A New Selection Rule for Isovector Reactions at Zero Degrees

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A general theorem, data, and nuclear reaction models are shown to demonstrate a new selection rule for isovector transitions from a 0^+;$T = 0$ target nucleus to a 0^+;$T = 1$ final state at zero degrees.

I. INTRODUCTION AND THE THEOREM

For a two-body nuclear reaction in which the ejectile is observed at zero (or 180) degrees, the beam direction is the only meaningful axis of quantization. For a spin zero projectile and ejectile, angular momentum and parity conservation are sufficient to demand that there be no transitions at zero degrees between nuclear states requiring a transfer of spin [1]. For example, the (α, α') reaction to the 1^+, $T = 0$ state of ^{12}C at 12.71 MeV shows an angular distribution dropping strongly towards small angles [2]. Several forms of reaction calculations reproduce this effect, since these conservation laws are included in those models [2].

Consideration of the Pauli Principle for the heavy system of zero spin also places a constraint on zero-degree cross sections for scattering or charge exchange reactions, not restricted to projectiles of zero spin, but dependent in a sensitive way to the isospin purity of the target system. Since the beam axis is the sole axis of quantization, also for the heavy system, the only spin coordinates that can be exchanged are among the projections along this axis. Let ΔS_z denote the algebraic change in that spin projection between any nucleon in the target nucleus and any nucleon in the final nucleus. For nuclear 0^+ to 0^+ transitions, ΔS and ΔS_z both must be zero, or more generally, even.

Under interchange of isospin coordinates between the target and final nucleus, let ΔT denote the algebraic difference between the initial and final nuclei. If the target nucleus has $T = 0$, with identical distributions of its neutrons and protons, the Pauli Principle demands that the reaction induce no net change in this symmetry under interchange of T and S for final states of positive parity. Hence, to retain the validity of the Pauli Principle for the heavy system, the only reactions that can give a cross section at zero (or 180) degrees are those that satisfy

$$(-1)^{\Delta S_z + \Delta T} = +1.$$

This is a hitherto unnoted selection rule, not included in considerations of the symmetry conditions for the projectile system alone. For 0^+ to 0^+ transtions from a $T = 0$ target, $\Delta T = 1$ (by charge exchange, for instance) cross sections must vanish at zero degrees, for a projectile of any spin. The symmetry between neutrons and protons under interchange is not unique for targets with a neutron excess, so this rule is not valid for all such transitions.

II. EXAMPLES AND CONNECTIONS

Clear examples of this zero degree rule are found for charge exchange and inelastic scattering on the $T = 0$ target ^{12}C, for which final states of spin zero are known. Consider the $(^3He, t)$ reaction leading to the $T = 1$ 0^+ state of ^{12}N at 2.43 MeV. The parity demands that $\Delta S = \Delta L$ be even, and the charge exchange ensures that ΔT=1. These conditions fail to meet the zero degree condition.

Data for this 0^+ to 0^+ ΔT=1 transition at 44 MeV indeed show a cross section dropping strongly towards zero degrees [3]. These data are shown in Figure 1. A $(^3He, t)$ experiment at 81 MeV used a wide angular acceptance for the detector, somewhat smearing the angular distributions, but also shows a decreasing cross section at zero degrees for the ^{12}N 2.43 MeV 0^+ state [4], as shown in Figure 2.

Selection rules similar to the one shown here are known to forbid or strongly retard Fermi beta decay 0^+ to 0^+ ΔT=1 transitions to a final $T = 0$ 0^+ state [5]. In the ($\overrightarrow{d}, \alpha$ reaction), where the deuteron spin projection can be selected to lie along the beam axis, a similar rule precludes the zero degree excitation of 0^+ states [6]. A specific model [7] destructively interfering $d_{3/2}$ and $f_{7/2}$ in a plane-wave two-step calculation was shown to yield the shape found for the $^{40}Ar(^3He, t)^{40}K$ reaction to the anti-analog state [8], which is another example of a transition forbidden by the present selection rule, producing a minimum, not the $L = 0$ maximum, at zero degrees.

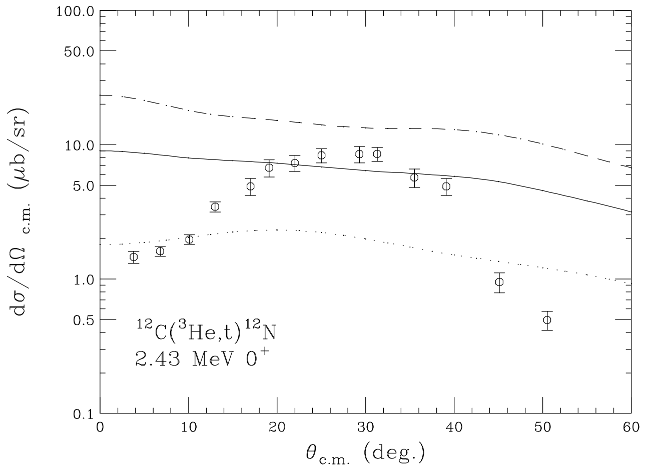

FIG. 1: Data shown are for the $^{12}C(^3He,t)^{12}N$ reaction at 44 MeV to the 0^+ state at 2.43 MeV[3]. The solid curve shows destructive interference between $p_{1/2}$ and $p_{3/2}$ contributions, taken to be of equal strength. The dashed curve shows the effect of a +10% change in the $p_{1/2}$ amplitude, and the dotted curve uses a 10% decrease in that amplitude.

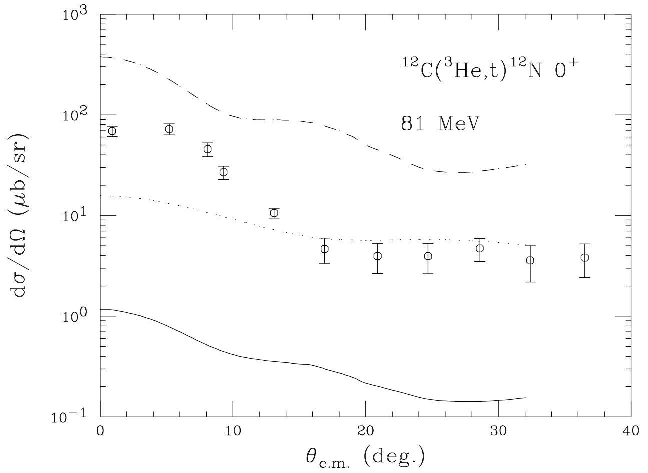

FIG. 2: As Figure 1, but for a beam energy of 81 MeV[4].

III. DESTRUCTIVE INTERFERENCE IN REACTION MODELS

In this Section reaction models will be used to demonstrate the mechanism by which the Pauli Principle for the heavy system is obeyed, and to examine the precison of the symmetry between nucleons in the heavy systems that must be found to bring agreement with the selection rule. The calculations are highly sensitive to the detailed assumptions, and the point of the calculations is to demonstrate effects, rather than achieve agreement with the data. The model uses the difference between $p_{1/2}$ and $p_{3/2}$ contribuions to the ^{12}C case, breaking the symmetry assumed for the model. This is in close analogy to deviations found in the beta decay examples.

The first vehicle used is the Distorted Wave Born Approximation calculation of the microscopic excitation of a 0^+ state coherently formed from $(p_{1/2}\ p^{-1}_{1/2})$ and $(p_{3/2}\ p^{-1}_{3/2})$ particle-hole excitations of ^{12}C. The DWBA code DWUCK4 [9] is used for these calculations. Nucleons were bound in potentials of radius 1.15 $A^{1/3} fm$, diffuseness a=0.80 fm and with 25 times the Thomas spin-orbit term. Optical potentials for the 44 MeV case are V = -106.5 MeV, $4W_D$=55.48 MeV, $r_{0R} = r_{oI} = 1.25 fm$ and $a_R = a_I = 0.80 fm$.

Figure 3 shows results at 44 MeV for either particle-hole excitation alone, as well as their constructively coherent sum. A microscopic projectile-nucleon Yukawa interation with a range parameter of $\mu = 0.7 fm^{-1}$ and a stength of 10 MeV was used for these demonstrations. The 2.43 MeV state of ^{12}N is bound to $p_{1/2}$ decay by 0.17 MeV, and unbound by 1.60 MeV to $p_{3/2}$ decay. The $p_{3/2}$ single particle state was thus treated as a resonance, with a single-particle decay width of 0.49 MeV. The physical width of the 0^+ state is 68 keV. Both components and their constructive sum show

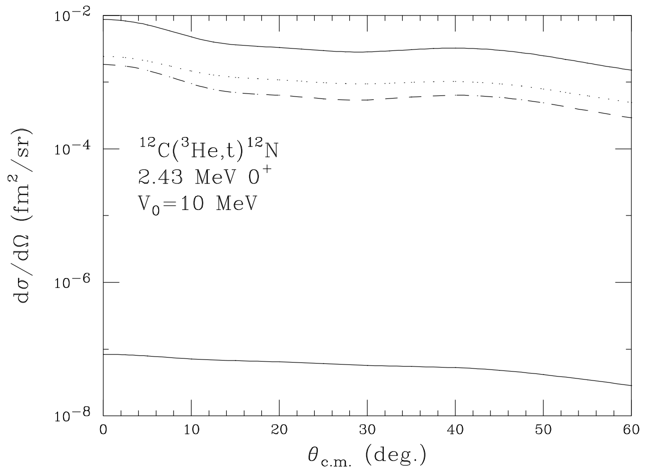

FIG. 3: DWBA calculations are shown for the 44 MeV $^{12}C(^3He,t)^{12}N$ 0^+ case, with $p_{1/2}$ contributions only shown as the dotted curve, $p_{3/2}$ contributions by the dashed curve, and their constructively coherent sum as the upper solid curve. The lower curve shows the striking effect of destructive interference, here for exactly the same form factors for the two shells.

the forward maximum expected for ΔL=0 transitions. Also shown in this figure is the destructively coherent sum, where exact symmetry was imposed upon the $p_{1/2}$ and $p_{3/2}$ components. That is, there was no spin-orbit potential in the bound states for either component, and all binding energies were taken to be the same. This symmetry and the destructive interference almost completely kill the cross sections.

Figure 1 shows DWBA destructively interfered cross sections for 44 MeV ($^3He,t$) on ^{12}C using a projectile-nucleon strength of 48 MeV for the Yukawa interaction The $p_{1/2}$ and $p_{3/2}$ wave functions were generated for actual binding energies, and do not show the extreme cancellation found for perfect symmetry. Equal potential strengths in this case mean equal contributions from the $p_{1/2}$ and $p_{3/2}$ participants. Changes of 10% for the $p_{1/2}$ interaction yield the dashed and dotted curves. Although the shape of the data is not matched, there is a strong suppression of the usual $L = 0$ forward maximum. Since the calculation is exquitely sensitive to many features of the model, it is difficult to identify the proper 'knob' to adjust to find a fit to the data.

Equal participation of $p_{1/2}$ and $p_{3/2}$ participants is used above. This is an artifact of the role of strong multistep processes not included in the DWBA. Analysis of the $^{12}C(^3He,\alpha)$ reaction [10] showed that the two-step path through the ground state of ^{11}C tended to weaken the yield to the 3/2- state and strengthen that to the 1/2- state, giving more nearly equal cross sections than would be expected from the occupation numbers alone. Since this coupling was not included in the present DWBA analysis, the $p_{3/2}$ component was weakened and the $p_{1/2}$ component strengthened to compensate for this effect.

Similar DWBA interference calculations are shown in Fig. 2 for the 81 MeV $^{12}C(3He,t)^{12}N$ reaction to the 0^+ state. The calculations were averaged over the angular acceptance of the detector for this experiment. Optical potentials were given in Ref. [4]. This time, 10% changes either way in the $p_{1/2}$ coupling cause increases in the cross sections, but again the effect of the destructive interference is seen. The forward peak in the data, averaged across the detector, is far below the usual $L = 0$ maximum.

Another reaction model used to study the selection rule was the two-step $^{12}C(^3He,\alpha)(\alpha,t)^{12}N$ reaction, using the Coupled Channel Born Approximation code CHUCK [11]. The strengths of the two pathways were set by matching known $^{12}C(^3He,\alpha)^{11}C$ cross sections [10], and assuming unit stripping spectroscopic factors. Results are shown in Fig. 4, with the two individual pathways alone, their constructive sum, and their destructive sum. Again, the destructive interference produces a minimum, not a maximum, at zero degrees. The search for the correct knob to adjust these calculations to match the data will be even more difficult than in the DWBA, and these curves should be used only as general demonstrations of the effect of the sign of the interference, as imposed by the Pauli Principle.

IV. A DIFFRACTIVE LESSON

In the adiabatic strong absorption limit, a 0^+ to 0^+ transition would have a shape given by $|J_0(kRsin\theta|^2$ [12]. Two destructively interfering amplitudes, with values of the wave number given by k and $k + \delta k$, would give the difference

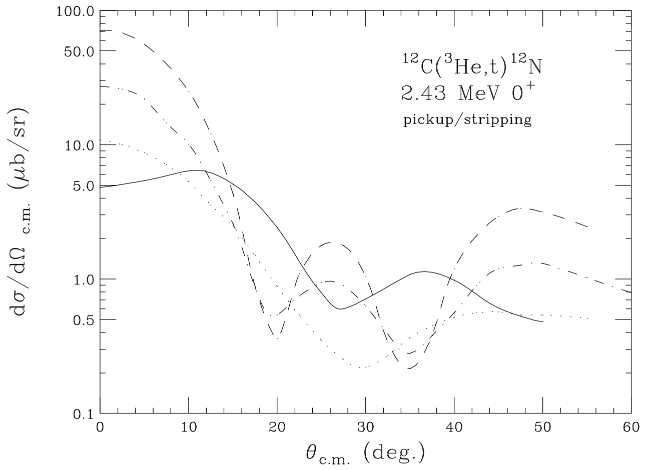

FIG. 4: A CCBA calculation for the 44 MeV $^{12}C(^3He,t)^{12}N$ reaction to the 0^+ state by way of sequential neutron pickup/proton stripping is shown. The dotted curve shows the $p_{1/2}$ contribution alone, the dashed curve the $p_{3/2}$ constribution alone, and the dot-dashed curve shows their constructively interfering sum. The solid curve shows the result of destructive interference.

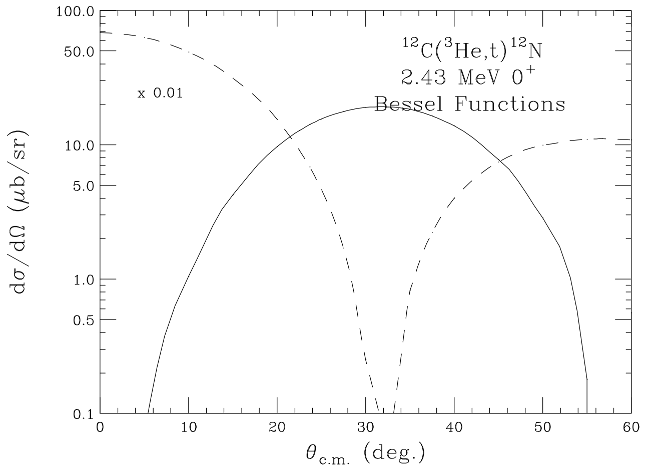

FIG. 5: The dashed curve shows the $|J_0|^2$ shape expected for diffractive scattering to the ^{12}N 0^+ state at 44 MeV. Destructive interference between two such Bessel functions, with a 2 MeV difference in Q value for the $p_{1/2}$ and $p_{3/2}$ portions, gives the solid curve, with good agreeement to the angular dependence of the 44 MeV $(^3He,t)$ data to the 0^+ state of ^{12}N.

of these $L = 0$ Bessel functions, or a cross section proportional to

$$ds/dW \sim (\delta kR sin\theta)^2 |J_1(\bar{k}R sin\theta)|^2,$$

with $\bar{k}$ the average of the projectile and ejectile wave numbers and δk given by the difference in reaction energies, here the binding energy difference of 2 MeV between the $p_{1/2}$ and $p_{3/2}$ components. A radius of 1.2 $A^{1/3}$ = 2.75 fm is used, and $\bar{k}$ is taken from the average of the 44 MeV ^3He beam energy and the outgoing triton energy, $\bar{k}$=1.98 fm^{-1}. The destructively interfered result is shown as the solid curve in Fig. 5. The dashed $|J_0|^2$ curve was matched to the constructive DWBA curve in Fig. 3 for normalization to give the absolute scale for this figure. Comparison to the data shows good agreement, especially considering the low degree of sophistication in this demonstration.

V. CONCLUSION

A new selection rule for ΔT=1 transitions between 0^+ states was derived for a strictly $T = 0$ target nucleus, and its zero degree result was seen to agree with 44 MeV $^{12}C(^3He,t)^{12}N$ data. Nuclear reaction models enforce this rule by way of destructive interference between the two possible single particle excitations, as shown by DWBA, CCBA and diffraction model examples. The high sensitivity of these calculations to the details of the assumptions make it difficult to match the data well, although these examples may provide a sharp test of more sophisticated reaction model calculations.

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