# Density dependence of the nuclear symmetry energy estimated from neutron skin thickness in finite nuclei

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M. Centelles, X. Roca-Maza, X. Viñas and M. Warda,
Phys. Rev. Lett. **102** 122502 (2009)
M. Warda, X. Viñas, X. Roca-Maza and M. Centelles,
Phys. Rev. **C80** 024316 (2009)

## Introducing myself...

- Degree in Physics: University of Barcelona from 2000 to 2005
- Phd thesis: University of Barcelona from 2005 to 2010
  - Title: Isospin asymmetry in stable and exotic nuclei.
  - Advisors: X. Viñas and M. Centelles
  - Defense: 6 May 2010
- Post-doc: University of Barcelona from May to October 2010.
- Post-doc: INFN Milano from November 2010

## Works in which I have participated

## The symmetry energy and the outer crust in colaboration with J. Piekarewicz

- Impact of the symmetry energy on the outer crust of non-accreting neutron stars.
  *Phys. Rev.* C **78** (2008) 025807.

## The symmetry energy and the neutron skin thickness of nuclei in colaboration with X. Viñas, M. Cenetelles and M. Warda

- Single particle shell effects in the neutron skin thickness of nuclei within mean-field models
  *In preparation, writing...*
- Origin of the neutron skin thickness of $^{208}$Pb in nuclear mean-field models
  Accepted in *Phys. Rev.* **C**.
- Analysis of bulk and surface contributions in the neutron skin of nuclei.
  *Phys. Rev.* **C81** (2010) 054309.
- Neutron skin thickness in droplet model with surface width dependence: indications of softness of the nuclear symmetry energy.
  *Phys. Rev.* **C80** (2009) 024316.
- Nuclear symmetry energy probed by neutron skin thickness of nuclei.
  *Phys. Rev. Lett.* **102** (2009) 122502.

## Electron scattering in colaboration with X. Viñas, M. Cenetelles and F. Salvat

- Parity violating electron scattering at the kinematics of the PREX experiment and the neutron skin thickness of $^{208}$Pb.
  *In preparation, writing...*
- Theoretical study of elastic electron scattering along $N = 16$, $N = 50$ and $N = 82$ isotonic chains.
  *In preparation, writing...*
- Theoretical study of elastic electron scattering off stable and exotic nuclei
  *Phys. Rev.* C **78** (2008) 044332.

## The symmetry energy and the GMR in colaboration with X. Viñas, M. Cenetelles, S.K. Patra, B.K. Sharma, P.D. Stevenson

- Influence of the symmetry energy on the giant monopole resonance of neutron-rich nuclei.
  *J. Phys. G.* **37** (2010) 075107.

## DDME$\delta$, new mean field effective interaction in colaboration with X. Viñas, M. Cenetelles, P. Ring and P. Schuck

- Relativistic mean field interaction with density dependent meson-nucleon vertices based on microscopical calculations. *In preparation, writing...*

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## Why is important the nuclear symmetry energy ?

The nuclear symmetry energy is a fundamental quantity in Nuclear Physics and Astrophysics because it governs, at the same time, important properties of very small entities like the atomic nucleus ( $R \sim 10^{-15}$ m ) and very large objects as neutron stars ( $R \sim 10^{4}$ m )

- Nuclear Physics: Neutron skin thickness in finite nuclei, stable nuclei, Heavy-Ion collisions, Giant Resonances...
- Astrophysics: Supernova explosion, Neutron emission and cooling of protoneutron stars, Mass-Radius relations in neutron stars, Composition of the crust of neutron stars...

## Equation of State in asymmetric matter

$$e(\rho, \delta) = e(\rho, 0) + c_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) \qquad \left(\delta = \frac{\rho_n - \rho_p}{\rho}\right)$$

Around the saturation density we can write

$$e(\rho, 0) \simeq a_v + \frac{1}{2}K_v\epsilon^2 \quad and \quad c_{sym}(\rho) \simeq J - L\epsilon + \frac{1}{2}K_{sym}\epsilon^2 \quad \left(\epsilon = \frac{\rho_0 - \rho}{3\rho_0}\right)$$

$$\rho_0 \approx 0.16 fm^{-3}, \quad a_v \approx -16 MeV, \quad K_v \approx 230 MeV, \quad J \approx 32 MeV$$

However, the values of

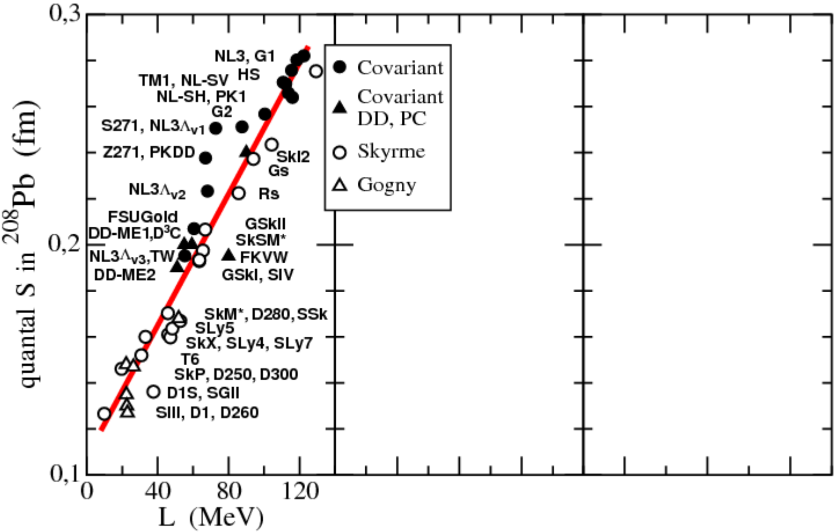
$$L = 3\rho\partial c_{sym}(\rho)/\partial\rho|_{\rho_0} \qquad and \qquad K_{sym} = 9\rho^2\partial^2 c_{sym}(\rho)/\partial\rho^2|_{\rho_0}$$

which govern the density dependence of $c_{sym}$ near $\rho_0$ are less certain and predictions vary largely among nuclear theories.

## Experimental constraints

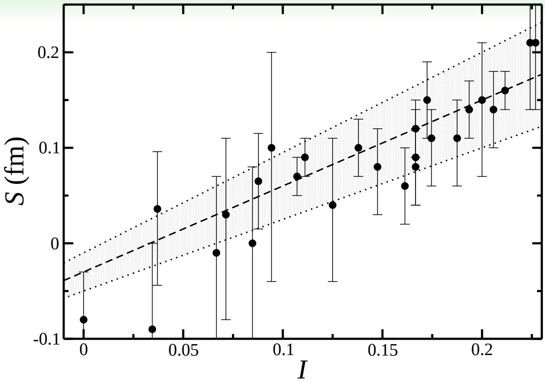
- Recent reseach in heavy-ion collisions at intermediate energy is consistent with $c_{sym}(\rho) = c_{sym}(\rho_0).(\rho/\rho_0)^\gamma$ at $\rho < \rho_0$.
- Isospin difussion $\gamma = 0.7$–$1.05$ ($L = 88 \pm 25$ MeV).
- Isoscaling $\gamma = 0.69$ ($L \sim 65$ MeV)
- Inferred from nucleon emision ratios $\gamma = 0.5$($L \sim 55$ MeV).
- The GDR of $^{208}$Pb analyzed with Skyrme forces suggests a constraint $c_{sym}(0.1\,\mathrm{fm}^{-3})$=23.3–24.9 MeV ($\gamma \sim 0.5$–$0.65$).
- The study of the PDR in $^{68}$Ni and $^{132}$Sn predicts $L$=49-80 MeV.
- The Thomas-Fermi model of Myers and Swiatecki fitted very precisely to binding energies of 1654 nuclei predicts an EOS that yields $\gamma = 0.51$
- NEUTRON SKIN THICKNESS ?

# Neutron skin thickness



## What is experimentally know about neutron skin thickness in nuclei ?

- The neutron skin thickness is defined as $S = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$, where $\langle r_n^2 \rangle^{1/2}$ and $\langle r_p^2 \rangle^{1/2}$ are the rms radii of the neutron and proton distributions respectively.
- $\langle r_p^2 \rangle^{1/2}$ is known very accurately from elastic electron scattering measurements (*e.g.* $r_{ch}(^{208}\mathrm{Pb}) = 5.5010 \pm 0.0009$ fm [Angeli (2004)]).
- $\langle r_n^2 \rangle^{1/2}$ has been obtained with hadronic probes such as:
  (a) Proton-nucleus elastic scattering ($5.522\mathrm{fm} < r_n(^{208}\mathrm{Pb}) < 5.550$ fm [Clark (2003)]).
  (b) Inelastic scattering excitation of the giant dipole and spin-dipole resonances ($r_n(^{208}\mathrm{Pb}) = 5.67 \pm 0.07$ fm [Krasznahorkay (1990)]).
  (c) Antiprotonic atoms: Data from antiprotonic X rays and radiochemical analysis of the yields after the antiproton annihilation ($r_n(^{208}\mathrm{Pb}) = 5.66 \pm 0.02$ fm) [Trzcińska (2001)].




$$S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02)\ \text{fm}$$

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

CAN $S$ OF 26 STABLE NUCLEI, FROM $^{40}$Ca TO $^{238}$U, ESTIMATED USING ANTIPROTONIC ATOMS DATA HELP IN CONSTRAINING THE SLOPE AND CURVATURE OF $c_{sym}$ ?

## Symmetry energy and neutron skin thickness in the Liquid Drop Model

- Symmetry Energy

$$a_{sym}(A) = \frac{J}{1 + x_A}, \quad x_A = \frac{9J}{4Q} A^{-1/3}$$

$$E_{sym}(A) = a_{sym}(A)(I + x_A I_C)^2 A$$

where

$$I = (N - Z)/A, \quad I_{\mathrm{C}} = e^2 Z/(20JR), \quad R = r_0 A^{1/3}$$

.

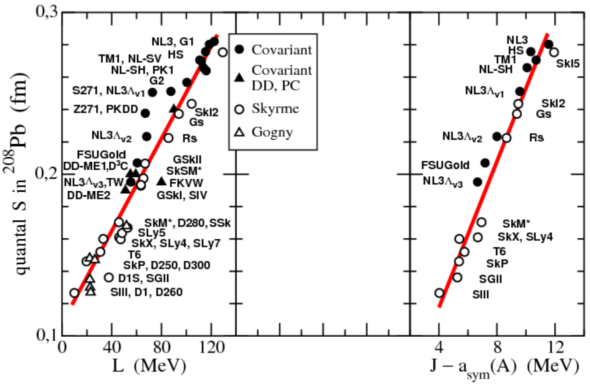
- Neutron skin thickness

$$S = \sqrt{3/5}\left[t - e^2 Z/(70J) + \frac{5}{2R}(b_n^2 - b_P^2)\right]$$

where

$$t = \frac{3r_0}{2}\,\frac{J/Q}{1 + x_A}\,(I - I_{\mathrm{C}})$$

## Neutron skin thickness




$$t = \frac{2r_0}{3J}\,[J - a_{sym}(A)]\,A^{1/3}\,(I - I_{\mathrm{C}})$$

Table: Value of $a_{sym}(A)$ and density $\rho$ that fulfils $c_{sym}(\rho) = a_{sym}(A)$ for $A = 208, 116$ and 40 in MF models. $J$ and $a_{sym}$ are in MeV and $\rho$ is in $\mathrm{fm}^{-3}$.

| | | $A = 208$ | | $A = 116$ | | $A = 40$ | |
|---|---|---|---|---|---|---|---|
| Model | $J$ | $a_{sym}$ | $\rho$ | $a_{sym}$ | $\rho$ | $a_{sym}$ | $\rho$ |
| NL3 | 37.4 | 25.8 | 0.103 | 24.2 | 0.096 | 21.1 | 0.083 |
| NL-SH | 36.1 | 25.8 | 0.105 | 24.6 | 0.099 | 21.3 | 0.086 |
| FSUGold | 32.6 | 25.4 | 0.098 | 24.2 | 0.090 | 21.9 | 0.075 |
| TF-MS | 32.6 | 24.2 | 0.093 | 22.9 | 0.085 | 20.3 | 0.068 |
| SLy4 | 32.0 | 25.3 | 0.100 | 24.2 | 0.091 | 22.0 | 0.075 |
| SkX | 31.1 | 25.7 | 0.102 | 24.8 | 0.096 | 22.8 | 0.082 |
| SkM* | 30.0 | 23.2 | 0.101 | 22.0 | 0.093 | 19.9 | 0.078 |
| SIII | 28.2 | 24.1 | 0.093 | 23.4 | 0.088 | 21.8 | 0.077 |
| SGII | 26.8 | 21.6 | 0.104 | 20.7 | 0.096 | 18.9 | 0.082 |

## The $c_{sym}(\rho)$-$a_{sym}(A)$ correlation

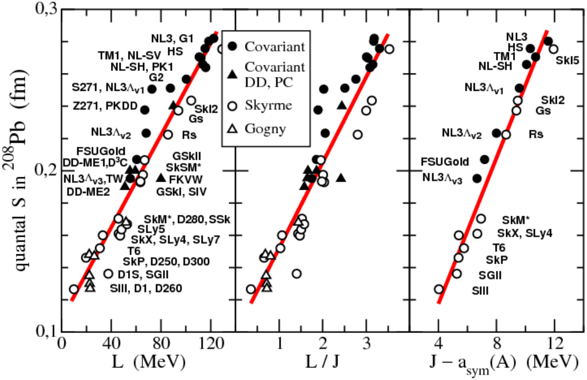
- There is a genuine relation between the symmetry energy coefficients of the EOS and of nuclei: $c_{sym}(\rho)$ equals $a_{sym}(A)$ of heavy nuclei like $^{208}$Pb at a density $\rho = 0.1 \pm 0.01\,\mathrm{fm}^{-3}$.
- A similar situation occurs down to medium mass numbers, at lower densities.
- We find that this density can be very well simulated by

$$\rho \approx \rho_A = \rho_0 - \rho_0/(1 + cA^{1/3})\,,$$

  where $c$ is fixed by the condition $\rho_{208} = 0.1\,\mathrm{fm}^{-3}$.
- Using the equality $c_{sym}(\rho) = a_{sym}(A)$ and the LDM , the neutron skin thickness can be finally written as:

$$t = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1 - \epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I - I_{\mathrm{C}})$$

# Neutron skin thickness




$$t = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1-\epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I-I_{\rm C})$$

## Fitting procedure and results

- We optimize

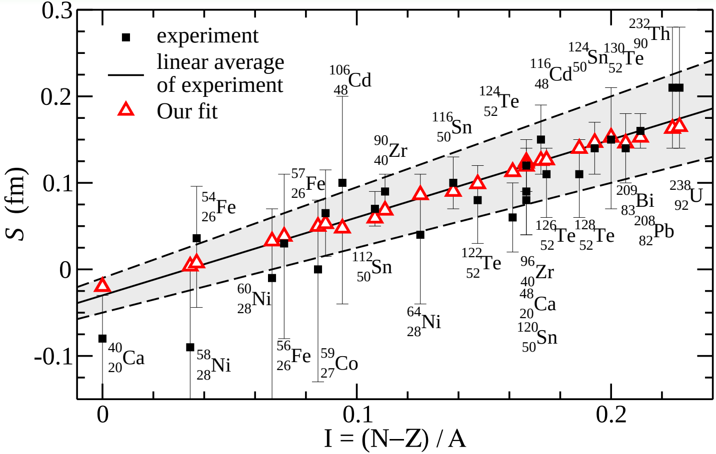
$$t = \sqrt{\frac{3}{5}}\frac{2r_0}{3}\frac{L}{J}\left(1 - \epsilon\frac{K_{sym}}{2L}\right)\epsilon A^{1/3}(I - I_{\rm C})$$

using

$$c_{sym} = 31.6(\frac{\rho}{\rho_0})^{\gamma} MeV, \quad \epsilon = \frac{1}{3(1 + cA^{1/3})}, \quad \rho_0 = 0.16 fm^{-3}$$

and taking as experimental baseline the neutron skins measured in 26 antiprotonic atoms.

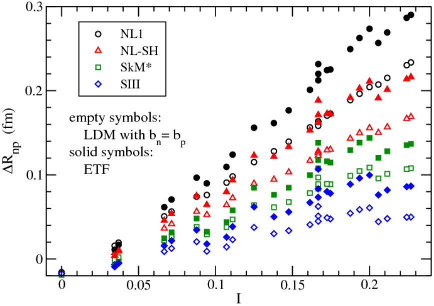
- We predict $(b_n \approx b_p)$: $L = 75 \pm 25$ MeV

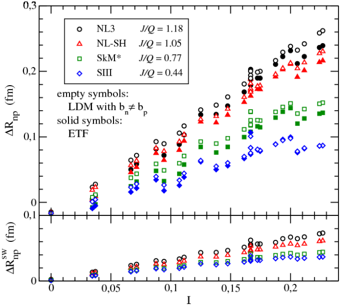



$$S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02)\ \mathrm{fm}$$

A. Trzcińska et al, Phys. Rev. Lett. **87**, 082501 (2001)

## Influence of the surface width ($b_n \neq b_p$)

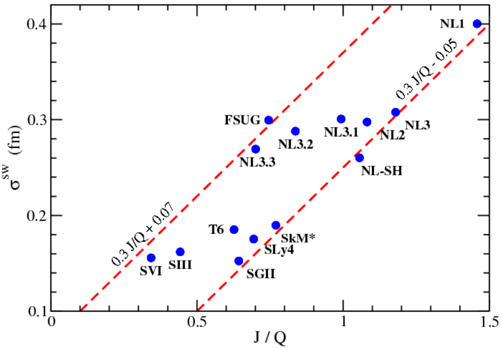



$$S = \sqrt{3/5}\left[t - e^2 Z/(70J) + \frac{5}{2R}(b_n^2 - b_P^2)\right]$$

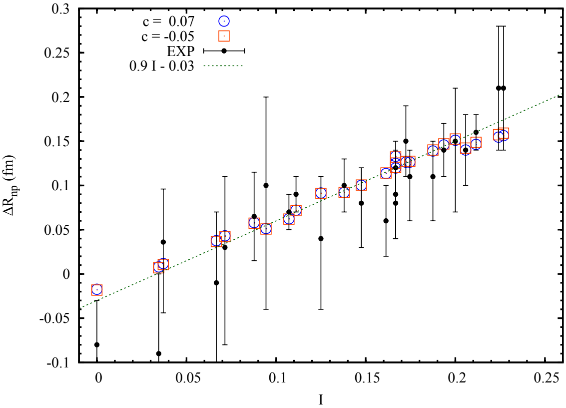
$b_n$ and $b_p$ are obtained at the ETF level.

# Surface contribution to the neutron skin thickness




$$\sqrt{\frac{3}{5}}\frac{5}{2R}(b_n^2 - b_p^2) = \sigma^{sw} I = (0.3\frac{J}{Q} + c) I$$

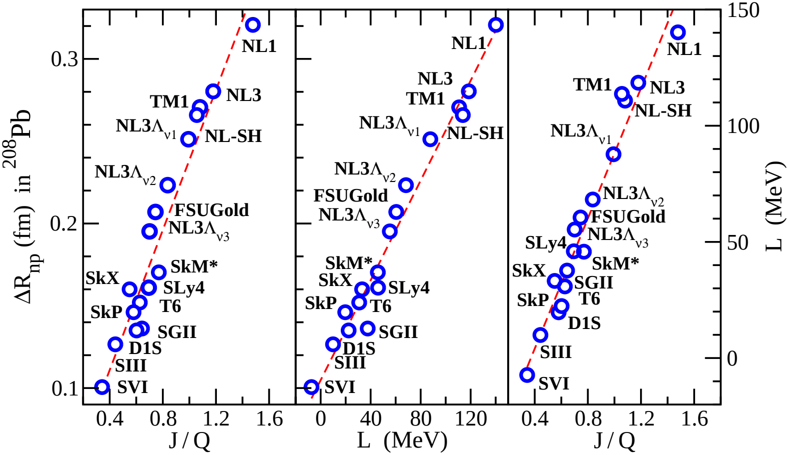
# Fit and results




With $\rho_0 = 0.16$ fm$^{-3}$ and $28 \lesssim J \lesssim 35$ MeV, and $-0.05 \lesssim c \lesssim 0.07$ fm

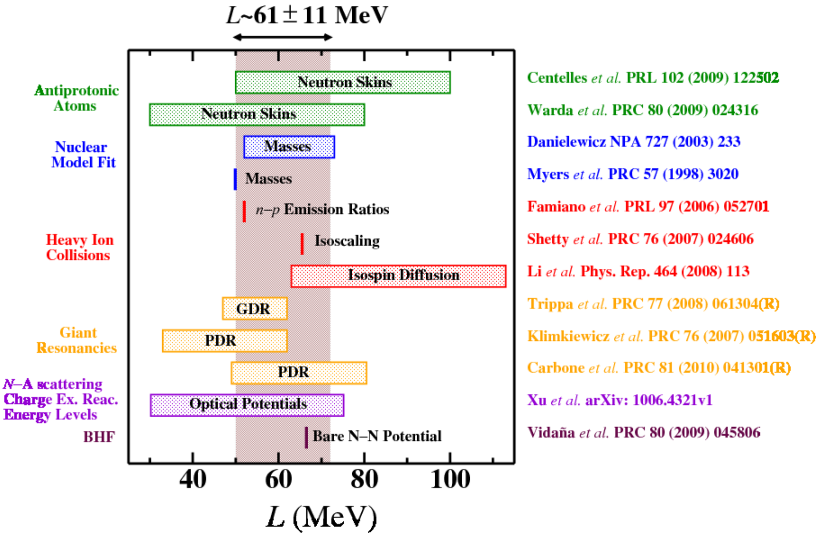
$$\frac{J}{Q} = 0.6 - 0.9$$

# Neutron skin thickness



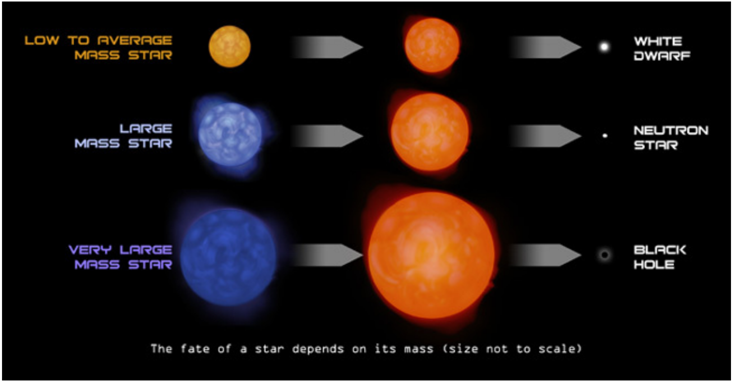

$L = 30 - 80$ MeV

# Constraints on the slope of the symmetry energy



## From the very small to the very big: the symmetry energy and the outer crust of a neutron star

# Introduction




| $R$ (Km) | $\bar{\rho}$ (gr/cm$^3$) | $v_{\rm scape}/c$ | $g/g_{\rm Earth}$ (surface) | $P$ (dyn/cm$^2$) |
|---|---|---|---|---|
| 10 | $10^{14} - 10^{15}$ | 0.5 | $10^{11}$ | $0 - 10^{35}$ |

*Orientative properties of a typical neutron star of mass $M = M_{\rm Sun}$.

## Formalism

Total energy per nucleon

$$e(A, Z, \rho = \rho_n + \rho_p) = e_N(A, Z) + e_{lat}(A, Z, \rho) + e_{el}(\rho)$$

The different contributions

$$e_N(A, Z) = \frac{M(A,Z)}{A}$$

$$e_{lat}(A, Z, \rho) = -C_{lat} \frac{Z^2}{A^{4/3} p_F}$$

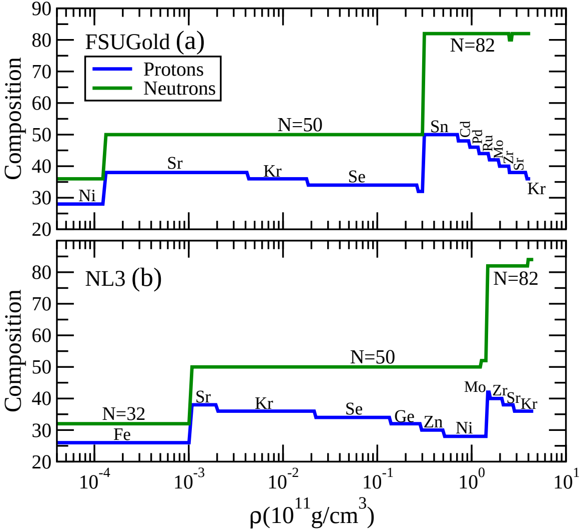
where $C_{lat} = 0.00341$ and

$p_F = (3\pi^2 \rho)^{1/3} = p_{F_{el}} (A/Z)^{1/3}$ $(N_{el} = Z)$

$$e_{el}(\rho) = \frac{m_{el}^4}{8\pi^2 \rho} \left( x_F y_F (x_F^2 + y_F^2) - \ln(x_F + y_F) \right)$$

where $x_F \equiv p_{F_{el}}$ and $y_F \equiv \frac{\epsilon_{F_{el}}}{m_{el}} = \sqrt{1 + x_F^2}$

# Composition of the outer crust



**The stiffer the symmetry energy the more exotic the composition of the outer crust and the larger the neutron skin of medium and heavy elements**

$\Delta R_{np}^{\mathrm{NL3}}(^{208}\mathrm{Pb}) = 0.28$ fm and $\Delta R_{np}^{\mathrm{FSUGold}}(^{208}\mathrm{Pb}) = 0.20$ fm

## Summary and Conclusions

- We have described a generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation.
- We take advantage of this relation to explore constraints on $c_{sym}(\rho)$ from neutron skins measured in antiprotonic atoms. These constraints points towards a soft symmetry energy.
- We discuss the $L$ values constrained by neutron skins in comparison with most recent observations from reactions and giant resonances.
- We learn that in spite of present error bars in the data of antiprotonic atoms, the size of the final uncertainties in $L$ is comparable to the other analyses.
- The generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation plausibly encompasses other prime correlations of nuclear observables with the density content of the symmetry energy as e.g. the constrains of $c_{sym}(0.1)$ from the GDR of $^{208}$Pb (L. Trippa et al. Phys. Rev. **C77**, 061304(R) (2008)).

**Thank you for your attention**

# Extra material

## Some technical details

- The surface stiffness coeficient $Q$ and the surface widths $b_n$ and $b_p$ are obtained from self-consistent calculations of the neutron and proton density profiles in asymmetric semi-infinite nuclear matter.
- To this end one has to minimize the total energy per unit area with the constraint of conservation of the number of protons and neutrons with respect to arbitrary variations of the densities.

$$\frac{E_{\rm const}}{S} = \int_{-\infty}^{\infty} \left[\varepsilon(z) - \mu_n \rho_n(z) - \mu_p \rho_p(z)\right] dz,$$

  where $\varepsilon(z)$ is the nuclear energy density functional.
- In the non-relativistic framework the densities $\rho_n$ and $\rho_p$ obey the coupled local Euler-Lagrange equations:

$$\frac{\delta\varepsilon(z)}{\delta\rho_n} - \mu_n = 0, \qquad \frac{\delta\varepsilon(z)}{\delta\rho_p} - \mu_p = 0.$$

  The relative neutron excess $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is a function of the $z$-coordinate. When $z \to -\infty$ , the densities $\rho_n$ and $\rho_p$ approach the values of asymmetric uniform nuclear matter in equilibrium with a bulk neutron excess $\delta_0$.

- From the calculated density profiles one computes:

$$z_{oq} = \frac{\int_{-\infty}^{\infty} z\rho'_q(z)dz}{\int_{-\infty}^{\infty} \rho'_q(z)dz},$$

$$b_q^2 = \frac{\int_{-\infty}^{\infty} (z - z_{0q})^2 \rho'_q(z)dz}{\int_{-\infty}^{\infty} \rho'_q(z)dz}.$$

- From the relation

$$t = z_{0n} - z_{0p} = \frac{3r_0}{2}\frac{J}{Q}\delta_0,$$

one can evaluate $Q$ from the slope of $t$ at $\delta_0 = 0$.

- The distance $t$ and the surface widths $b_n$ and $b_p$ in finite nuclei with neutron excess $I = (N - Z)/A$ are obtained using $\delta_0$ given by:

$$\delta_0 = \frac{I + \frac{3}{8}\frac{c_1}{Q}\frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4}\frac{J}{Q}A^{-1/3}}.$$