

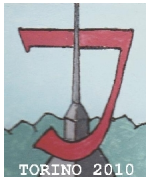
# Density dependence of the nuclear symmetry energy estimated from neutron skin thickness in finite nuclei

X. Viñas<sup>a</sup>

M. Centelles<sup>a</sup>

M. Warda<sup>a,b</sup>

X. Roca-Maza<sup>a,c</sup>



<sup>a</sup>Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Universitat de Barcelona, Barcelona, Spain

<sup>b</sup>Katedra Fizyki Teoretycznej, Uniwersytet Marii Curie-Skłodowskiej, Poland

<sup>c</sup>INFN sezione di Milano. Via Celoria 16, 20133 Milano, Italy

M. Centelles, X. Roca-Maza, X. Viñas and M. Warda,  
*Phys. Rev. Lett.* **102** 122502 (2009)

M. Warda, X. Viñas, X. Roca-Maza and M. Centelles,  
*Phys. Rev.* **C80** 024316 (2009)

M. Warda, X. Viñas, X. Roca-Maza and M. Centelles,  
*Phys. Rev.* **C81** 054309 (2010)

## Why is important the nuclear symmetry energy ?

The **nuclear symmetry energy** is a fundamental quantity in **Nuclear Physics** and **Astrophysics** because it governs, at the same time, important properties of very small entities like the atomic nucleus (  $R \sim 10^{-15}$  m ) and very large objects as neutron stars (  $R \sim 10^4$  m )

- **Nuclear Physics:** Neutron skin thickness in finite nuclei, stable nuclei, Heavy-Ion collisions, Giant Resonances...
- **High-Energy Physics:** Test of the Standard Model through atomic parity non-conservation observables.
- **Astrophysics:** Supernova explosion, Neutron emission and cooling of protoneutron stars, Mass-Radius relations in neutron stars, Composition of the crust of neutron stars...

## Equation of State in asymmetric matter

$$e(\rho, \delta) = e(\rho, 0) + c_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) \quad \left( \delta = \frac{\rho_n - \rho_p}{\rho} \right)$$

Around the saturation density we can write

$$e(\rho, 0) \simeq a_v + \frac{1}{2}K_v\epsilon^2 \quad \text{and} \quad c_{sym}(\rho) \simeq J - L\epsilon + \frac{1}{2}K_{sym}\epsilon^2 \quad \left( \epsilon = \frac{\rho_0 - \rho}{3\rho_0} \right)$$

$$\rho_0 \approx 0.16 \text{fm}^{-3}, \quad a_v \approx -16 \text{MeV}, \quad K_v \approx 230 \text{MeV}, \quad J \approx 32 \text{MeV}$$

However, the values of

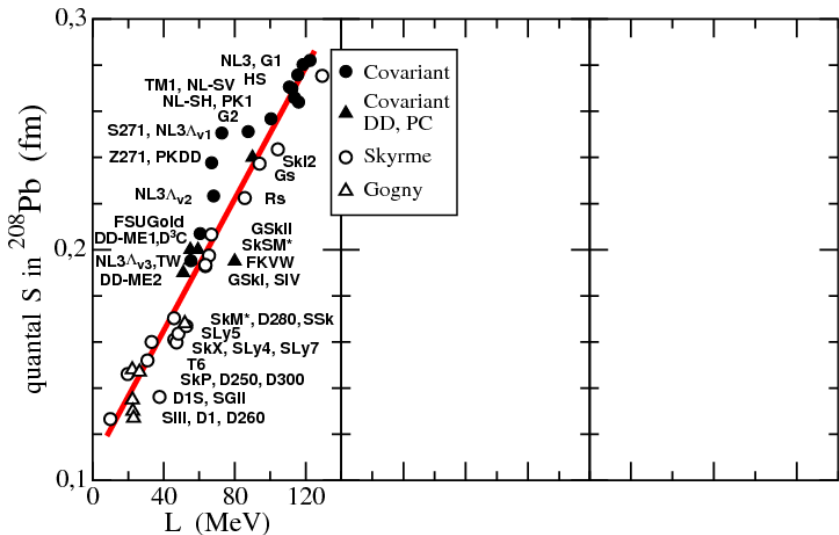
$$L = 3\rho \partial c_{sym}(\rho) / \partial \rho |_{\rho_0} \quad \text{and} \quad K_{sym} = 9\rho^2 \partial^2 c_{sym}(\rho) / \partial \rho^2 |_{\rho_0}$$

which govern the density dependence of  $c_{sym}$  near  $\rho_0$  are less certain and predictions vary largely among nuclear theories.

## Experimental constraints

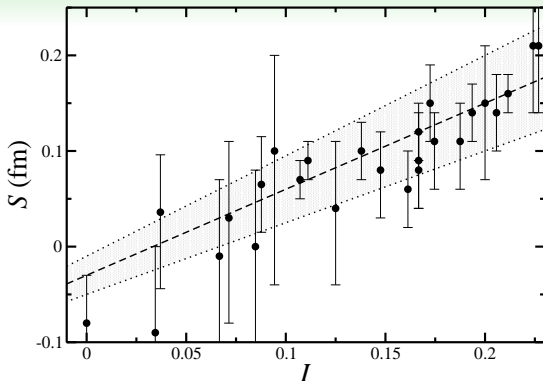
- Recent research in heavy-ion collisions at intermediate energy is consistent with  $c_{sym}(\rho) = c_{sym}(\rho_0) \cdot (\rho/\rho_0)^\gamma$  at  $\rho < \rho_0$ .
- Isospin diffusion  $\gamma = 0.7-1.05$  ( $L = 88 \pm 25$  MeV).
- Isoscaling  $\gamma = 0.69$  ( $L \sim 65$  MeV)
- Inferred from nucleon emission ratios  $\gamma = 0.5$  ( $L \sim 55$  MeV).
- The GDR of  $^{208}\text{Pb}$  analyzed with Skyrme forces suggests a constraint  $c_{sym}(0.1 \text{ fm}^{-3}) = 23.3-24.9$  MeV ( $\gamma \sim 0.5-0.65$ ).
- The study of the PDR in  $^{68}\text{Ni}$  and  $^{132}\text{Sn}$  predicts  $L=49-80$  MeV.
- The Thomas-Fermi model of Myers and Swiatecki fitted very precisely to binding energies of 1654 nuclei predicts an EOS that yields  $\gamma = 0.51$
- NEUTRON SKIN THICKNESS ?

## Neutron skin thickness



## What is experimentally known about neutron skin thickness in nuclei ?

- The neutron skin thickness is defined as  $S = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$ , where  $\langle r_n^2 \rangle^{1/2}$  and  $\langle r_p^2 \rangle^{1/2}$  are the rms radii of the neutron and proton distributions respectively.
- $\langle r_p^2 \rangle^{1/2}$  is known very accurately from elastic electron scattering measurements (e.g.  $r_{ch}(^{208}\text{Pb}) = 5.5010 \pm 0.0009$  fm [Angeli (2004)]).
- $\langle r_n^2 \rangle^{1/2}$  has been obtained with hadronic probes such as:
  - (a) Proton-nucleus elastic scattering ( $5.522\text{fm} < r_n(^{208}\text{Pb}) < 5.550$  fm [Clark (2003)]).
  - (b) Inelastic scattering excitation of the giant dipole and spin-dipole resonances ( $r_n(^{208}\text{Pb}) = 5.67 \pm 0.07$  fm [Krasznahorkay (1990)]).
  - (c) Antiprotonic atoms: Data from antiprotonic X rays and radiochemical analysis of the yields after the antiproton annihilation ( $r_n(^{208}\text{Pb}) = 5.66 \pm 0.02$  fm) [Trzcińska (2001)].



$$S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02) \text{ fm}$$

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

CAN  $S$  OF 26 STABLE NUCLEI, FROM  $^{40}\text{Ca}$  TO  $^{238}\text{U}$ , ESTIMATED USING ANTIPROTONIC ATOMS DATA HELP IN CONSTRAINING THE SLOPE AND CURVATURE OF  $c_{sym}$  ?

## Symmetry energy and neutron skin thickness in the Liquid Drop Model

- Symmetry Energy

$$a_{sym}(A) = \frac{J}{1 + x_A}, \quad x_A = \frac{9J}{4Q} A^{-1/3}$$

$$E_{sym}(A) = a_{sym}(A)(I + x_A I_C)^2 A$$

where

$$I = (N - Z)/A, \quad I_C = e^2 Z / (20JR), \quad R = r_0 A^{1/3}$$

- Neutron skin thickness

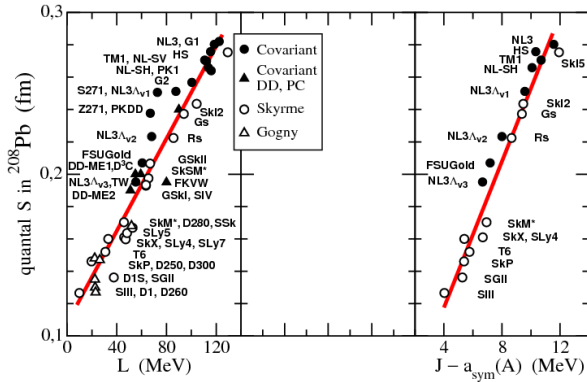
$$S = \sqrt{3/5} \left[ t - e^2 Z / (70J) + \frac{5}{2R} (b_n^2 - b_p^2) \right]$$

where

$$t = \frac{3r_0}{2} \frac{J/Q}{1 + x_A} (I - I_C)$$



## Neutron skin thickness



$$t = \frac{2r_0}{3J} [J - a_{sym}(A)] A^{1/3} (I - I_c)$$

Table: Value of  $a_{sym}(A)$  and density  $\rho$  that fulfils  $c_{sym}(\rho) = a_{sym}(A)$  for  $A = 208, 116$  and  $40$  in MF models.  $J$  and  $a_{sym}$  are in MeV and  $\rho$  is in  $\text{fm}^{-3}$ .

Model	$J$	$A = 208$		$A = 116$		$A = 40$	
		$a_{sym}$	$\rho$	$a_{sym}$	$\rho$	$a_{sym}$	$\rho$
NL3	37.4	25.8	0.103	24.2	0.096	21.1	0.083
NL-SH	36.1	25.8	0.105	24.6	0.099	21.3	0.086
FSUGold	32.6	25.4	0.098	24.2	0.090	21.9	0.075
TF-MS	32.6	24.2	0.093	22.9	0.085	20.3	0.068
SLy4	32.0	25.3	0.100	24.2	0.091	22.0	0.075
SkX	31.1	25.7	0.102	24.8	0.096	22.8	0.082
SkM*	30.0	23.2	0.101	22.0	0.093	19.9	0.078
SIII	28.2	24.1	0.093	23.4	0.088	21.8	0.077
SGII	26.8	21.6	0.104	20.7	0.096	18.9	0.082

## The $c_{sym}(\rho)$ - $a_{sym}(A)$ correlation

- There is a genuine relation between the symmetry energy coefficients of the EOS and of nuclei:  $c_{sym}(\rho)$  equals  $a_{sym}(A)$  of heavy nuclei like  $^{208}\text{Pb}$  at a density  $\rho = 0.1 \pm 0.01 \text{ fm}^{-3}$ .
- A similar situation occurs down to medium mass numbers, at lower densities.
- We find that this density can be very well simulated by

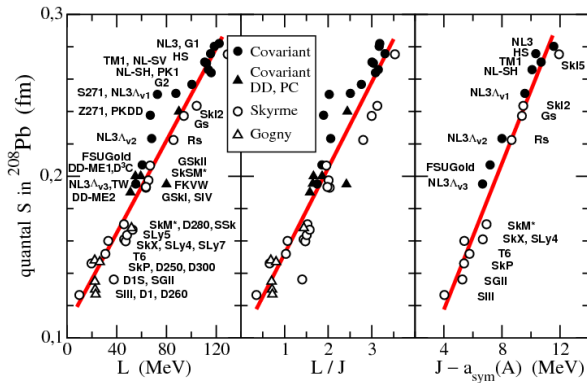
$$\rho \approx \rho_A = \rho_0 - \rho_0 / (1 + cA^{1/3}),$$

where  $c$  is fixed by the condition  $\rho_{208} = 0.1 \text{ fm}^{-3}$ .

- Using the equality  $c_{sym}(\rho) = a_{sym}(A)$  and the LDM, the neutron skin thickness can be finally written as:

$$t = \sqrt{\frac{3}{5}} \frac{2r_0}{3} \frac{L}{J} \left( 1 - \epsilon \frac{K_{sym}}{2L} \right) \epsilon A^{1/3} (I - I_C)$$

## Neutron skin thickness



$$t = \sqrt{\frac{3}{5}} \frac{2r_0}{3} \frac{L}{J} \left(1 - \epsilon \frac{K_{\text{sym}}}{2L}\right) \epsilon A^{1/3} (I - I_C)$$

## Fitting procedure and results

- We optimize

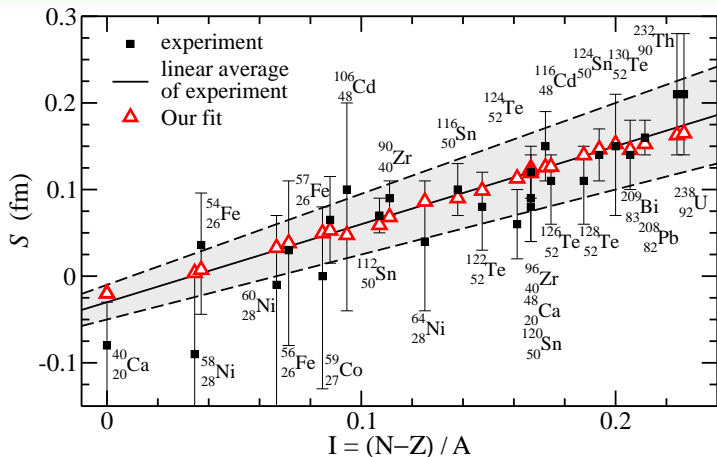
$$t = \sqrt{\frac{3}{5}} \frac{2r_0}{3} \frac{L}{J} \left(1 - \epsilon \frac{K_{sym}}{2L}\right) \epsilon A^{1/3} (I - I_C)$$

using

$$c_{sym} = 31.6 \left(\frac{\rho}{\rho_0}\right)^\gamma \text{MeV}, \quad \epsilon = \frac{1}{3(1 + cA^{1/3})}, \quad \rho_0 = 0.16 \text{fm}^{-3}$$

and taking as experimental baseline the neutron skins measured in 26 antiprotonic atoms.

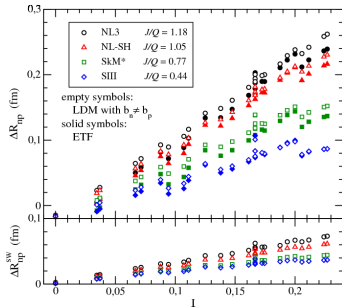
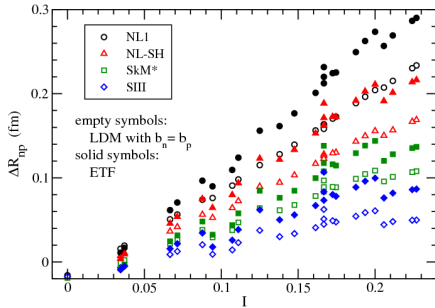
- We predict ( $b_n \approx b_p$ ):  $L = 75 \pm 25 \text{ MeV}$



$$S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02) \text{ fm}$$

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

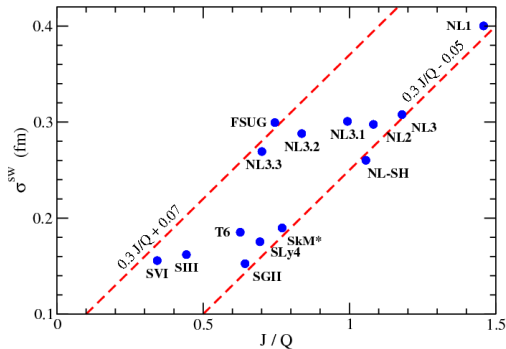
## Influence of the surface width ( $b_n \neq b_p$ )



$$S = \sqrt{3/5} \left[ t - e^2 Z / (70J) + \frac{5}{2R} (b_n^2 - b_p^2) \right]$$

$b_n$  and  $b_p$  are obtained at the ETF level.

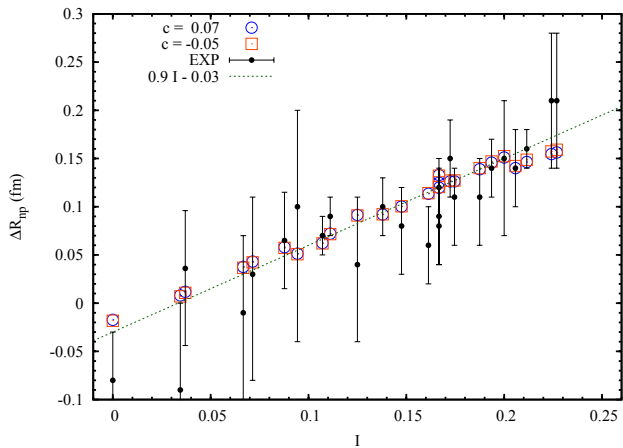
## Surface contribution to the neutron skin thickness



$$\sqrt{\frac{3}{5}} \frac{5}{2R} (b_n^2 - b_p^2) = \sigma^{sw} l = \left(0.3 \frac{J}{Q} + c\right) l$$

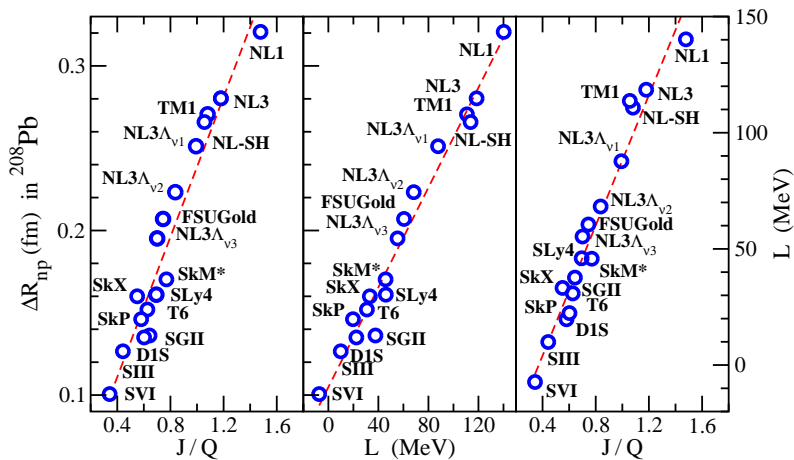


## Fit and results



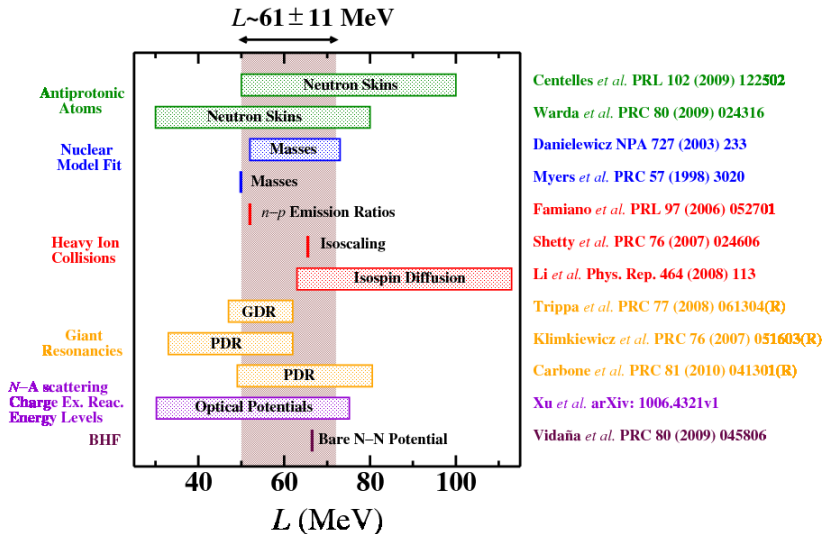
$$\frac{J}{Q} = 0.6 - 0.9$$

## Neutron skin thickness



$$L = 30 - 80 \text{ MeV}$$

## Constraints on the slope of the symmetry energy



## Summary and Conclusions

- We have described a generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation.
- We take advantage of this relation to explore constraints on  $c_{sym}(\rho)$  from neutron skins measured in antiprotonic atoms. These constraints points towards a **soft symmetry energy**.
- We discuss the  $L$  values constrained by neutron skins in comparison with most recent observations from reactions and giant resonances.
- We learn that in spite of present error bars in the data of antiprotonic atoms, the size of the final uncertainties in  $L$  is comparable to the other analyses.
- The generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation plausibly encompasses other prime correlations of nuclear observables with the density content of the symmetry energy as e.g. the constrains of  $c_{sym}(0.1)$  from the GDR of  $^{208}\text{Pb}$  (L. Trippa et al. Phys. Rev. **C77**, 061304(R) (2008)).

**Thank you for your attention**

**Extra material**

## Some technical details

- The surface stiffness coefficient  $Q$  and the surface widths  $b_n$  and  $b_p$  are obtained from self-consistent calculations of the neutron and proton density profiles in **asymmetric semi-infinite nuclear matter**.
- To this end one has to minimize **the total energy per unit area** with the constraint of conservation of **the number of protons and neutrons** with respect to arbitrary variations of the densities.

$$\frac{E_{\text{const}}}{S} = \int_{-\infty}^{\infty} [\varepsilon(z) - \mu_n \rho_n(z) - \mu_p \rho_p(z)] dz,$$

where  $\varepsilon(z)$  is the nuclear energy density functional.

- In the non-relativistic framework the densities  $\rho_n$  and  $\rho_p$  obey the coupled local Euler-Lagrange equations:

$$\frac{\delta \varepsilon(z)}{\delta \rho_n} - \mu_n = 0, \quad \frac{\delta \varepsilon(z)}{\delta \rho_p} - \mu_p = 0.$$

The relative neutron excess  $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$  is a function of the  **$z$ -coordinate**. When  $z \rightarrow -\infty$ , the densities  $\rho_n$  and  $\rho_p$  approach the values of asymmetric uniform nuclear matter in equilibrium with a bulk neutron excess  $\delta_0$ .

- From the calculated density profiles one computes:

$$z_{0q} = \frac{\int_{-\infty}^{\infty} z \rho'_q(z) dz}{\int_{-\infty}^{\infty} \rho'_q(z) dz},$$

$$b_q^2 = \frac{\int_{-\infty}^{\infty} (z - z_{0q})^2 \rho'_q(z) dz}{\int_{-\infty}^{\infty} \rho'_q(z) dz}.$$

- From the relation

$$t = z_{0n} - z_{0p} = \frac{3r_0}{2} \frac{J}{Q} \delta_0,$$

one can evaluate  $Q$  from the slope of  $t$  at  $\delta_0 = 0$ .

- The distance  $t$  and the surface widths  $b_n$  and  $b_p$  in finite nuclei with neutron excess  $I = (N - Z)/A$  are obtained using  $\delta_0$  given by:

$$\delta_0 = \frac{I + \frac{3}{8} \frac{c_1}{Q} \frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4} \frac{J}{Q} A^{-1/3}}.$$