QCD-view on hadron form factors in space-like and time-like momentum transfer regions

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Abstract

QCD gives definite predictions for hadron electromagnetic form factors in space-like and time-like momentum transfer regions, such as the quark counting rules, the hypothesis of hadron helicity conservation, and the relations between nucleon and deuteron form factors in the formalism of reduced nuclear matrix elements. Recent precise data about these form factors, obtained in polarization experiments at the Jefferson Laboratory, have essentially changed our view on this subject. QCD-predictions do not apply to these data up to Q^2=5-6 GeV^2 for deuteron and for the electric form factor of proton. An analysis of these data suggests that the asymptotic region will more probably start at Q^2=20-25 GeV^2. We show that the separation of magnetic and electric proton form factors in the time-like region represents the most stringent test of the asymptotic regime and QCD-predictions.

In this talk we will discuss the recent developments in the field of hadron electromagnetic form factors (FFs), due to the very precise and surprising data obtained at the Jefferson Laboratory (JLab), in $\vec{e}+p\to e+\vec{p}$ elastic scattering.

The application of the polarization transfer method, proposed about 30 years ago [1] has been possible only recently, as it needs high intensity polarized beams, large solid angle spectrometers and advanced techniques of polarimetry [2–4] in the GeV range.

At these energies, one probes distances of the order of the nucleon size or less, and would expect expect the manifestation of quark degrees of freedom.

FFs, which characterize the internal structure of composite particles, can be experimentally measured and theoretically calculated, thus providing a good playground for the models of nucleon structure.

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In a P and T invariant theory, a particle with spin S is characterized by $2S+1$ electromagnetic form factors, which are complex (real) functions in the time(space)-like region. The nucleon has two FFs, called electric G_{EN} and magnetic G_{MN}, which, a priori, are different.

Before the new, precise data appeared in the space-like (SL) region, the dipole approximation, $G_D = [1 + Q^2/a]^{-2}$ with $a = 0.71$ GeV^2, has beeen considered a good approximation for three of the four nucleon FFs, G_{En} being considered $\simeq 0$ or according to [5].

In the Breit system, FFs are related to the Fourier transform of the charge and magnetic moment distribution and the dipole approximation results from an exponential distribution, the coefficient $a = 0.71$ GeV^2 corresponding to a root mean squared radius $\sqrt{< r^2 >}$=0.81 fm.

Perturbative QCD gives definite rules about the scaling behavior of the form factors and about helicity conservation, indicating where quark degrees of freedom should be taken explicitely into account. Scaling laws, which give the probability that a hadron remains intact after absorbing a photon of momentum Q^2, have been formulated in [6] as $F = [1 + Q^2/(n\beta_n^2)]^{n-1}$, where n is the number of quarks and β_n the quark average momentum. When applied to proton, they give the same power law as dipole, including the coefficient $a = 0.71$, derived from a fit to pion form factors.

The traditional way to access FFs in the SL region, is the elastic scattering of electrons on hadrons $e+h \to e+h$, and in the time-like (TL) region, the annihilation processes $e^+ + e^- \leftrightarrow p + \overline{p}$. The unpolarized elastic ep cross section, in one-photon exchange approximation, can be written as a function of the electric G_{Ep} and magnetic G_{Mp} proton form factors [7]:

$$\frac{d\sigma}{d\Omega}(ep \to ep) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_{Ep}^2 + \tau G_{Mp}^2}{1+\tau} + 2\tau G_{Mp}^2 \tan^2\frac{\theta_e}{2}\right],\ \tau = \frac{Q^2}{4M^2} \tag{1}$$

with

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2\ \cos^2(\theta_e/2)E'}{4E^3\sin^4(\theta_e/2)} \text{ and } E' = \frac{E}{1 + 2\dfrac{E}{M}\sin^2(\theta_e/2)}$$

where M is the proton mass, E is the energy of the incident electron, E' and θ_e are the energy and scattering angle of the outgoing electron, α is the fine structure constant. The momentum of the virtual photon is $Q^2 = 4EE'\sin^2(\theta_e/2)$ and it is positive in the SL region.

In the TL region the cross section can be expressed as a function of FFs according to the following formula [8]:

$$\frac{d\sigma}{d(cos\theta)}(e^+e^- \to p\bar{p}) = \frac{\pi\alpha^2}{8M^2\tau\sqrt{\tau(\tau-1)}}\left[\tau|G_{Mp}|^2(1+\cos^2\theta) + |G_{Ep}|^2\sin^2\theta\right], \tag{2}$$

where θ is the angle between the electron and the antiproton in the center of mass frame.

Eqs. (1) and (2), contain the moduli squared of the FFs, therefore one can not access their sign. Moreover the contribution of G_{Mp} appears weighted by the factor τ: as Q^2 increases, it becomes the dominant term, making the extraction of G_{Ep} very imprecise.

In the SL region the collisions of polarized electron and polarized target (or measuring the polarization of the scattered proton), induce an interference term proportional to the product $G_{Ep}G_{Mp}$. It is therefore more sensitive to a small contribution of G_{Ep} .

Data have been obtained at JLab up to $Q^2 = 5.6$ GeV^2 and an extension up to 9 GeV^2 is in preparation [9]. The measurement of the polarization of protons at momentum up to $P_p = 5.3$ GeV/c can be done with a POMME-like polarimeter, as it has been proved at the JINR-LHE, in Dubna [10]. Although the analyzing powers for the inclusive reaction $p + CH_2 \rightarrow \text{one charged particle} + X$ decreases with increasing incident momentum, it is still sizeable at a proton momentum of 5.3 GeV.

The recent data [3, 4] show a linear deviation form the dipole behavior and can be parametrized by $\mu\ G_{Ep}\ /\ G_{Mp} = 1.0 - 0.130(Q^2/\text{GeV}^2 - 0.04)$ [4] (μ is the proton magnetic moment), definitely proving that the electric and magnetic distributions in the proton are different.

Although different models can reproduce this trend, (as constituent quark models [11], soliton model [12], diquark model [13] ..) few of them give a satisfactory description of all four electromagnetic form factors and even fewer of them are applicable in SL and TL regions. I would like to quote here the models based on vector meson dominance (VMD)[14]. The most recent one [15] contains several parameters to fit the world data and includes the asymptotic QCD behavior. Note that 'Il Nuovo Cimento' reported the first studies, in this field. In Ref. [16] the right trend for the G_{Ep} was already proposed, and in [17] a 'one parameter fit' gave a good qualitative description of the present data and prescriptions for TL region as well. The best VMD-predictions, concerning the proton FFs, are given by [18] (Fig. 1). A compilation of the world data reported in the figure can be found in [15].

The nucleon FFs are important ingredients for the calculations of the light nuclei structure. One of the consequences of the new data is the revision of the models of the deuteron structure. Following [6], let us introduce a generalized deuteron FF, $F_D(Q^2)$, $F_D(Q^2) = \sqrt{A(Q^2)}$, where $A(Q^2)$ is the structure function related to the forward deuteron

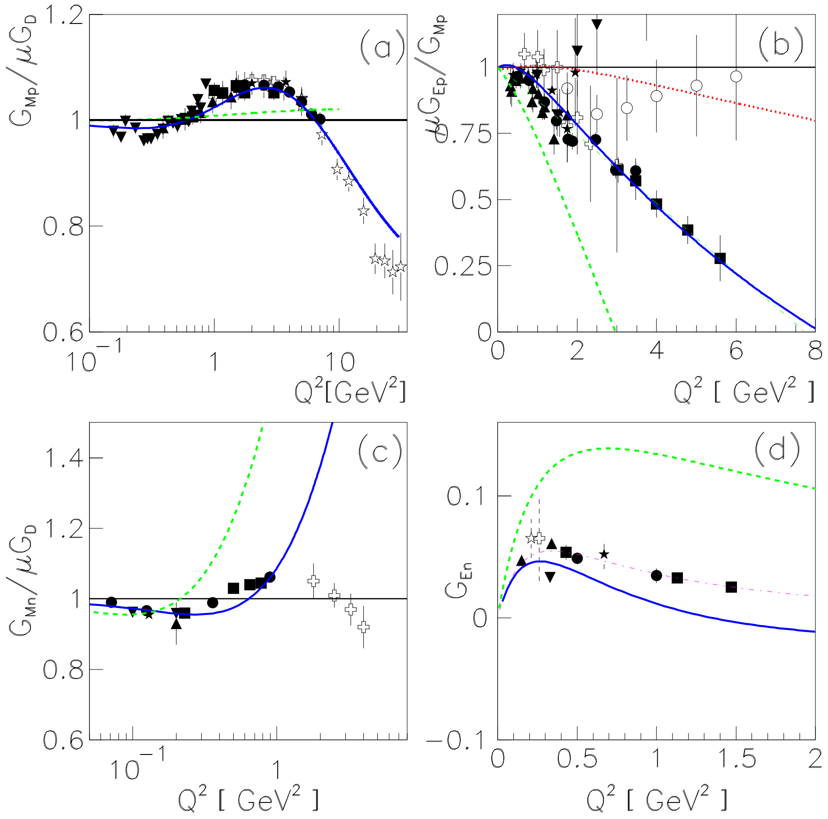

FIG. 1: Nucleon FFs in the SL region. The magnetic FF for proton (a) and for neutron (c) are normalized to one and divided by G_D. The proton electric FF is shown as the ratio $\mu G_{Ep}\,/G_{Mp}$ (b), the dotted line is from [16]. The neutron electric FF (c) is plotted with the [5] parametrization (dash-dotted line). The solid line is from [18], the dashed line from [17].

cross section, and a reduced deuteron FF $f_D(Q^2)$:

$$f_D(Q^2) = \frac{F_D(Q^2)}{F_N^2(Q^2/4)}, \tag{3}$$

where F_N is the nucleon electromagnetic FF. The Q^2-behavior of $f_D(Q^2)$ (at large Q^2) can be predicted in the framework of pQCD, in the following form:

$$f_D(Q^2) = N\frac{\alpha_s(Q^2)}{Q^2}\left(ln\frac{Q^2}{\Lambda^2}\right)^{-\Gamma}, \tag{4}$$

where N is the normalization factor (which is not predicted by QCD), α_s is the running QCD strong interaction coupling constant, Λ is the scale QCD parameter, and Γ is determined by the leading anomalous dimension, here $\Gamma = -\dfrac{8}{145}$.

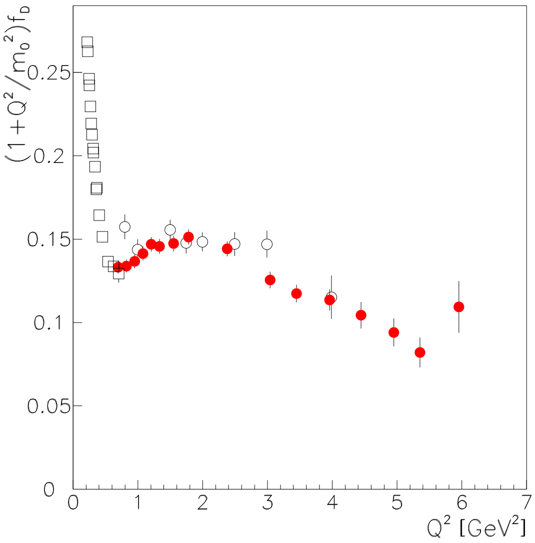

FIG. 2: Data set corresponding to the reduced deuteron FFs multiplied by $(1 + Q^2/m_0^2)$. Open circles are from [21], open squares from [22], solid circles from [19].

In [19] it was shown that the QCD prediction (4), which can be applied to asymptotic momentum transfer, is working well already for $Q^2 \geq 2$ GeV^2, with a plausible value of the parameter $\Lambda \simeq 100$ MeV, in agreement with the values determined by many other possible methods [20].

In [6] another interesting prediction, concerning the scaling behavior of the reduced deuteron FF, was done:

$$f_R = \left(1 + \frac{Q^2}{m_0^2}\right) f_D(Q^2) \simeq const, \tag{5}$$

where $m_0^2 = 0.28$ GeV^2 is a parameter related to the Q^2-behavior of the pion FF. The same data from [19], if plotted in the representation of the reduced deuteron FFs, should illustrate the Q^2-independence of this product.

This result was confirmed by the previous $A(Q^2)$ data [21], in the limit of their accuracy. In Fig. (2) we show that the new, more precise data about $A(Q^2)$ [19], are not consistent with the prediction (5) as they show an evident dependence of the product f_R on Q^2, even for Q^2 $\leq$2 GeV^2. This behavior can not be changed by varying the parameter m_0.

Although the scaling laws seem to be consistent with cross section measurements, up to 6 GeV^2, if one replaces the dipole approximation with other descriptions of the nucleon FFs,

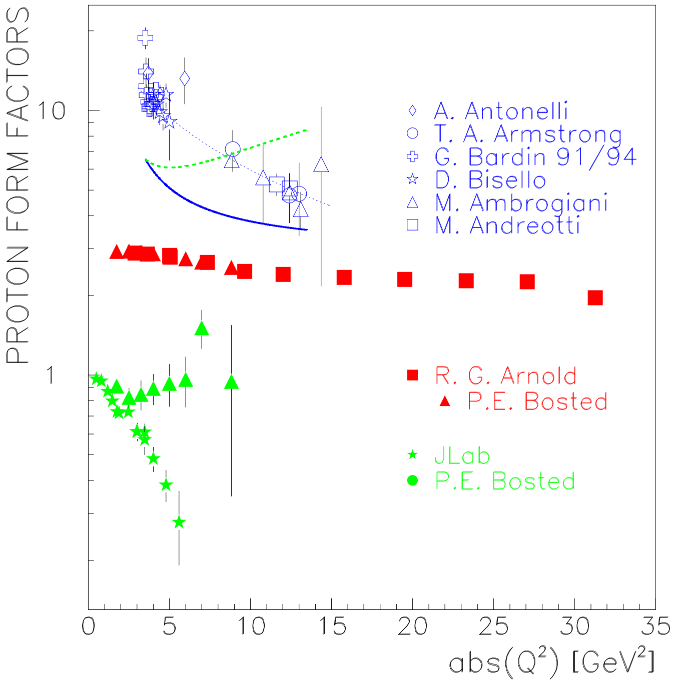

FIG. 3: Data for electric and magnetic FFs in SL (solid symbols) and TL (open symbols) regions, scaled by dipole, as functions of the modulus of Q^2. The dotted line is a fit to the TL data, according to the functional form described in the text.The solid (dashed) line is the prediction for G_{Mp} (G_{Ep}) from [17].

taking into account the deviation from dipole for G_{Ep} , a fit following Eqs. (3) and (4) shows a large instability for Λ [23].

One should mention that a calculation based on the impulse approximation, where the nucleon FF are taken form [14], satisfactorily reproduces the existing data on the three deuteron FFS [24].

We will go a step further, and look for a definition of the asymptotic region with respect to the analyticity properties of complex functions.

FFs must obey the Phràgmen-Lindelöf theorem [25], which gives a rigorous prescription for the asymptotic behavior of analytical functions: $\lim_{Q^2\to -\infty} F^{(SL)}(Q^2) = \lim_{Q^2\to\infty} F^{(TL)}(Q^2)$. This means that, asymptotically, the TL phase vanishes and the real part of the FFs, $\mathcal{R}e F^{(TL)}(Q^2)$, coincides with the corresponding value $F^{(SL)}(Q^2)$.

The Rosenbluth separation has not yet been realized in the TL region. In order to extract the FFs, due to the poor statistics, it is necessary to integrate the differential cross section over a wide angular range. One assumes that the G_{Ep} -contribution plays a minor role in the cross section and the experimental results are usually given in terms of $|G_{Mp}|$, under the hypothesis that G_{Ep} =0 or $|G_{Ep}| = |G_{Mp}|$. The first hypothesis is arbitrary. The second hypothesis is strictly valid at threshold only, but there is no theoretical argument which justifies its validity at any other momentum transfer, where $s \neq 4M^2$. The prediction for the TL region in [8] shows that if G_{Ep} $/G_D$ decreases, G_{Mp} $/G_D$ increases, due to the definitions (dashed and solid line, respectively in Fig. 3).

Therefore a comparison of data in TL and SL region should give an unambiguous indication on the asymptotic region. The experimental data are shown in Fig. 3, normalized to the function G_d. For a compilation see [26], here updated with recent data [28] (open squares).

The values of G_{Mp} in the TL region, obtained under the assumption that $|G_{Ep}| = |G_{Mp}|$ (open symbols), are larger than the corresponding SL values (solid squares and solid triangles). This has been considered as a proof of the non applicability of the Phràgmen-Lindelöf theorem, or as an evidence that the asymptotic regime is not reached [27].

The magnetic form factor of the proton in the TL region (which is deduced from the hypothesis G_{Ep} =0 (case 1) or G_{Ep} =G_{Mp} (case 2), can be parametrized as: $G_M^{(TL)} = G_d a/\left(1+s/m_{nd}^2\right)$, where a is a normalization parameter and $m^2_{nd} = 3.6 \pm 0.9$ GeV^2 characterizes the deviation from the usual dipole s-dependence. The extrapolation to higher s based on this formula (Fig. 3, dotted line), indicates that the Phràgmen-Lindelöf theorem will be satisfied by this FF, only for $s(Q^2) \geq 20$ GeV^2.

Let us assume now that one of the two proton electromagnetic FFs has reached the asymptotic regime and apply the Phràgmen-Lindelöf theorem to extract the other. This looks as a reasonable hypothesis for G_M, which shows an early scaling behavior, in accordance with quark counting rules. From Eq. 2 we can deduce $|G_E|$, using the existing experimental data about $\overline{p} + p \leftrightarrow e^+ + e^-$ (case 3). We report, in Fig. 4, the recent data in TL region, reanalized following the possibilities suggested above. Fig. 4a shows the values of the form factors taking $G_E = 0$ (circles) and $|G_E| = |G_M|$ (squares) respectively). For case 3, where $G_M = G_D$, the values of $|G_E|$ (triangles) are larger than in cases 1 and 2. This suggests that asymptotics are not reached for G_E, as the values in the SL and TL regions get more apart. A fourth possibility is taking for G_E in the TL region the values from [3, 4] and calculate

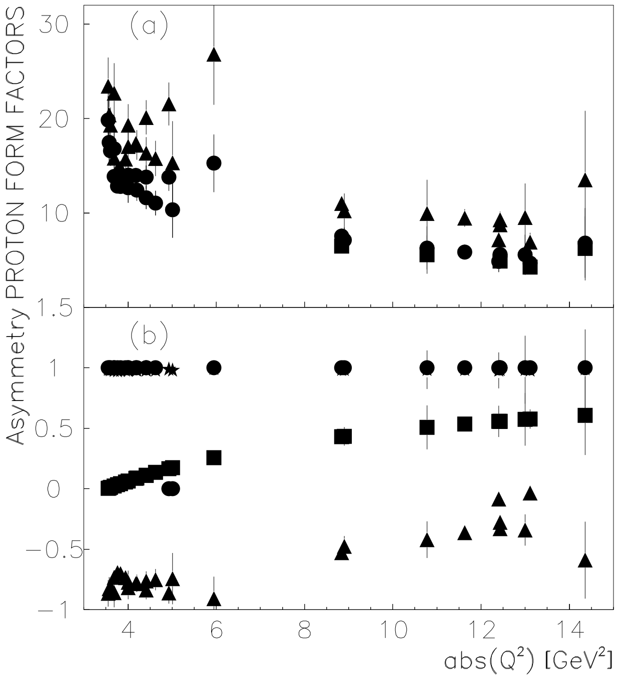

FIG. 4: Nucleon form factors (a) and angular asymmetry (b) in TL region, deduced from the data according to different assumptions (see text).

$|G_M|$ (case 4). This affects very little the values of G_M, due to the kinematical factor τ, which weights the magnetic contribution to the differential cross section (stars).

One can express the angular dependence of the differential cross section as a function of the angular asymmetry $\mathcal{A}$ as:

$$\frac{d\sigma}{d(\cos\theta)} = \sigma_0 \left[1 + \mathcal{A}\cos^2\theta\right], \text{ with } \mathcal{A} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2}, \tag{6}$$

where σ_0 is the value of the differential cross section at $\theta = \pi/2$. Fig. 4b shows the angular asymmetry for the different cases. Case 1 and case 2 give, respectively, $\mathcal{A} = 1$ and $\mathcal{A} = (\tau - 1)/(\tau + 1)$. The calculated asymmetries are very sensitive to the different underlying assumptions, therefore a precise measurement of this quantity would be very interesting.

Finally, we note that the angular dependence of the cross section, Eq. (2), results directly from the assumption of one-photon exchange, where the spin of the photon is equal 1 and the electromagnetic hadron interaction satisfies the $C-$invariance. Therefore the measurement of the differential cross section at three angles (or more) would also allow to test the presence of 2γ exchange. The relative role of the 2γ mechanism can increase at relatively large momentum transfer in SL and TL regions, for the same physical reasons, which are related to the steep decreasing of the hadronic electromagnetic FFs, as recently discussed in [29].

...instead of Conclusions

It is for me a great pleasure and honour to express my gratitude to Prof. R. A. Ricci, to whom this Conference is dedicated. As supervisor of my 'Tesi di Laurea, Università di Padova, Anno Accademico DCCLVI', he gave me the basis to progress in the field of Nuclear Physics. Since that time, his enthusiasm, his dynamic personality, his optimism in affording a variety of problems of different nature, his broad interests and the ability to listen, to understand and synthesize information very rapidly, have always been for me a source of motivation and example. I take this opportunity to thank Prof. Ricci, and to wish to him, besides the recognizements due by the italian and world-wide nuclear physics community, to fully enjoy a time (may be) more quiet and to have a lot of satisfaction in his family life.

E. T.-G.

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