	4.70	2.15	4.81	6.5	5.6	
^{40}Ca	3.06	2.08	3.42	4.4	3.9	4.8 ± 0.1 [a])
	3.47	1.89	3.67	4.8	4.3	
^{208}Pb	1.76	1.90	2.69	3.0	2.6	2.6 ± 0.2 [b])
	2.00	1.69	2.73	3.3	2.8	

The Goldhaber–Teller (GT), Migdal and droplet model values are compared with RPA and constrained Hartree–Fock (CHF) results extracted from ref. [18]). Experimental values are also listed [a]): ref. [1]) and [b]): ref. [2]). The upper (lower) line displays the theoretical results obtained with the SIII (SkM) force. The fact that we take the Hartree–Fock value for r and not a liquid-drop estimate $r_0A^{1/3}$ explains the variation of the Migdal $\sigma_{-2}A^{-5/3}$ value with A.

J. Meyer, P. Quentin, and B. K. Jennings, Nucl. Phys. A **386** (1982) 269

K_τ versus L from isospin difusion, GMR and neutron skin data



Note: $K_\tau \approx Ksym - 6L$ has been estimated as in isospin difussion and GMR data only for comparison purposes

M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. **102**, 122502 (2009)
L. W. Chen, C. M. Ko, and B. A. Li, Phys. Rev. Lett. **94**, 032701 (2005); Phys. Rev. C72, 064309 (2005)
B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008)

T. Li et al, Phys. Rev. Lett. **99**, 162503 (2007)

Covariance analysis: χ^2 test

Observables $\mathcal{O}$ are used to calibrate the parameters $\mathbf{p}$ of a given model. The optimum parametrization $\mathbf{p}_0$ is determined by a least-squares fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{\imath=1}^{m} \left(\frac{\mathcal{O}_\imath^{\text{theo.}} - \mathcal{O}_\imath^{\text{ref.}}}{\Delta\mathcal{O}_\imath^{\text{ref.}}} \right)^2$$

Assuming that the χ^2 is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned} \chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{\imath,\jmath}^{n} (p_\imath - p_{0\imath}) \partial_{p_\imath} \partial_{p_\jmath} \chi^2 (p_\jmath - p_{0\jmath}) \\ &\equiv \sum_{\imath,\jmath}^{n} (p_\imath - p_{0\imath}) \mathcal{M}_{\imath\jmath} (p_\jmath - p_{0\jmath}) \end{aligned}$$

where $\mathcal{M}$ is the curvature matrix.

Covariance analysis: χ^2 test

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(\mathbf{p})$),

$$\Delta\mathcal{A} = \sqrt{\sum_{\imath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\imath} \partial_{p_\imath} A} \quad (1)$$

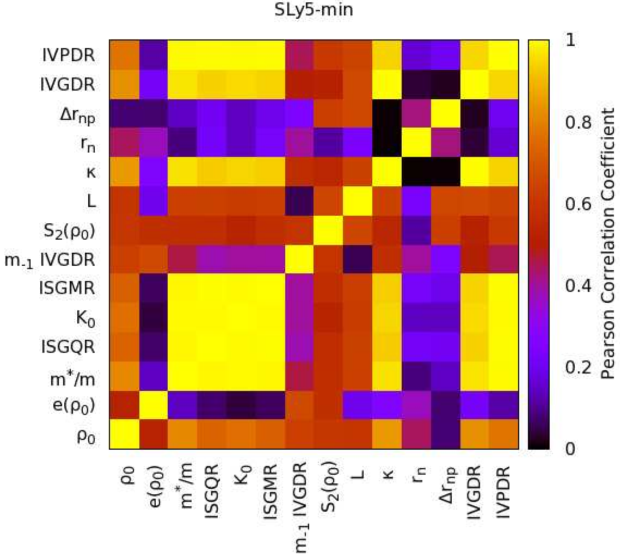
$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}} \quad (2)$$

where,

$$C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{\imath\jmath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\jmath} \partial_{p_\jmath} B$$

Covariance analysis: SLy5-min as an example



Covariance analysis: SLy5-min as an example

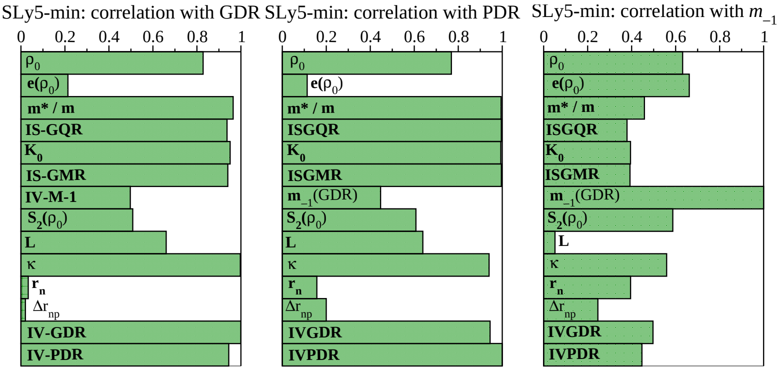


Figure: Pearson product-moment correlation coefficient for the IVGDR (left panel), IVPDR (middle panel) and m_{-1}(IVGDR) (right panel) with all other studied properties as predicted by the covariance analysis of SLy5.