Dipole Polarizability and Parity Violating Asymmetry in ^{208}Pb

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Motivation:

The importance of determining isovector properties in nuclei

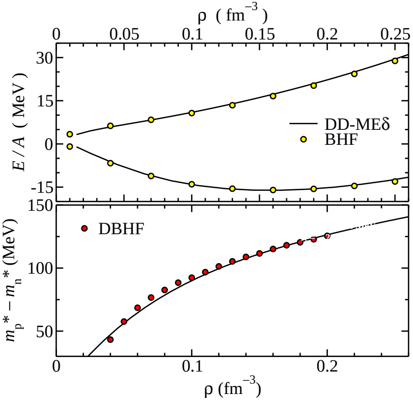
In the past (and also in the present), **neutron properties** in stable medium and heavy nuclei have been mainly measured by using **strongly interacting probes**.

$\Downarrow$

Limited knowledge of isovector properties

Isovector channel of current **effective theories** need to be **fixed by other means**.
Example: fixed to *ab-initio* calculations of the N-N inmedium interaction.

X. Roca-Maza, X. Viñas, M. Centelles, P. Ring and P. Schuck, Phys. Rev. C **84** 054309 (2011).

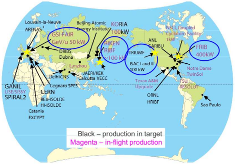


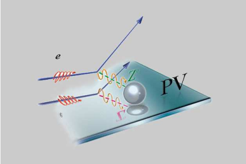
Motivation:

At present,
The use of **rare ion beams** has opened the possibility of measuring properties of **exotic nuclei**.
Parity violating elastic electron scattering (PVES), a **model independent technique**, has allowed to estimate the **neutron radius** of a stable heavy nucleus like ^{208}Pb (PREx@JLab).

$\Downarrow$

Promising perspectives for the near future.





http://hallaweb.jlab.org/experiment/HAPPEX/

http://www.lsw.uni-heidelberg.de/nic2010/talks/Kruecken.pdf

Motivation:

It is possible to connect observables with general isovector properties of the nuclear effective interaction?

Example:

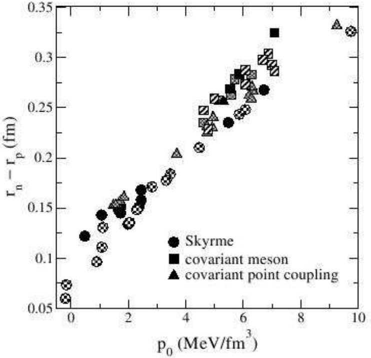
Mean-Field predictions show a clear **correlation** between

$\Delta r_{np} = \langle r_n \rangle^{1/2} - \langle r_p \rangle^{1/2}$

of a medium and heavy nucleus and the density slope L of the symmetry energy,

$S(\rho) \approx e_{\rm neut}(\rho) - e_{\rm sym}(\rho)$:

$L = 3\rho_0 \partial_\rho S(\rho)|_{\rho_0} = 3\rho_0 p_0$



R.J. Furnstahl, NPA, **706**, 85 (2002)

Motivation:

Observables, processes and observations known to be correlated with the isovector properties of the nuclear effective interaction

- **Binding energies**
- **Neutron distributions** (proton elastic scattering, antiprotonic atoms, parity violating asymmetry,...)
- **Giant Resonances**: Giant Dipole, Gamow-Teller, Isobaric Analog, Spin Dipole and Anti-analog of the Giant Dipole Resonances (inelastic hadron-nucleus, nucleus-nucleus and γ-nucleus scattering).
- **Heavy Ion Collisions** (EoS — transport models)
- **Neutron Star properties**: mass-radius relation, transition density crust-core, composition,... (observational data).
- Low-energy dipole response (?)
- Isovector Giant Quadrupole Resonance (?)
- Isoscalar Giant Resonances along isotopic chains (?)
- ...

Isovector static dipole polarizability

Definition: α_D

- The linear response or dynamic polarizability of a nuclear system excited from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the action of an external oscillating dipolar field of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_D = \frac{Z}{A}\sum_i^N r_n Y_{1M}(\hat{r}_n) - \frac{N}{A}\sum_i^Z r_p Y_{1M}(\hat{r}_p)$$

- is proportional to the **static dipole polarizability, α_D,** for small oscillations

$$\alpha_D = \frac{8\pi}{9}e^2 m_{-1} = \frac{8\pi}{9}e^2 \sum_\nu \frac{|\langle\nu|F_D|0\rangle|^2}{E}$$

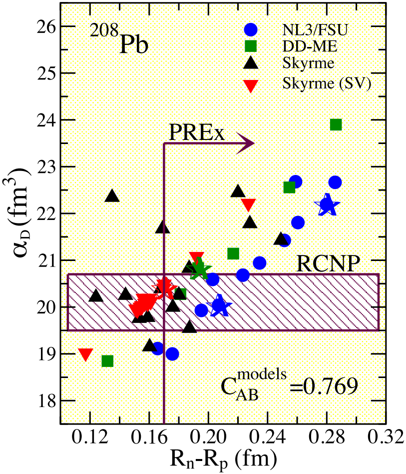
where m_{-1} is the inverse energy weighted moment of the strength function,

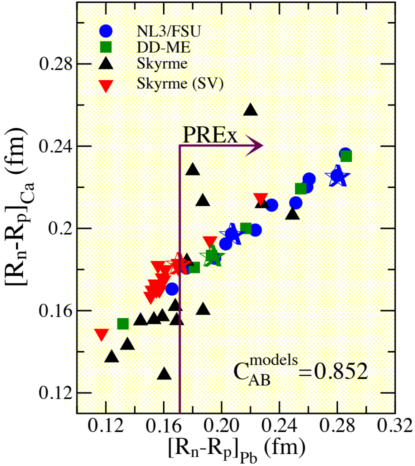
$$S_D(E) = \sum_\nu |\langle\nu|F_D|0\rangle|^2 \delta(E - E_\nu)$$

Some considerations on the α_D in nuclei

- The **restoring force** in the isovector dipole response is proportional to the **symmetry energy**, **S**.
- Larger symmetry energies at saturation shift the excitation energies to lower values (MF).
- The **strength increases** as a consequence of previous point and the **conserved** m_1 sum rule.
- Since m_{-1} is **not a conserved** quantity, and **weights more the low energy** region, the **effect** of the the **low-lying** states on the α_D is **not negligible**.
- "$E_{\mathrm{dipole}} \sim \left(E^2_{\mathrm{unpert}} + c \times S_{\mathrm{pot.}}\right)^{1/2}$" (Bohr & Mottelson) $\rightarrow$ E_{dipole} expected to be **correlated** with the m^* (E_{unpert} depends on the level density and, therefore, on the effective mass) and the symmetry energy parameters J and/or L.
- **Qualitatively**: assuming **small variations** of m^* and J in MF calculations when compared to the theoretical **spread** in L $\rightarrow$ suggest a **correlation** between α_D and Δr_{np}.

Mean-Field + RPA results for ^{208}Pb





$\Delta r_{np}(^{208}\text{Pb}) \approx 0.168 \pm 0.022$ fm

$\Delta r_{np}(^{48}\text{Ca}) \approx 0.176 \pm 0.018$ fm and

$\alpha_D(^{48}\text{Ca}) \approx 2.3 \pm 0.1$ fm^3

J. Piekarewicz, B. K. Agrawal, G. Colò, W. Nazarewicz, N. Paar, P.-G. Reinhard, X. Roca-Maza and D. Vretenar, Phys. Rev. C **85** 041302 (2012) (R).

Parity violating elastic electron scattering in ^{208}Pb

Theoretical bases of PVES:

- **Electrons** interact by exchanging a γ or a Z_0 boson.
- While **protons** couple basically to γ, **neutrons** do it to Z_0.
- **Ultra-relativistic electrons**, depending on their helicity, interact with the nucleons $V_{\pm} = V_{\rm Coulomb} \pm V_{\rm Weak}$.
- **Coulomb distortions** should be taken into account: **DWBA** calculations give **$\sim$30%** correction with respect to PWBA.

Refs: C. J. Horowitz, Phys. Rev. C **57** 3430 (1998); C. J. Horowitz, S. J. Pollock, P. A. Souder, and R. Michaels, Phys. Rev. C **63**, 025501 (2001); M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C **82**, 054314 (2010); X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, Phys. Rev. Lett. **106** 252501 (2011) and (for the electric proton and neutron form factors) J. Friedrich and Th. Walcher, Eur. Phys. J. A **17**, 607623 (2003)

PREx data analysis:

- **PREx** measures, model-independently, the **parity violating asymmetry** at 1.06 GeV and for a single angle ($\sim$ 5 deg.) in ^{208}Pb,

$$\mathbf{A_{pv}} = \left(\frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega}\right) \Big/ \left(\frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega}\right)$$

- **Input for the calculation: ρ_n and ρ_p**
- ρ_p of ^{208}Pb is well known from other experiments
- ρ_n of ^{208}Pb is the quantity to be determined

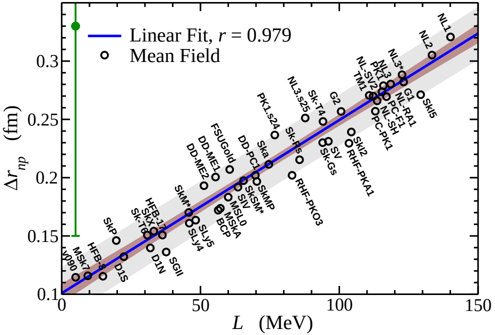
Problem: In the analysis, one can only fix one paramter of the adopted neutron distribution to the data on A_{pv}.
Solution: Fix a range for the other parameter/s based on theoretical calculations.
Problem: Model dependence is introduced.
Solution: measurements of A_{pv} at different angles (measuring more nuclei would also help).

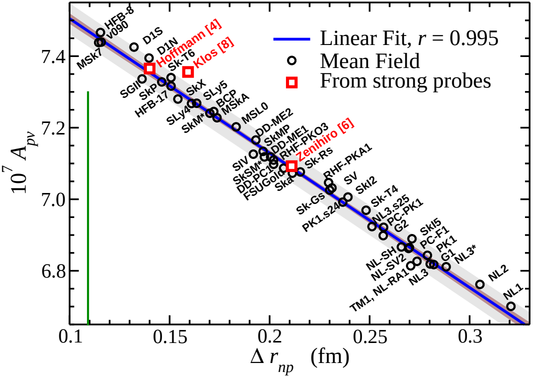
In case in which a measurement of A_{pv} at different angles is not possible/available,

we propose the following analysis:

Direct correlations within MF

X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, Phys. Rev. Lett. **106** 252501 (2011)

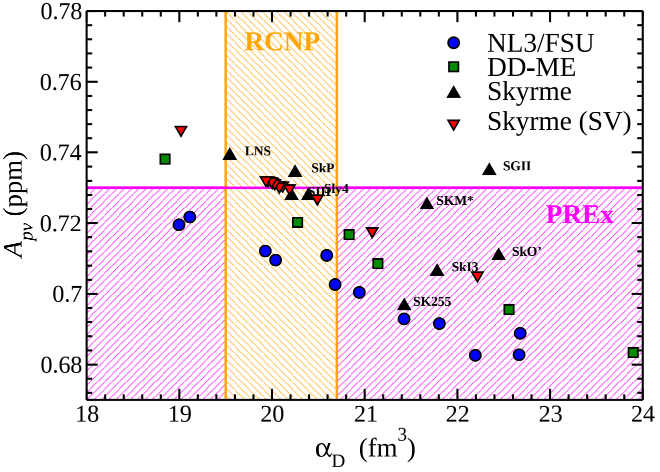




MF correlations allows to determine Δr_{np} and L without direct assumptions on ρ

Different experiments on proton elastic scattering and antirpotonic atoms agrees with the correlation

Constraints set by A_{pv} measured at JLab and α_D measured at RCNP on MF calculations.



Conclusions:

- **Families of MF** models predict a high linear **correlation** between α_D and Δr_{np} in ^{208}Pb (m^* and other properties except J and L have been fixed).
- **Further** exeprimental and theoretical **studies** on α_D are needed for a **better physical understanding** on the **properties** of the nuclear **effective interaction** (m^*, J, L,...) that are determing this observable.

- A **model-independent** determination of Δr_{np} in ^{208}Pb via PVES experiments would need a measurement of A_{pv} at **different scattering angles.**
- We demonstrate a **linear correlation** between A_{pv} and Δr_{np}.
- Other **experiments** fairly **agree** with the **correlation** between A_{pv} and Δr_{np}.
- A_{pv} measured by the PREx collaboration at JLab and α_D measured at RCNP are complementary **observables** that may set tight **constraints** on the **density dependence of the symmetry energy.**

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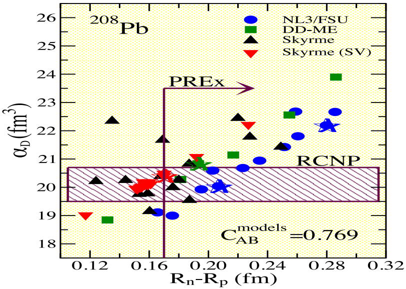
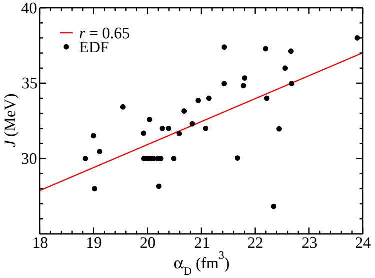
Extra Material

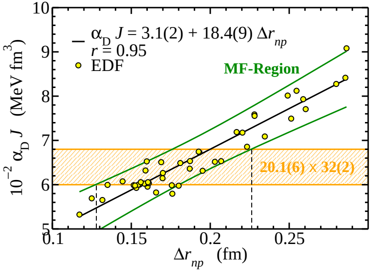
Why ^{208}Pb?

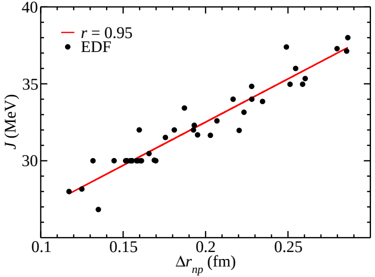
- ^{208}Pb is a **stable**, spin zero nucleus with **large neutron asymmetry**
- ^{208}Pb has the advantage that it has the **largest known splitting to the first excited** of any heavy nucleus.
- **Well known structure** since it has been extensively studied (**spherical, no pairing, no deformation...**)
- **EDF** are expected to be **accurate** in the description of **average porperties** (Kohn-Sham)
- **Charge radii** (average property) in the region of **Pb** are **well described by EDF**.
- The **correlation** between the **neutron skin thickness** and the slope of the **nuclear symmetry energy** have been demosntrated to **exist within the EDF** framework.
- Most of the existent **EDF** have been **fitted to spherical** and also quite frequently semi- or **double-magic** nuclei.

Correlations: J, α_D and Δr_{np}

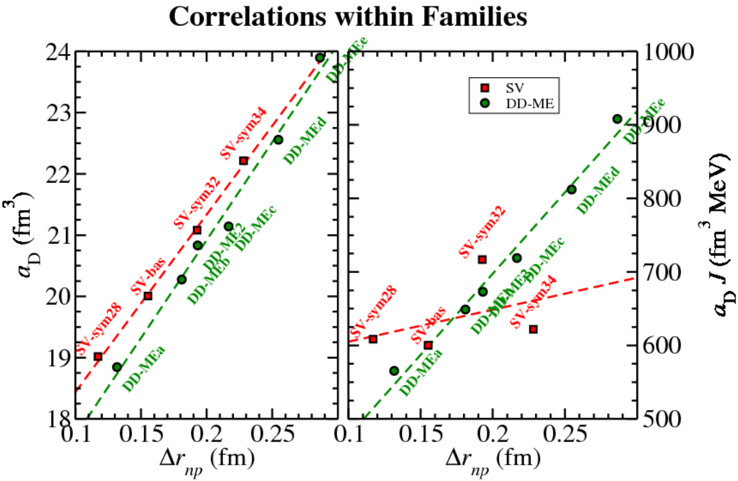








Correlations: J, α_D and Δr_{np}



Covariance analysis: χ^2 test

Observables $\mathcal{O}$ are used to calibrate the parameters $\mathbf{p}$ of a given model. The optimum parametrization $\mathbf{p}_0$ is determined by a least-squares fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{\imath=1}^{m} \left(\frac{\mathcal{O}_\imath^{\text{theo.}} - \mathcal{O}_\imath^{\text{ref.}}}{\Delta \mathcal{O}_\imath^{\text{ref.}}} \right)^2$$

Assuming that the χ^2 is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\begin{aligned} \chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) &\approx \frac{1}{2} \sum_{\imath,\jmath}^{n} (p_\imath - p_{0\imath}) \partial_{p_\imath} \partial_{p_\jmath} \chi^2 (p_\jmath - p_{0\jmath}) \\ &\equiv \sum_{\imath,\jmath}^{n} (p_\imath - p_{0\imath}) \mathcal{M}_{\imath\jmath} (p_\jmath - p_{0\jmath}) \end{aligned}$$

where $\mathcal{M}$ is the curvature matrix.

Covariance analysis: χ^2 test

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(\mathbf{p})$),

$$\Delta \mathcal{A} = \sqrt{\sum_{\imath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\imath} \partial_{p_\imath} A} \tag{1}$$

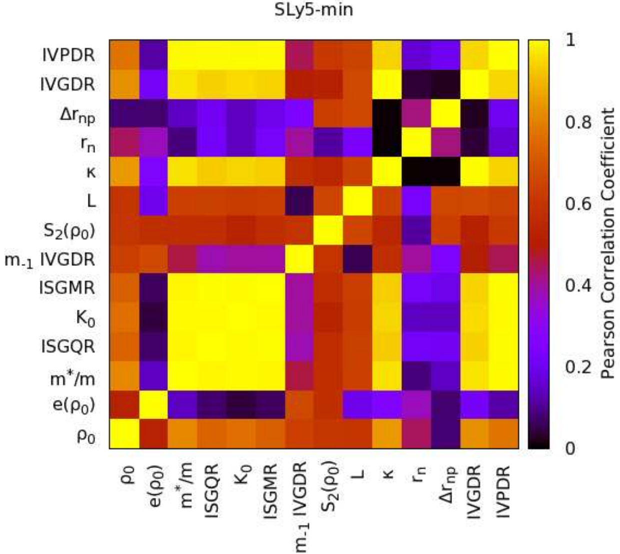
$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA} C_{BB}}} \tag{2}$$

where,

$$C_{AB} = \overline{(A(\mathbf{p}) - \overline{A})(B(\mathbf{p}) - \overline{B})} \approx \sum_{\imath\jmath}^{n} \partial_{p_\imath} A \mathcal{E}_{\imath\jmath} \partial_{p_\jmath} B$$

Covariance analysis: SLy5-min as an example



Covariance analysis: SLy5-min as an example

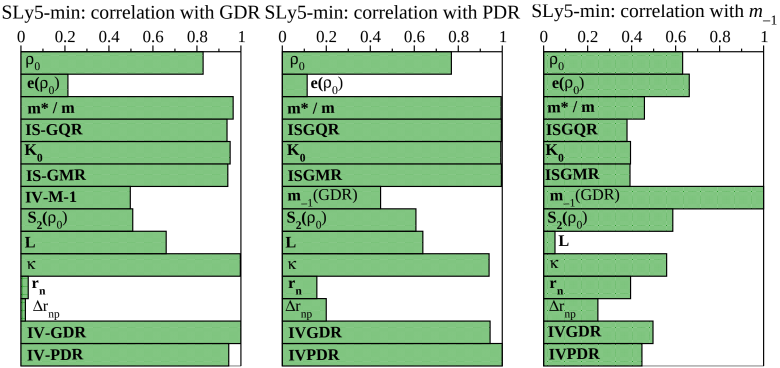


Figure: Pearson product-moment correlation coefficient for the IVGDR (left panel), IVPDR (middle panel) and m_{-1}(IVGDR) (right panel) with all other studied properties as predicted by the covariance analysis of SLy5.