The Nuclear Symmetry Energy: constraints from Giant Resonances

Xavier Roca-Maza

Dipartimento di Fisica, Università degli Studi di Milano
and
INFN, Sezione di Milano

Trobada de Nadal 2012

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INTRODUCTION

The Nuclear Many-Body Problem:

- **Nucleus:** from few to more than 200 strongly interacting and **self-bound fermions**.
- **Underlying interaction** is **not perturbative** at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: **spin, isospin, pairing, deformation,** ...
- **Many-body** calculations based on **NN scattering data** in the vacuum are **not conclusive** yet:
  - **different nuclear interactions in the medium** are found **depending** on the **approach**
  - only EoS, **no extensive calculations for nuclei** are available
- Based on effective interactions, **Nuclear Energy Density Functionals** are **successful** in the description of **masses, nuclear sizes, deformations, Giant Resonances,...**

Approximate realization of an exact Nuclear Energy Density Functionals:

Kohn-Sham iterative scheme (static approximation)

- Determine a good $E[\rho]$
- Initial guess ρ_0
- Calculate potential V_{eff} from ρ_0
- Solve single particle (Schrödinger) equation and find single particle wave functions ϕ_i
- Use ϕ_i for calculating new $\rho_1 = \sum_i^A |\phi_i|^2$
- Repeat until convergence

Runge-Gross Theorem: dynamic generalization of the static EDFs.

$$\int dt \, \{\langle \Phi(t) | i\partial_t | \Phi(t) \rangle - E[\rho(t), t]\} = 0$$

Giant Resonances well described within the small amplitude limit (known as RPA approach)

Nuclear Energy Density Functionals:

Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

$$\begin{aligned}\mathcal{L}_{int} \quad &= \bar{\mathbf{\Psi}}\Gamma_\sigma(\bar{\mathbf{\Psi}},\mathbf{\Psi})\mathbf{\Psi}\Phi_\sigma \quad +\bar{\mathbf{\Psi}}\Gamma_\delta(\bar{\mathbf{\Psi}},\mathbf{\Psi})\boldsymbol{\tau}\mathbf{\Psi}\mathbf{\Phi}_\delta \\ &-\bar{\mathbf{\Psi}}\Gamma_\omega(\bar{\mathbf{\Psi}},\mathbf{\Psi})\gamma_\mu\mathbf{\Psi}A^{(\omega)\mu} \quad -\bar{\mathbf{\Psi}}\Gamma_\rho(\bar{\mathbf{\Psi}},\mathbf{\Psi})\gamma_\mu\tau\mathbf{\Psi}\mathbf{A}^{(\rho)\mu} \\ &-e\bar{\mathbf{\Psi}}\hat{Q}\gamma_\mu\mathbf{\Psi}A^{(\gamma)\mu}\end{aligned}$$

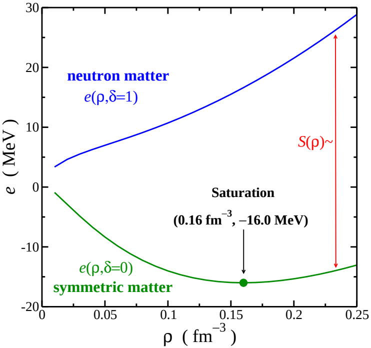
Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

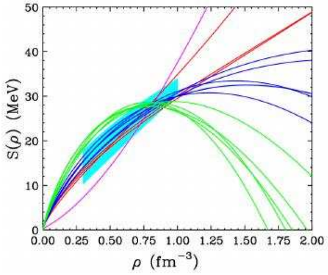
$$V^{\mathrm{eff}}_{\mathrm{Nucl}} = V^{\mathrm{long-range}}_{\mathrm{attractive}} + V^{\mathrm{short-range}}_{\mathrm{repulsive}} + V_{\mathrm{SO}} + V_{\mathrm{pair}}$$

- Fitted **parameters contain** (important) **correlations beyond the mean-field**
- Nuclear energy functionals are **phenomenological** $\rightarrow$ **not directly connected to any NN** (or NNN) **interaction**

The Nuclear Equation of State: Infinite System

(Independent particle picture: nucleons move in an effective mean field potential generated by all of them)





Physical Review C **86** *015803 (2012)*

$$e(\rho,\delta) = \mathbf{e}(\rho,\delta=\mathbf{0}) + \mathbf{S}(\rho)\delta^2 + \mathcal{O}\left[\delta^4\right] \text{ with } \delta = \frac{\rho_n-\rho_p}{\rho_n+\rho_p}$$

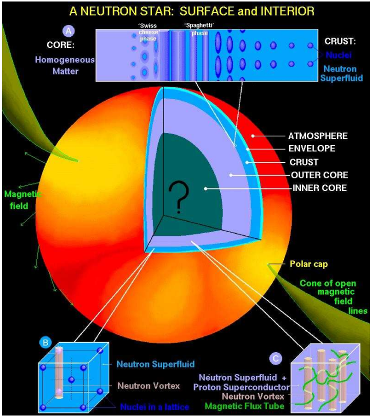
$$\mathbf{S}(\rho) = \mathbf{J} + \mathbf{L}\frac{\rho-\rho_0}{3\rho_0} + K_{\rm sym}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \mathcal{O}\left[(\rho-\rho_0)^3\right]$$

The uncertainties in $S(\rho)$ **impacts** on many **observables** and, therefore, on our understanding on the **nuclear effective interaction**.

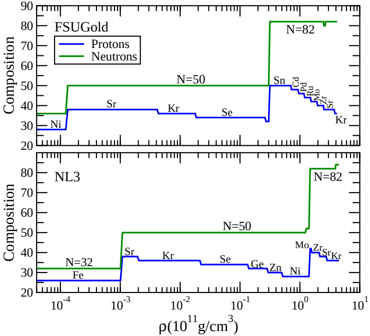
MOTIVATION

Motivation:

The symmetry energy and the structure and composition of a neutron star crust



Courtesy of Dany Page



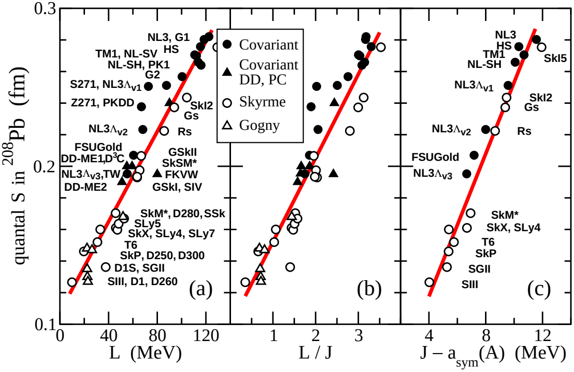
Physical Review C **78**, *025807 (2008)*

The faster the symmetry energy increases with density, the more exotic the composition of the outer crust.

Motivation:

The symmetry energy and the neutron distribution in nuclei

(quantal S: difference between the neutron and proton root mean square radius)



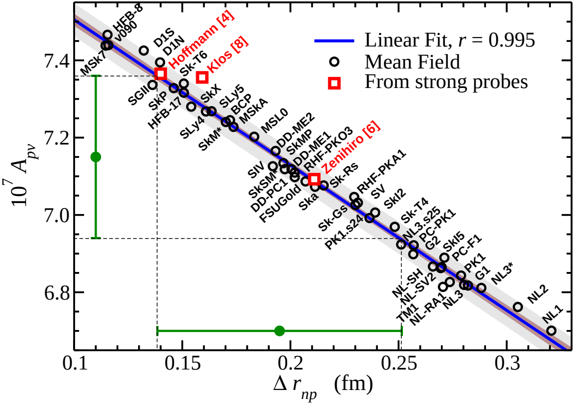
Physical Review Letters **102**, *122502 (2009)*

The faster the symmetry energy increases with density, the largest the size of the neutron distribution in nuclei. **[Exp. from strongly interacting probes:** 0.18 ± 0.03 **fm (***Physical Review C* **86** *015803 (2012)***)].**

Motivation:

The symmetry energy and parity violating electron scattering

(A_{pv}: relative difference between the elastic cross sections of right- and left-handed electrons)



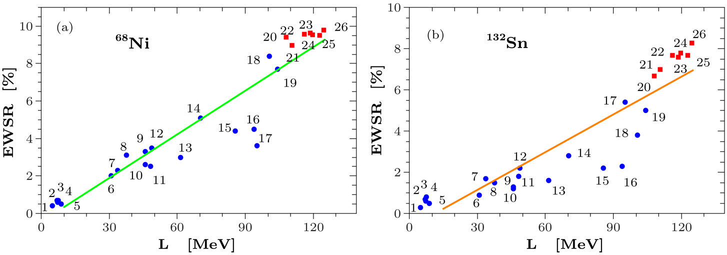
Physical Review Letters **106**, *252501 (2011)*

The largest the size of the neutron distribution in nuclei, the smaller the elastic electron parity violating asymmetry. **[Exp. from ew probes:** 0.302 ± 0.175 **fm (***Physical Review C* **85**, *032501 (2012)***)].**

Motivation:

The symmetry energy and the Pygmy Dipole Resonance

(Pygmy: low-energy excited state appearing in the dipole response of $N \neq Z$ nuclei)



Physical Review C **81**, *041301 (2010)*

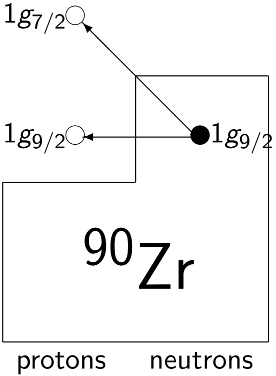
The faster the symmetry energy increases with density, the larger is the energy (E) and the larger is the probability (P) of exciting the Pygmy state ⇒ larger the Energy Weighted Sum Rule (EWSR) $\propto E \times P$.

Motivation:

The symmetry energy and the Gamow-Teller Resonance

(GT transitions change spin and isospin of the initial quantum state and favors $\Delta n \to 0$)

- Spin orbit splittings around the Fermi surface
- Proton and neutron single particle potentials $\Rightarrow V_{\rm sym}$



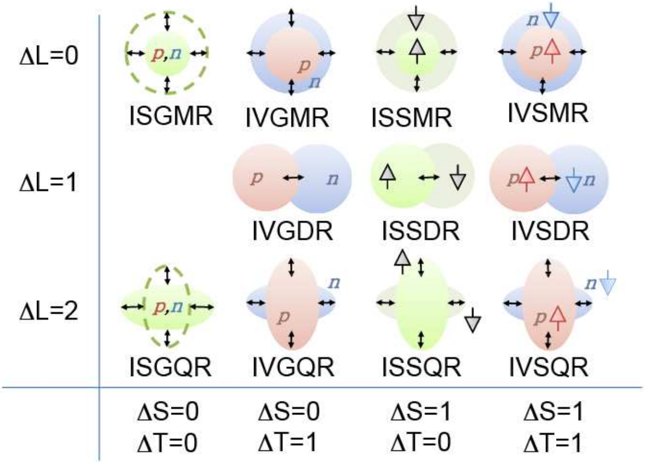
Gamow-Teller transitions determine:

- weak interaction rates essential in core-collapse dynamics of massive stars
- in neutron-rich environment, neutrino-induced nucleosynthesis
- studies of double-β decay
- useful in the calibration of detectors used to measure neutrinos

Physical Review C **86** *031306 (2012)*

GIANT RESONANCES

Giant Resonances



http://www.majimak.com/wordpress/

$\Delta T = 1$, $\Delta S = 0$ and $\Delta L = 1 \rightarrow$ **I**so**V**ector **GDR**

$\Delta T = 1$, $\Delta S = 0$ and $\Delta L = 2 \rightarrow$ **I**so**V**ector **GQR**

Isovector Giant Resonances

- In **isovector** giant resonances **neutrons and protons "oscillate" out of phase**
  e.g. within a classical picture: **"e-m interacting probes basically excite protons, protons drag neutrons thanks to the nuclear strong interaction, when neutrons approach too much to protons, they are pushed out"**
- **Isovector** resonances will depend on oscillations of the density $\rho_{\rm iv} \equiv \rho_{\bf n} - \rho_{\bf p} \Rightarrow \mathbf{S}(\rho)$ will drive such "oscillations"
- The **excitation energy (E_x)** within a **Harmonic Oscillator** approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m}\frac{d^2U}{dx^2}} \propto \sqrt{k} \to E_x \sim \sqrt{\frac{\delta^2 e}{\delta\beta^2}} \propto \sqrt{S(\rho)}$$

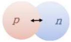
where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$

Polarizability, Strength distribution and its moments

- The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar/quadrupolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:
  $F_{JM} = \sum_i^A r^J Y_{JM}(\hat{r})\tau_z(i)$ $(\Delta L = 1, 2 \rightarrow Dipole, Quadrupole)$
- is proportional to the **static polarizability** for small oscillations
  $\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_\nu |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_\nu |\langle\nu|F|0\rangle|^2 \delta(E - E_\nu)$
- **Isovector energy weighted sum rules (EWSR)** are:
  $m_1 = \frac{\hbar^2}{2m}\frac{NZ}{A}(1 + \kappa_D)$ $\qquad$ $m_1 = \frac{\hbar^2}{2m}\frac{50}{4\pi}A\langle r^2\rangle(1 + \kappa_Q)$

  equal to one half of the HF expectation value of $[\hat{F}, [H, \hat{F}]]$ (Thouless theorem) and where κ is the dipole/quadrupole enhancement factor

Isovector Giant Dipole Resonance:

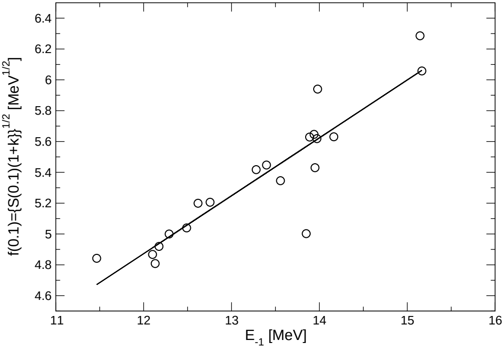


Excitation energy

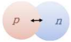
$S(\rho = 0.1\ \mathbf{fm}^{-1})$ is **correlated** with the value of the excitation energy (E_{-1}) of the IVGDR in spherical nuclei $\Rightarrow$ experimental data on $E_{-1}(^{208}\text{Pb})$ leads to **23.3 MeV** $< S(\rho = 0.1\ \text{fm}^{-3}) <$ **24.9 MeV**.

Physical Review C **77** *061304 (2008)*

Based on the hydrodynamical model of Ref. [*Physical Reports* **175** *103-261 (1989)*], it can be written in good approximation within EDF [*Physical Review Letters* **102**, *122502 (2009)*] calculations that: $\mathbf{E_{-1}} = \sqrt{\frac{m_1}{m_{-1}}} \approx \sqrt{\frac{6\hbar^2}{m\langle r^2\rangle}\mathbf{S}(\rho = \mathbf{0.1})(\mathbf{1} + \kappa_{\mathbf{D}})}$



Isovector Giant Dipole Resonance:



Dipole polarizability: macroscopic approach

E the energy within a Liquid Drop based Model; F_{ext} an external field (dipole operator) → constrained calculation keeping N and Z fixed:

$$\delta \left\{ E(\rho,\delta) + F_{ext}(\Lambda,\rho,\delta) - \lambda_n \int_0^\infty \rho_n(r) d\mathbf{r} - \lambda_p \int_0^\infty \rho_p(r) d\mathbf{r} \right\} = 0$$

and find:

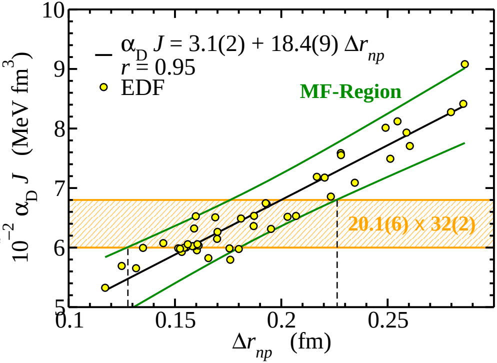
$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J} \left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

within the same model:

$$\mathbf{\Delta r_{np}} = \sqrt{\frac{3}{5}}\left[\mathbf{t} - \frac{e^2 Z}{70J}\right] + \Delta r_{np}^{\text{surface}}$$

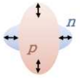
$$\mathbf{t} \equiv \frac{3r_0}{2}\frac{\mathbf{J/Q}}{1+\frac{9}{4}\frac{\mathbf{J}}{\mathbf{Q}}A^{-1/3}}(I - I_C)$$

using these expressions:



$$\alpha_{\mathbf{D}} \approx \frac{A\langle r^2\rangle}{12\mathbf{J}}\left[1 + \frac{5}{2}\frac{\mathbf{\Delta r_{np}} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

Isovector Giant Quadrupole Resonance:



Within the Quantum Harmonic Oscillator approach

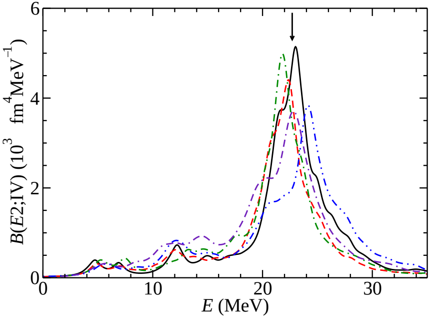
$E_x^{\mathrm{IV}} = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m}\frac{V_{\mathrm{sym}}\langle r^2\rangle}{(\hbar\omega_0)^2\langle r^4\rangle}}$

and EDF calculations, one can deduce

$V_{\mathrm{sym}} \approx 8(S(\rho_A) - S^{\mathrm{kin}}(\rho_0))$

$S^{\mathrm{kin}}(\rho_0) \approx \varepsilon_{\mathrm{F}_0}/3$

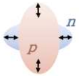
and the expression one finds depend on well known quantities from the theory and observable GQR energies.

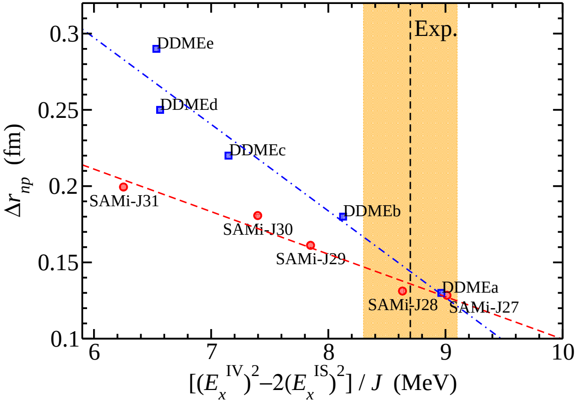


$$S(\rho_A) = \frac{\varepsilon_{\mathrm{F}_0}}{3}\left\{\frac{A^{2/3}}{8\varepsilon_{\mathrm{F}_0}^2}\left[\left(E_x^{\mathrm{IV}}\right)^2 - 2\left(E_x^{\mathrm{IS}}\right)^2\right] + 1\right\}$$

$$S(\rho = 0.1\ \mathbf{fm}^{-3}\ \mathbf{for}\ ^{208}\mathbf{Pb}) = 23.3 \pm 0.6\ \mathbf{MeV}$$

Isovector Giant Quadrupole Resonance:

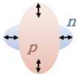




$$\frac{\mathbf{\Delta r_{np}} - \Delta r_{np}^{\mathrm{surf}}}{\langle r^2\rangle^{1/2}} = \frac{2}{3}\,(I - I_C)\left\{1 - \frac{\varepsilon_{F\infty}}{3J} - \frac{3}{7}\,\frac{I_C}{I - I_C} - \frac{A^{2/3}}{24\varepsilon_{F\infty}}\left[\frac{\left(\mathbf{E_x^{IV}}\right)^2 - 2\left(\mathbf{E_x^{IS}}\right)^2}{J}\right]\right\}$$

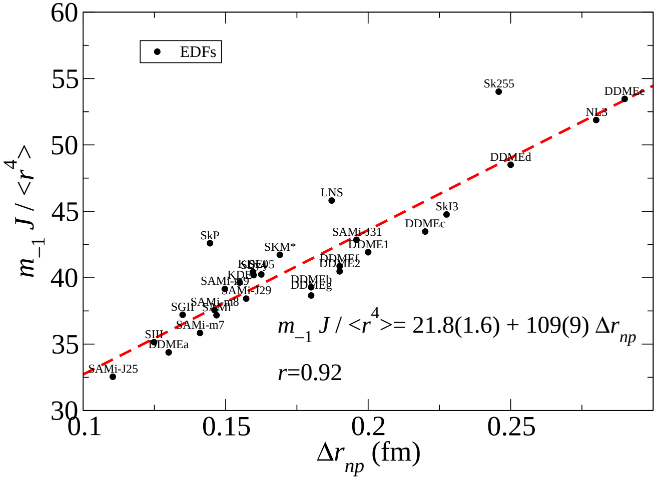
$$\Delta r_{np} \approx 0.14 \pm 0.03 \ \mathbf{fm}$$

Isovector Giant Quadrupole Resonance:



Quadrupole polarizability

$$\alpha_{\mathbf{Q}} \approx \frac{A\langle \mathbf{r^4}\rangle}{16\pi \mathbf{J}}\left[1+\frac{7}{2}\,\frac{\boldsymbol{\Delta r_{np}}+\sqrt{\frac{3}{5}}\frac{e^2 Z}{70J}-\Delta r_{np}^{\mathrm{surface}}}{\langle r^2\rangle^{1/2}(I-I_C)}\right]$$



CONCLUSIONS

Conclusions:

For heavy nuclei:

- the **macroscopic model** presented here contains **relevant physics** for the description of the different observables.
- the excitation **energy** of the **IVGDR** depend on $S(0.1\ \mathbf{fm}^{-3})$.
- within the **macroscopic** approach $S(0.1\ \mathbf{fm}^{-3})$ (not observable) or the Δr_{np} (observable) can be **determined by empirical data** on the excitation energies of the **IS-** and **IV-GQRs**.
- the dipole and quadrupole **polarizabilities** depend in an analogous way on the Δr_{np}.
- within the macroscopic model presented here, one can build the model independent quantity:

$$\frac{3}{5}\frac{\alpha_D J}{\langle r^2\rangle} - \frac{4\pi}{7}\frac{\alpha_Q J}{\langle r^4\rangle} = \frac{A}{70}$$

Co-workers:

G. Colò, P. F. Bortignon and M. Brenna (University of Milan)

M. Centelles and X. Viñas (University of Barcelona)

N. Paar and D. Vretenar (University of Zagreb, Croatia)

B. K. Agrawal (SINP, Kolkata, India)

L. Cao (IMP-CAS, Lanzhou, P.R. China)