Relativistic mean field interaction with density dependent meson-nucleon vertices based on microscopical calculations

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General Motivation

- **One of the main goals in Nuclear Physics is to build a universal density functional based on microscopical calculations**
  - **Able to explain as many as possible measured data**
  - **Provide reliable predictions for properties of nuclei far from stability**
- **Problem: Attempts to derive such a functional in a fully-microscopic way provide only qualitative results** (for two reasons)
  - **The three-body term of the bare interaction is not known well enough**
  - **The methods to derive such a functional are not precise enough to achieve the required accuracy, *e.g.*, for astrophysical applications**

General Motivation

- **Most successful functionals are either:**
  - **fully phenomenological, *i.e.*, fitted to large set of experimental data**
  - **fitted to a combination of fully-microscopic results and experimental data**
- **We propose a different strategy to build a new relativistic functional (similar to the one used for the non-relativistic BCP functional)**
  - **Bulk part based on and consistent with BHF and DBHF calculations** (we include the scalar isovector channel, compatible meson-nucleon verteces,...)
  - **Surface part based on experimental data**

General Motivation

Why we use a relativistic functional?

- **Pros: QCD is Lorentz invariant $\rightarrow$ this symmetry allows to describe: the spin-orbit coupling, preserve causality, consistent description of currents and time-odd fields** (important for the calculation of odd nuclei)
- **Cons: In relativistic MF models, bulk and surface contributions are mixed** (only the exchanged meson masses influences the surface part without affecting the bulk part, in practice, only m_σ**!**)

General Motivation

Why we include the scalar isovector channel in our description?

- Ab-initio calculations lead to large scalar isovector fields in nuclear matter $\rightarrow$ exchange of the δ meson
- Microscopic investigations and phenomenological studies stressed that MF models which neglect the δ-meson miss important ingredients in describing very asymmetric nuclear matter at high densities.
- Problems: Considering ρ- and δ-mesons leads to redundancies in a fit to experimental data. An alternative way of constraining one of the two channels is needed

Contents:

- **The model: a brief introduction**
- **Density dependence of the meson-nucleon verteces: consistent with DBHF calculations**
- **Fit: DD-MEδ protocol**
- **Results: comparison with the accurate**
  - **BCP** (similar strategy / very different model)
  - **DD-ME2** (purely empirical strategy / same model without δ).

The Model: Density Dependent Hadron Field theory

Lagrangian $\mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{int}$

$\mathcal{L}_N$ is the Lagrangian of free nucleons $\mathcal{L}_N = \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi$

$\mathcal{L}_M$ is the Lagrangian of free mesons

$$\mathcal{L}_M = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}\left(\partial_\mu\vec{\delta}\partial^\mu\vec{\delta} - m_\sigma^2\vec{\delta}^2\right) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$- \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} - \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$\mathcal{L}_{int}$ is the Lagrangian describing the interactions
$\mathcal{L}_{int} = g_\sigma\bar{\psi}\sigma\psi + g_\delta\bar{\psi}\vec{\tau}\vec{\delta}\psi - g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi - g_\rho\bar{\psi}\gamma_\mu\vec{\tau}\vec{\rho}^\mu\psi - e\bar{\psi}\gamma_\mu A^\mu$

where the field strength tensors for the vector fields $\Omega^{\mu\nu}$, $\vec{R}^{\mu\nu}$ and $F^{\mu\nu}$ take the usual form $\Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$

The Model: Density Dependent Hadron Field theory

Equations of motion: classical variational principle

Dirac equation $\left[\gamma_\mu(i\partial^\mu - \Sigma^\mu) - m^*\right]\psi = 0$ **where** $m^* \equiv m - \Sigma^s$

Self-energies $\Sigma^s \equiv g_\sigma(\rho)\sigma + g_\delta(\rho)\vec{\tau}\vec{\delta}$ $\quad \Sigma^\mu \equiv \Sigma^{(0)\mu} + \delta_{\mu 0}\Sigma^{(r)}$

$\Sigma^{(0)\mu} \equiv g_\omega(\rho)\omega^\mu + g_\rho(\rho)\tau_3\rho_3^\mu + eA^\mu$ (usual definition)

$\Sigma^{(r)} \equiv -\frac{dg_\sigma}{d\rho}\sigma\rho^s + \frac{dg_\omega}{d\rho}\omega^0\rho + \frac{dg_\delta}{d\rho}\delta_3\rho_3^s + \frac{dg_\rho}{d\rho}\rho_3^0\rho_3$ (rearrangement)

Meson equations $\quad \partial_\nu\partial^\nu A^\mu = +e\, j_p^\mu$

$\left(\partial_\nu\partial^\nu + m_\sigma^2\right)\sigma = -g_\sigma(\rho)\ \rho^s \qquad \left(\partial_\nu\partial^\nu + m_\omega^2\right)\omega^\mu = +g_\omega(\rho)\ j^\mu$

$\left(\partial_\nu\partial^\nu + m_\delta^2\right)\delta_3 = -g_\delta(\rho)\rho_3^s \qquad \left(\partial_\nu\partial^\nu + m_\rho^2\right)\rho_3^\mu = +g_\rho(\rho)\ j_3^\mu$

(densities and currents at the ground-state expectation values)

The Model: Density Dependent Hadron Field theory

Asymmetric infinite nuclear matter

Energy

$$\epsilon = \frac{1}{4}\left[3E_{Fn}\rho_n + m_n^*\rho_n^s\right] + \frac{1}{4}\left[3E_{Fp}\rho_p + m_p^*\rho_p^s\right] + \frac{1}{2}\left[m_\sigma^2\sigma^2 + m_\omega^2(\omega^0)^2 + m_\delta^2\delta_3^2 + m_\rho^2(\rho_3^0)^2\right]$$

Pressure

$$P = \frac{1}{4}\left[E_{Fn}\rho_n - m_n^*\rho_n^s\right] + \frac{1}{4}\left[E_{Fp}\rho_p - m_p^*\rho_p^s\right] - \frac{1}{2}\left[m_\sigma^2\sigma^2 - m_\omega^2(\omega^0)^2 + m_\delta^2\delta_3^2 - m_\rho^2(\rho_3^0)^2\right] + (\rho_n + \rho_p)\,\Sigma^{(r)0}$$

Effective mass splitting $m_n^* - m_p^* = -2g_\delta(\rho)\delta(\rho)$

(therefore, the δ meson could be independently determined)

The Model: Density Dependent Hadron Field theory

The Symmetry energy

$$S_2(\rho) = \frac{1}{2}\left(\frac{\partial^2 e(\rho,\alpha)}{\partial\alpha^2}\right)_{\alpha=0} =$$

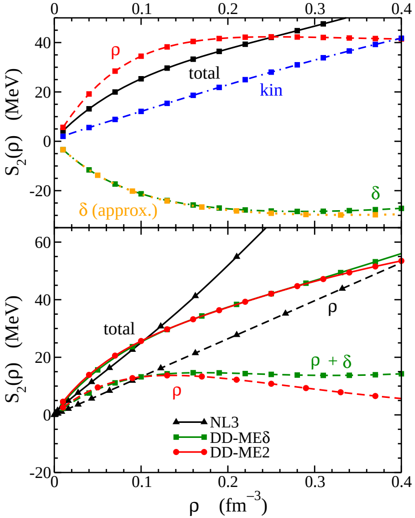
$$S_2^{\rm kin} + S_2^{\rho} + S_2^{\delta}$$

where,

$$S_2^{\rm kin}(\rho) = \frac{k_F^2}{6E_F}$$

$$S_2^{\rho}(\rho) = \frac{1}{2}\rho\frac{g_\rho^2(\rho)}{m_\rho^2}$$

$$S_2^{\delta}(\rho) \approx -\frac{1}{2}\rho\frac{g_\delta^2(\rho)}{m_\delta^2}\left(\frac{m^*}{E_F}\right)^2$$



Density dependence of the meson-nucleon verteces:

Consistency with DBHF calculations

- **The meson-nucleon vertices have been mapped to the scalar and vector self-energies obtained from Dirac-Brueckner calculations. The deduced density dependence is**

$$g_i(\rho) = g_i(\rho_{\mathsf{sat}}) f_i(x) \quad \text{for} \quad i = \sigma, \omega, \delta, \rho$$

where $x \equiv \rho/\rho_{\rm sat}$ and,

$$f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+e_i)^2}$$

- **The same density dependence for all vertices!**
- **By definition:** $f(1) = 1$ **and we impose:** $e_\sigma = d_\sigma$, $e_\omega = d_\omega$, $f''_\sigma(x=1) = f''_\omega(x=1)$ **and** $f''_i(x=0) = 0$.
- **Meson masses are fixed** $m_\omega = 783$ **MeV,** $m_\delta = 983$ **MeV and** $m_\rho = 763$ **MeV. The nucleon mass is** $m = 939$ **MeV.**
- **14 adjustable parameters: 4** $g_i(\rho_{\mathsf{sat}})$**, 9 in** $f_i(x)$**, and** m_σ

Fit: strategy

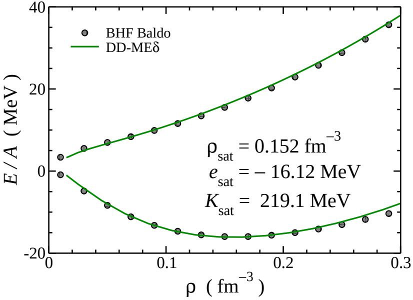
- **Fit to binding energies and charge radii do not allow to determine more than 7 or 8 parameters**
- **To calibrate the 14 free parameters of the DD-MEδ functional we use finite nuclei data (fixes 4 p.) and modern microscopic non-relativistic and relativistic Brueckner calculations (fixes 10 p.):**
  - **161 binding energies and 86 charge radii**(spherical)
    $g_\sigma(\rho_{\text{sat}})$, $g_\omega(\rho_{\text{sat}})$, $g_\rho(\rho_{\text{sat}})$, **and** m_σ
  - **BHF-EoSs from the Catania group (M. Baldo)**
    b_σ, c_σ, c_ω, c_ρ, d_ρ **and** e_ρ
  - **DBHF-($m_p^* - m_n^*$) from the Tübingen Group (A. Faessler)**
    $g_\delta(\rho_{\text{sat}})$, c_δ, d_δ **and** e_δ

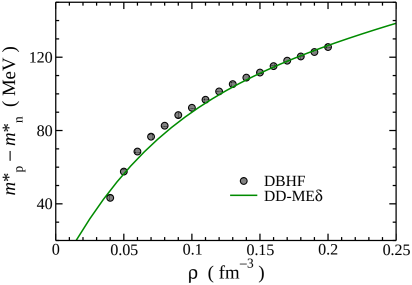
Fit: DD-MEδ versus DD-ME2 parameters

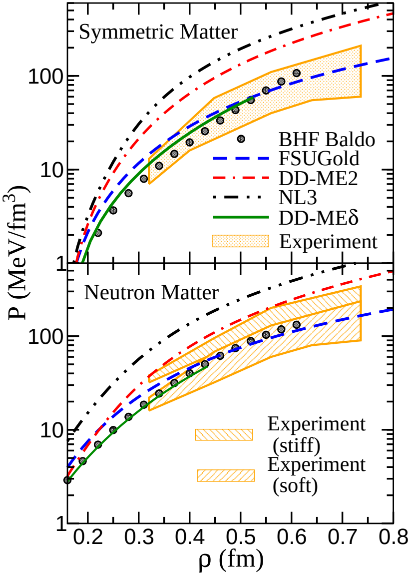
i	m_i (MeV)	$g_i(\rho_{\rm sat})$	a_i	b_i	c_i	d_i	e_i
σ	566.1577	10.3325	1.3927	0.1901	0.3679	0.9519	0.9519
σ	550.1238	10.5396	1.0943	0.1901	1.7057	0.4421	0.4421
ω	783.0000	12.2904	1.4089	0.1698	0.3429	0.9860	0.9860
ω	783.0000	13.0189	1.3892	0.1698	0.9240	1.4620	1.4620
δ	983.0000	7.1520	1.5178	0.3262	0.6041	0.4257	0.5885
ρ	763.0000	6.3128	1.8877	0.0651	0.3469	0.9417	0.9737
ρ	763.0000	3.6836	0.5647				

- In DD-ME2 the density dependence of the ρ-meson is given by $g_\rho(\rho) = g_\rho(\rho_{\rm sat})\exp(-a_\rho(x-1))$
- ρ-meson in DD-ME2 “absorbe” the effects due to the δ-meson.

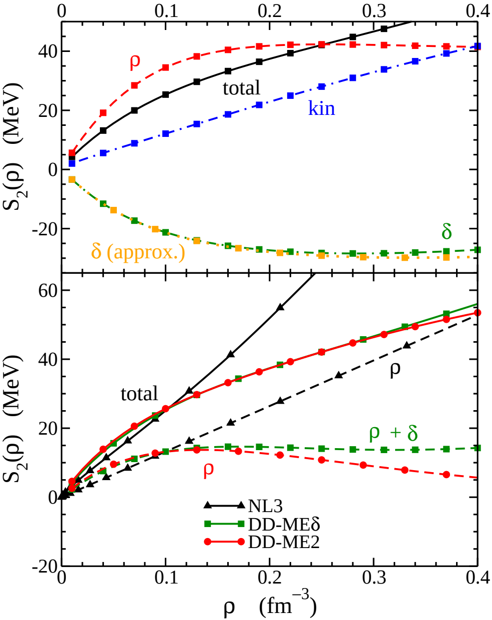
Results: EoS and $m_p^* - m_n^*$



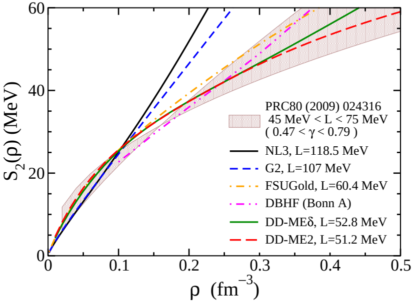


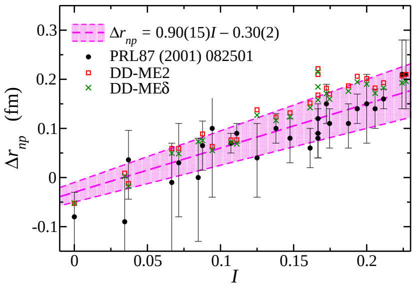


Results: symmetry energy



Results: symmetry energy and experimental Δr_{np}

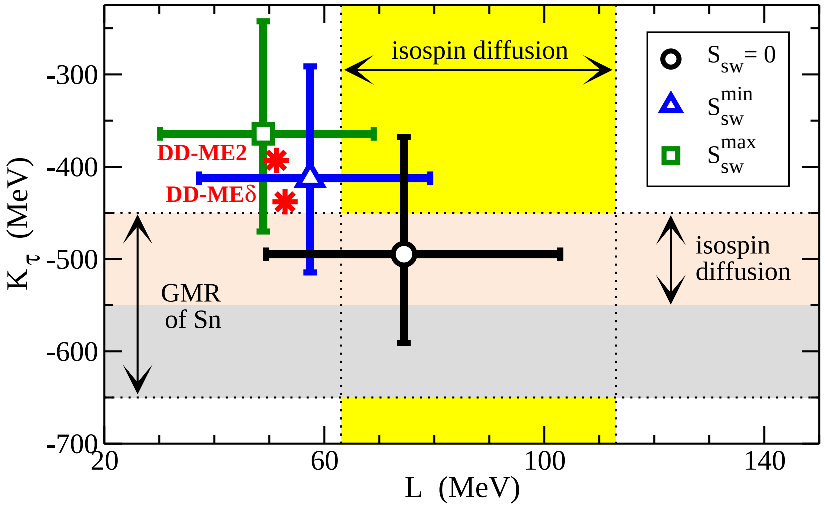




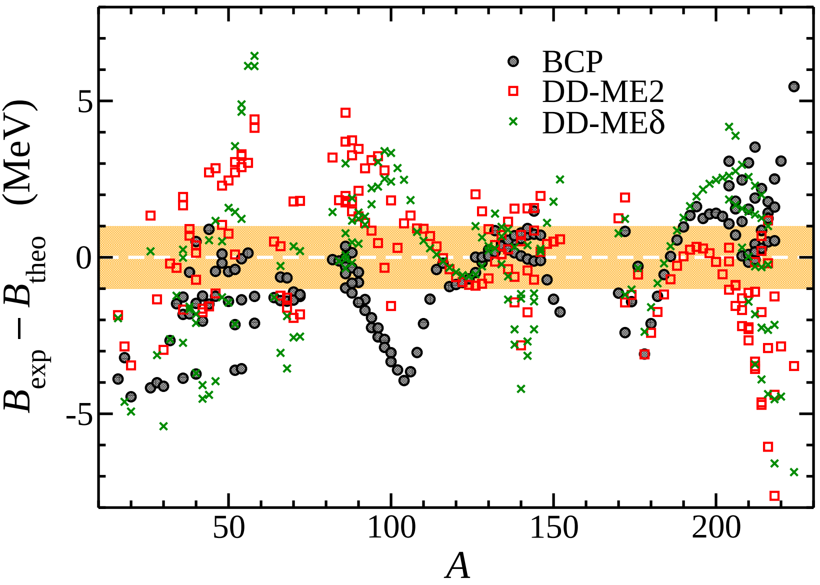
	J	L	$K_{\rm sym}$	$Q_{\rm sym}$	K_τ
DD-ME2	32.30	51.26	−85	808	−393
DD-MEδ	32.35	52.85	−120	884	−438

$$K_\tau \equiv K_{\rm sym} - 6L \qquad \left(-\frac{Q}{K}L\right)$$

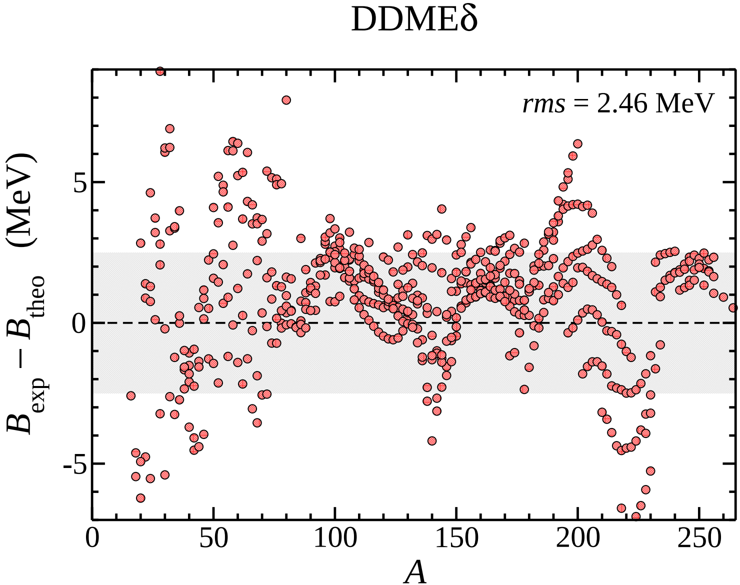
Results: symmetry energy, experimental indications versus model predictions



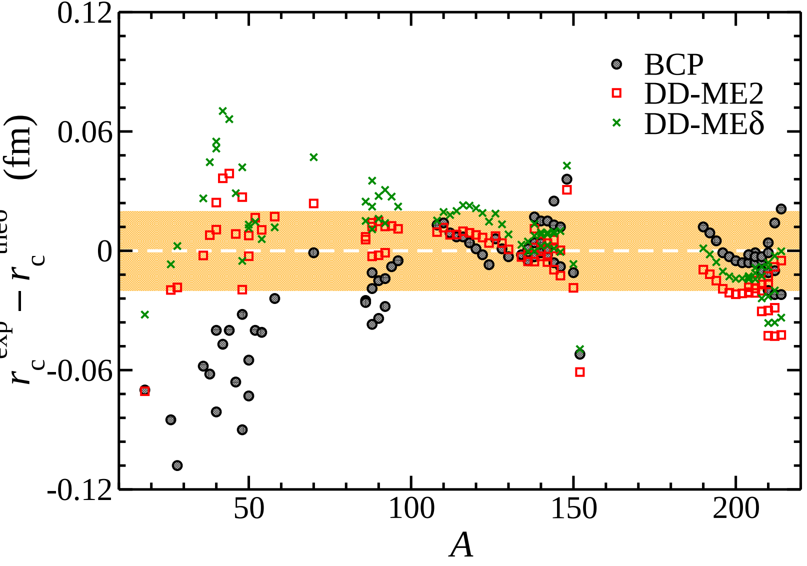
Results: fitted binding energies



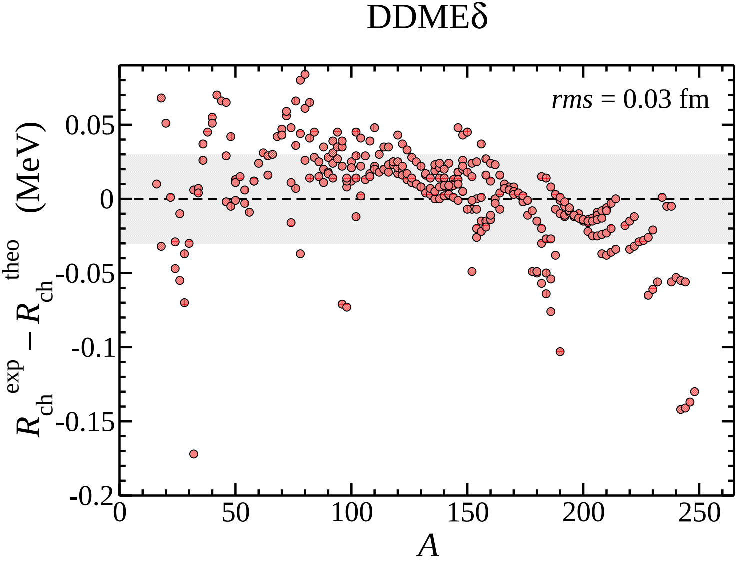
Results: binding energies of all measured even-even nuclei



Results: fitted charge radii



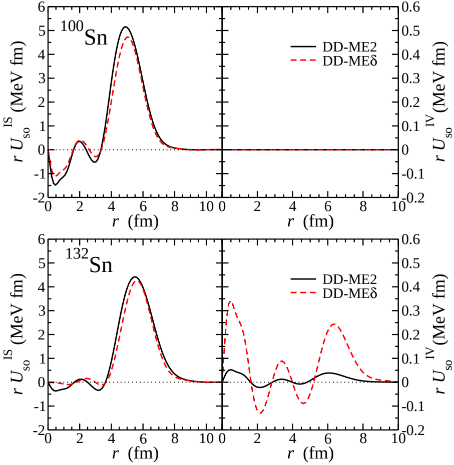
Results: charge radii of all measured even-even nuclei

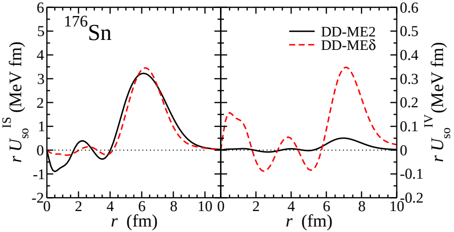


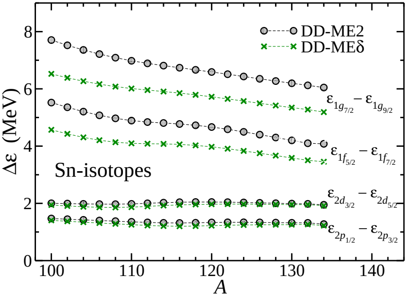
Results: the influence of the δ-meson on measured finite nuclei properties

- **We have see that:**
  - **the influence of the δ-meson on the symmetry energy can be compensated by renormalizing the ρ-meson**
  - **the same seems to be true for masses, radii and neutron skin thicknesses**
- **Spin-orbit splittings:**
  - **isoscalar and isovector parts of the spin-orbit potential are enhanced with respect the normal potential since there is no destructive interference between the terms comming from σ and ω not from the δ and ρ.**
  - **therefore, the effects of the δ-meson cannot be compensated by renormilizing the ρ-meson** (one would destroy the other properties)
- **Essential difference of a theory containing the δ-meson. It may impact on finite nuclei properties.**

Results: Spin orbit splitings







Conclusions and general remarks

- **Relativistic models**
  - **provide a consistent treatment of the spin degrees of freedom and velocity dependent terms**
  - **account for the large Lorentz scalar and vector self-energies interplay** (induced on the QCD level)
- **On the way to build a "more microscopic" energy density functional:**
  - **we started with BHF calculations for the EoS**
  - **and with DBHF calculations for** $m_n^* - m_p^*$
  - **we adopted a density dependence of the meson-nucleon vertices derived from DBHF calculations** ("proper density powers")
  - **we added experimental data in finite nuclei** (it is well known that all attempts to derive the functionals directly from bare forces do not reach the required accuracy)
  - **by including the** δ**-meson we took into account that in DBHF calculations the scalar self-energies show a strong isovector part** (usually neglected in relativistic models)

Conclusions and general remarks

- DD-MEδ and DD-ME2 predict similar properties for finite nuclei
- DD-MEδ is based to a large extent on mic. calculations
- Only 4 parameters adjusted to finite nuclei
- The δ-meson does not improve the accuracy of finite nuclei properties
- The δ-meson renders the model much more physical
- The mass splitting of neutrons and protons in DD-MEδ is correctly incorporated
- The DD-MEδ-EoS at high densities agree with exp. data derived from HIC
- DD-MEδ may be a reliable interaction for applications to neutron stars
- DD-MEδ may provide a better description of spin-isospin resonances than conventional RMF (Hartree) models

Thank you for your attention!

Extra material:

Fit: 3-steps

1. **Search of a good starting point**: **DD-ME2 protocol + δ-nucleon vertex fitted to DBHF** (4-δ parameters fixed from this step to the end)
$$\chi^2_{(1)} = \chi^2_B + \chi^2_{r_c} + \chi^2_{\rm sym} + \chi^2_{\rm neut} + \chi^2_{\Delta {\rm m}^*}$$
2. **Density dependent parameters (6): fitted to BHF-EoS** (up to 0.3 fm^{-3})
$$\chi^2_{(2)} = \chi^2_{\rm sym} + \chi^2_{\rm neut} + \chi^2_{\Delta {\rm m}^*}$$
3. **Strength parameters (3) + m_σ: fitted to 161 binding energies and 86 charge radii**
$$\chi^2_{(3)} = \chi^2_B + \chi^2_{r_c} + \chi^2_{\rm sym} + \chi^2_{\rm neut} + \chi^2_{\Delta {\rm m}^*}$$

process converged—we do not have to repeat steps 2 and 3

Fit: χ^2 *definition*

$$\chi^2 = \frac{1}{n_{\rm data}} \sum_{i=1}^{n_{\rm data}} w_i^2 \left(\mathcal{O}_i^{\rm model} - \mathcal{O}_i^{\rm ref}\right)^2$$

where $n_{\rm data}$ is the number of data points and w_i the weight associated to each data point.

$\mathcal{O}_i$	w_i	$n_{\rm data}$	ρ (fm^{-3})	χ^2
B	$1/0.50$ MeV^{-1}	161		23.40
$r_{\rm c}$	$1/0.01$ fm^{-1}	86		2.90
$e(\rho, \alpha = 1)$	$1/(0.03 \times \mathcal{O}_i)$	30	$0.01 - 0.30$	3.42
$e(\rho, \alpha = 0)$	$1/(0.03 \times \mathcal{O}_i)$	30	$0.01 - 0.30$	7.03
$m_p^* - m_n^*$	$1/(0.03 \times \mathcal{O}_i)$	25	$0.04 - 0.20$	0.39

Table: Partial contributions to the total χ^2, fifth column.