Determination of S_{17} from systematic analyses on ^{8}B Coulomb breakup with the Eikonal-CDCC method

Kazuyuki Ogata, M. Yahiro[A], Y. Iseri[B], T. Matsumoto, N. Yamashita and M. Kamimura

Department of Physics, Kyushu University
[A]*Department of Physics and Earth Sciences, University of the Ryukyus*
[B]*Department of Physics, Chiba-Keizai College*

1 Introduction

The solar neutrino problem is one of the central issues in the neutrino physics [1]. Nowadays, the neutrino oscillation is assumed to be the solution of the problem and the solar neutrino physics moves its focus on determining oscillation parameters: the mass difference among ν_e, ν_μ and ν_τ, and their mixing angles [2]. The astrophysical factor S_{17}, defined by $S_{17}(E) \equiv \sigma_{p\gamma}(E)E\exp[2\pi\eta]$ with $\sigma_{p\gamma}$ the cross section of the p-capture reaction $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ and η the Sommerfeld parameter, plays an essential role in the parameter-search procedure, since the prediction value for the flux of ^{8}B neutrino, which is intensively being detected on the earth, is proportional to $S_{17}(0)$. The required accuracy from astrophysics is about 5% in errors.

Because of difficulties of direct measurements for the p-capture reaction at very low energies, alternative indirect measurements were proposed. p-transfer reaction [3, 4, 5] with the Asymptotic Normalization Coefficient (ANC) method [6] and ^{8}B Coulomb dissociation [7, 8, 9] are typical examples of them; we concentrate on the latter in the present paper. ^{8}B Coulomb dissociation can be assumed to be the inverse reaction of $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ provided the ^{8}B is dissociated through only its E1 transition by absorption of virtual photons. Once this condition is satisfied, cross section of the p-capture reaction, thus $S_{17}(E)$, can easily be obtained by applying the principle of detailed balance to the measured dissociation cross sections. Intensive measurements of the ^{8}B Coulomb dissociation are being made in RIKEN [7], GSI [8] and MSU [9]; the extracted $S_{17}(0)$ are almost consistent each others. There is, however, contradiction between MSU and RIKEN data about the contribution of the E2 component. Additionally, role of nuclear interaction, interference between nuclear and Coulomb interactions in particular, has not yet clarified quantitatively. Therefore, it can be said that the extracted $S_{17}(0)$ from those ^{8}B Coulomb dissociation measurements contain uncertainties to some extent.

In order to determine $S_{17}(0)$ accurately, careful analysis of the ^{8}B dissociation including both nuclear and Coulomb interactions is necessary. The method of Continuum-Discretized Coupled-Channels (CDCC) [10], which was proposed and developed by Kyushu group, is suitable for that purpose. In fact, it was shown in Ref. [9] that CDCC can very well reproduce the MSU data. It is not straightforward, however, to extract the component corresponding to the inverse reaction of $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ out of the total measured spectra.

Very recently [11], we proposed a procedure that determines $S_{17}(0)$ from the ^{8}B dissociation measurements by using the ANC method, which is free from uncertainties by the use of the detailed balance. An important advantage of the CDCC + ANC analysis is that one can quantitatively evaluate the accuracy of the extracted $S_{17}(0)$; the fluctuation of ANC by changing the ^{8}B single particle wave functions is interpreted as the error of $S_{17}(0)$ coming from the use of the ANC method. We analyzed $^{58}\mathrm{Ni}(^8\mathrm{B},^7\mathrm{Be}+p)^{58}\mathrm{Ni}$ at 25.8 MeV measured at Notre Dame [12], for which detailed balance was found to fail to obtain $S_{17}(0)$ because of its low incident energy [13]. The extracted $S_{17}(0)$ is 22.83 $\pm$ 0.51 (theo) $\pm$ 2.28 (expt) eVb and the ANC method turned out to work very well, i.e., less than 1% of error; the remaining theoretical error comes from the choice of the modelspace of CDCC calculation. Although quite large systematic error (10%) of the experimental data prevents one from determining $S_{17}(0)$ with the required accuracy, this method significantly reduces theoretical errors of $S_{17}(0)$.

The CDCC + ANC analysis is expected to be applicable to the RIKEN and MSU data at several tens of MeV/nucleon. From practical point of view, however, CDCC calculation including long-ranged Coulomb coupling-potentials requires extremely large modelspace rather difficult to handle; typically the required number of partial waves is 15,000 for the MSU data [9]. Although interpolation technique for angular momentum reduces the number of Coupled-Channels (CC) equations to be solved, CC equation with huge angular momentum L is rather unstable and careful treatment is necessary. In this sense, it seems almost impossible to apply CDCC to the GSI data at 250 MeV/nucleon, where more than 100,000 partial waves will be required.

In the present paper we propose a new version of CDCC; the Eikonal-CDCC method (E-CDCC) being based on eikonal and non-adiabatic calculation. Since E-CDCC equations contain no angular momentum for the relative motion of a projectile and a target nucleus, the S matrix is obtained easily and safely. Another important feature of the method is that the resultant scattering amplitude has the form which is very similar to that obtained by CDCC, by using the relation between impact parameter b and L. This allows one *hybrid* calculation, i.e., one can adopt the amplitude of E-CDCC only for larger L, where classical picture is expected to be valid, and *connect* them with that of the standard, fully quantum mechanical, CDCC for smaller L. The hybrid calculation includes all quantum-mechanical effects necessary through the CDCC amplitudes. In addition to that, the use of the E-CDCC amplitudes removes all problems concerning with huge angular momenta and drastically reduces computation time. From theoretical point of view, the hybrid calculation will give an insight into the *connection* between quantum-mechanical and classical pictures.

In Sec. 2 formalism of E-CDCC and the construction of hybrid scattering amplitude are described. We show in Sec. 3 the validity of the hybrid calculation for $^{58}\mathrm{Ni}(^8\mathrm{B},^7\mathrm{Be}+p)^{58}\mathrm{Ni}$ at 240 MeV; results for the elastic and total breakup cross sections are compared with those of fully quantum-mechanical calculation. Effects of adiabatic approximation and importance of the interference between the larger- and smaller-L regions are also discussed. Finally summary and conclusions are given in Sec. 4.

2 Formalism

In this section we describe formalism of E-CDCC and how to construct hybrid scattering amplitude by using a result of (standard) CDCC partly. Detailed formalism and theoretical foundation of CDCC are shown elsewhere [10, 14, 15].

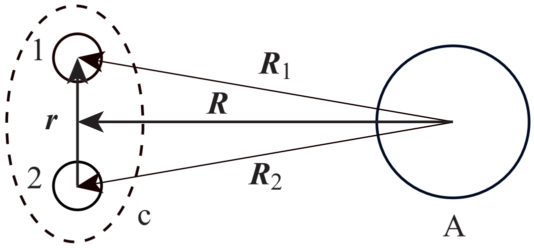

Figure 1: Schematic illustration of the system concerned in the present paper.

The system that we treat in the present study is illustrated in Fig. 1. We start with the expansion of the total wave function Ψ:

$$\begin{aligned}\Psi(\mathbf{R},\mathbf{r}) &= \sum_{i\ell m}\Phi_{i,\ell m}(\mathbf{r})\sum_{JM} e^{-i(m-m_0)\phi_R}\chi^J_{i\ell m}(R,\theta_R), \\ \chi^J_{i\ell m}(R,\theta_R) &\equiv \sum_L (\ell m L\, m_0{-}\, m|Jm_0) i^L P_{L\, m_0-m}(\theta_R)\chi_{i\ell,L}(R),\end{aligned} \tag{1}$$

where ℓ, L, and J are the total spin of c, the orbital angular momentum between c and A, and the total angular momentum of the system, respectively. m is the projection of ℓ on the z-axis taken to be parallel to the incident beam. m_0 is m in the initial state and P_{lm} is the associated Legendre polynomial. $\Phi_{i,\ell m}$ is the channel wave function of c satisfying

$$h\Phi_{i,\ell m}(\mathbf{r}) = \epsilon_{i,\ell}\Phi_{i,\ell m}(\mathbf{r}), \tag{2}$$

where h is the internal Hamiltonian of c and $\epsilon_{i,\ell}$ is its eigenvalue corresponding to channel i. $\Phi_{i,\ell m}$ contains both bound and scattering states; for the latter discretization of the continuum is made. $\{\Phi_{i,\ell m}\}$ is assumed to form an approximate complete set for a finite configuration space being significant for a reaction concerned.

Ψ satisfies the following three-body Schrödinger equation:

$$[T_{\mathbf{R}} + V_1(R_1) + V_2(R_2) + h - E]\Psi(\mathbf{R},\mathbf{r}) = 0, \tag{3}$$

where E is the total energy and V_j $(j = 1, 2)$ is the interaction between A and each constituent of c containing both nuclear and Coulomb parts. We assume a central interaction as V_j, which is justified as follows. We use the result of E-CDCC only for large L value, i.e., large impact parameter. Since spin-orbit interaction is localized at surface regions of a nucleus, it has no contribution to the result of E-CDCC adopted to construct hybrid scattering amplitude $f^{\rm H}$. It should be noted that spin-orbit interaction can entirely be included in CDCC component of $f^{\rm H}$, i.e., smaller L regions.

Taking the cylindrical coordinate, $T_{\mathbf{R}}\Psi$ is given by

$$T_{\mathbf{R}}\Psi(\mathbf{R},\mathbf{r}) = \sum_{i\ell m}\varphi_{i,\ell}(r) i^\ell Y_{\ell m}(\hat{\mathbf{r}})\sum_J e^{-i(m-m_0)\phi_R}\left[T_{\mathbf{R}}\chi^J_{i\ell m}(R,\theta_R)\right]$$

$$+\frac{\hbar^2}{2\mu}\sum_{i\ell m}\varphi_{i,\ell}(r)i^\ell Y_{\ell m}(\hat{\mathbf{r}})\sum_J e^{-i(m-m_0)\phi_R}\frac{(m-m_0)^2}{b^2}\chi^J_{i\ell m}(R,\theta_R), \quad (4)$$

where μ is the reduced mass of c-A system and b is the impact parameter. Inserting Eqs. (2) and (4) into Eq. (3), multiplying $\Phi^*_{i',\ell' m'}(\mathbf{r})$ from left, and integrating over $\mathbf{r}$, one can obtain

$$\sum_J e^{-i(m'-m_0)\phi_R}\left[-\frac{\hbar^2}{2\mu}\nabla^2_R+\epsilon_{i',\ell'}+\frac{\hbar^2}{2\mu}\frac{(m'-m_0)^2}{b^2}-E\right]\chi^J_{i'\ell' m'}(R,\theta_R)$$
$$=-\sum_{i\ell m}F_{i'\ell' m';i\ell m}(\mathbf{R})\sum_J e^{-i(m-m_0)\phi_R}\chi^J_{i\ell m}(R,\theta_R), \quad (5)$$

where $F_{i'\ell' m';i\ell m}$ is the form factor defined by

$$F_{i'\ell' m';i\ell m}(\mathbf{R})\equiv\langle\Phi_{i',\ell' m'}(\mathbf{r})|V_1(R_1)+V_2(R_2)|\Phi_{i,\ell m}(\mathbf{r})\rangle_{\mathbf{r}}\,. \quad (6)$$

After manipulation one finds

$$\begin{aligned}F_{i'\ell' m';i\ell m}(\mathbf{R}) &= \left[\sum_\lambda i^{\ell-\ell'}\sqrt{4\pi}\frac{\hat{\ell}}{\hat{\lambda}\hat{\ell'}}(\ell m\lambda\ m'{-}m|\ell' m')(\ell 0\lambda 0|\ell' 0)C_{\lambda\, m'-m}P_{\lambda\, m'-m}(\cos\theta_R)\right.\\ &\quad\left.\times\left\{\int\varphi^*_{i',\ell'}(r)\mathcal{V}^{(\lambda)}(R,r)\varphi_{i,\ell}(r)r^2dr\right\}\right]e^{-i(m'-m)\phi_R}\\ &\equiv \mathcal{F}_{i'\ell' m';i\ell m}(R,\theta_R)e^{-i(m'-m)\phi_R}, \end{aligned} \quad (7)$$

where $\hat{\xi}\equiv\sqrt{2\xi+1}$ and

$$C_{\lambda\mu}\equiv(-)^{(\mu+|\mu|)/2}\sqrt{\frac{2\lambda+1}{4\pi}\frac{(\lambda-|\mu|)!}{(\lambda+|\mu|)!}}.$$

$\mathcal{V}^{(\lambda)}$ is defined by $\mathcal{V}^{(\lambda)}(R,r)\equiv v_1^{(\lambda)}(R,\alpha r)+v_2^{(\lambda)}(R,\beta r)$ with

$$V_j(R_j)=\sum_\lambda v_1^{(\lambda)}(R,\alpha_j r)\frac{4\pi}{\hat{\lambda}^2}\sum_\mu Y^*_{\lambda\mu}(\hat{\mathbf{R}})Y_{\lambda\mu}(\hat{\mathbf{r}}) \qquad (j=1,2), \quad (8)$$

where $\alpha_j=1-m_j/(m_1+m_2)$. So far no classical approximation is made.

Here we make the eikonal approximation to χ^J_c:

$$\chi^J_c(R,\theta_R)\approx\chi'^J_c(b,z)\frac{1}{(2\pi)^{3/2}}e^{i\mathbf{K}_c(b)\cdot\mathbf{R}}, \quad (9)$$

where c denotes channels $\{i,\,\ell,\,m\}$ together and the wave number K_c is defined by

$$\frac{\hbar^2}{2\mu}\left(K_c(b)\right)^2=E-\epsilon_{i,\ell}-\frac{\hbar^2}{2\mu}\frac{(m-m_0)^2}{b^2}; \quad (10)$$

the direction of $\mathbf{K}_c$ is assumed to be parallel to the z-axis. The second order derivative of χ'^J_c can be neglected since it is a slowly varying function. Then inserting Eqs. (7) and (10) into Eq. (5) one obtains

$$\frac{i\hbar^2}{\mu}K_c(b)\left(\frac{\partial}{\partial z}\psi_c(b,z)\right)e^{iK_c(b)z}=\sum_{c'}\mathcal{F}_{cc'}(R,\theta_R)\,\psi_{c'}(b,z)e^{iK_{c'}(b)z}, \quad (11)$$

where $\psi_c(b,z) \equiv \sum_J \chi_c'^J(b,z)$ and we commuted c and c'. In Eq. (11) b is not a dynamical variable but an input parameter. We thus rewrite the E-CDCC equation as

$$\frac{i\hbar^2}{\mu} K_c^{(b)} \frac{d}{dz}\psi_c^{(b)}(z) = \sum_{c'} \mathcal{F}_{cc'}^{(b)}(z)\ \psi_{c'}^{(b)}(z) e^{i\left(K_{c'}^{(b)} - K_c^{(b)}\right)z}, \tag{12}$$

which is solved with the boundary condition $\psi_c^{(b)}(-\infty) = \delta_{c0}$ where 0 denotes the incident channel. Since this E-CDCC equation is first-order differential equation and contains no coefficient with huge angular momentum, one can easily and safely solve the equation.

Using the solution of the E-CDCC equation, the scattering amplitude with E-CDCC is given by

$$f_{c0}^{\mathrm{E}} = -\frac{\mu}{2\pi\hbar^2} \int \sum_{c'} \mathcal{F}_{cc'}^{(b)}(z)\, e^{-i(m-m_0)\phi_R} e^{i(\mathbf{K}_{\mathbf{c}'}^{(\mathbf{b})} - \mathbf{K}_{\mathbf{c}}'^{(\mathbf{b})})\cdot\mathbf{R}}\, \psi_{c'}^{(b)}(z) d\mathbf{R}. \tag{13}$$

Making use of forward-scattering approximation following Glauber theory, one can rewrite the phase factor in Eq. (13) as follows:

$$\begin{aligned} e^{i(\mathbf{K}_{\mathbf{c}'}^{(\mathbf{b})} - \mathbf{K}_{\mathbf{c}}'^{(\mathbf{b})})\cdot\mathbf{R}} &= -K_c^{(b)} \sin\theta_f b\cos\phi_R + (K_{c'}^{(b)} - K_c^{(b)}\cos\theta_f) z \\ &\approx -K_c^{(b)}\theta_f b\cos\phi_R + (K_{c'}^{(b)} - K_c^{(b)}) z, \end{aligned} \tag{14}$$

where the scattering plane is defined following Madison convention. Inserting Eq. (14) into Eq. (13) leads one to

$$f_{c0}^{\mathrm{E}} = -\frac{\mu}{2\pi\hbar^2} \iint e^{-i(m-m_0)\phi_R} e^{-iK_c^{(b)}\theta_f b\cos\phi_R}\, bdbd\phi_R \int \sum_{c'} \mathcal{F}_{cc'}^{(b)}(z)\, e^{i(K_{c'}^{(b)} - K_c^{(b)})z}\, \psi_{c'}^{(b)}(z) dz. \tag{15}$$

Integration over z can analytically be done by using Eq. (12) and one obtains

$$f_{i\ell m, i_0\ell_0 m_0}^{\mathrm{E}} = \frac{1}{2\pi i} \iint K_{i\ell m}^{(b)} e^{-i(m-m_0)\phi_R} e^{-iK_{i\ell m}^{(b)}\theta_f b\cos\phi_R} \left(\mathcal{S}_{i\ell m, i_0\ell_0 m_0}^{(b)} - \delta_{ii_0}\delta_{\ell\ell_0}\delta_{mm_0}\right) bdbd\phi_R, \tag{16}$$

where channel indices are explicitly shown and the eikonal S-matrix elements are defined by $\mathcal{S}_{i\ell m, i_0\ell_0 m_0}^{(b)} \equiv \psi_{i\ell m}^{(b)}(\infty)$.

We here divide the continuous b into finite regions:

$$\begin{aligned} f_{i\ell m, i_0\ell_0 m_0}^{\mathrm{E}} &= \frac{1}{2\pi i} \sum_L \iint_{b_L^{\min}}^{b_L^{\max}} K_{i\ell m}^{(b)} e^{-i(m-m_0)\phi_R} e^{-iK_{i\ell m}^{(b)}\theta_f b\cos\phi_R} \\ &\quad \times \left(\mathcal{S}_{i\ell m, i_0\ell_0 m_0}^{(b)} - \delta_{ii_0}\delta_{\ell\ell_0}\delta_{mm_0}\right) bdbd\phi_R, \end{aligned} \tag{17}$$

where $b_L^{\min}$ and $b_L^{\max}$ are defined through $K_{i\ell m}^{(b_L^{\min})} b_L^{\min} = L$ and $K_{i\ell m}^{(b_L^{\max})} b_L^{\max} = L+1$, respectively. Neglecting the b-dependence of $K_{i\ell m}^{(b)}$, $\exp[-iK_{i\ell m}^{(b)}\theta_f b\cos\phi_R]$ and $\mathcal{S}_{i\ell m, i_0\ell_0 m_0}^{(b)}$ within small size bin of b corresponding to each L, one can replace b by b_L^{mid} obtained from $K_{i\ell m}^{(b_L^{\mathrm{mid}})} b_L^{\mathrm{mid}} = L + 1/2$, i.e.,

$$\begin{aligned} f_{i\ell m, i_0\ell_0 m_0}^{\mathrm{E}} &= \frac{1}{2\pi i} \sum_L K_{i\ell m}^{(b_L^{\mathrm{mid}})} \left[\int e^{-i(m-m_0)\phi_R} e^{-iK_{i\ell m}^{(b_L^{\mathrm{mid}})}\theta_f b_L\cos\phi_R} d\phi_R\right] \\ &\quad \times \left(\mathcal{S}_{i\ell m, i_0\ell_0 m_0}^{(b_L^{\mathrm{mid}})} - \delta_{ii_0}\delta_{\ell\ell_0}\delta_{mm_0}\right) \int_{b_L^{\min}}^{b_L^{\max}} bdb \end{aligned} \tag{18}$$

Integration over ϕ_R can be done by using the following approximate formula:

$$\int e^{-i(m-m_0)\phi_R} e^{-iK_{i\ell m}^{(b_L^{\rm mid})}\theta_f b_L^{\rm mid}\cos\phi_R} d\phi_R \approx 2\pi i^{(m-m_0)}\sqrt{\frac{4\pi}{2L+1}} Y_{L\, m-m_0}(\hat{\mathbf{K}}'), \tag{19}$$

where $\phi_{K'} = 0$ as mentioned above. The resultant $f^{\rm E}$ is given by

$$f^{\rm E}_{i\ell m, i_0\ell_0 m_0} = \frac{2\pi}{iK_0}\sum_L \frac{K_{i\ell m}^{(b_L^{\rm mid})}}{K_0}\sqrt{\frac{2L+1}{4\pi}} i^{(m-m_0)} Y_{L\, m-m_0}(\hat{\mathbf{K}}')\left(\mathcal{S}^{(b_L^{\rm mid})}_{i\ell m, i_0\ell_0 m_0} - \delta_{ii_0}\delta_{\ell\ell_0}\delta_{mm_0}\right), \tag{20}$$

which has a similar form to that of standard CDCC:

$$f^{\rm Q}_{i\ell m, i_0\ell_0 m_0} = \frac{2\pi}{iK_0}\sum_L \sum_{J=|L-\ell|}^{L-\ell} \sum_{L_0=|J-\ell_0|}^{J-\ell_0} \sqrt{\frac{2L_0+1}{4\pi}} (L_0 0 \ell_0 m_0 | J m_0)(L\, m_0 - m\, \ell m | J m_0)$$
$$\times\, (S^J_{iL\ell, i_0 L_0\ell_0} - \delta_{ii_0}\delta_{LL_0}\delta_{\ell\ell_0})(-)^{m-m_0} Y_{L\, m-m_0}(\hat{\mathbf{K}}'). \tag{21}$$

The construction of the hybrid scattering amplitude $f^{\rm H}$ is straightforward:

$$f^{\rm H}_{i\ell m, i_0\ell_0 m_0} \equiv \sum_{L\le L_{\rm C}} f^{\rm Q}_L + \sum_{L > L_{\rm C}} f^{\rm E}_L, \tag{22}$$

where $f^{\rm Q}_L$ ($f^{\rm E}_L$) is the L-component of $f^{\rm Q}$ ($f^{\rm E}$). $L_{\rm C}$ represents the connecting point between quantum-mechanical and classical pictures, which is chosen so that $f^{\rm E}_L$ coincides with $f^{\rm Q}_L$ for $L > L_{\rm C}$. It should be noted that Eq. (22) includes all quantum-mechanical effects being necessary and also interference between the two L-regions.

In the above formulation we neglect the Coulomb distortion. In order to include it, we use [16]

$$\chi^J_c(R,\theta_R) \approx \chi'^J_c(b,z)\phi^{\rm C}_c(b,z) \tag{23}$$

instead of Eq. (9), where $\phi^{\rm C}_c$ is the Coulomb wave function:

$$\phi^{\rm C}_c(b,z) = \frac{1}{(2\pi)^{3/2}} e^{-\pi\eta_c/2}\Gamma(1+i\eta_c) e^{i\mathbf{K}^{(b)}_c\cdot\mathbf{R}} F(-i\eta_c, 1, iK^{(b)}_c(R-z)) \tag{24}$$

with Γ and F be the Gamma function and the confluent hypergeometric function, respectively. We here make the following three approximations: (i) For sufficiently large impact parameters, the flux of $\phi^{\rm C}_c$ is parallel to the z-axis. (ii) Since F is a slowly varying function, the local semiclassical approximation

$$\frac{\partial\phi^{\rm C}_c(b,z)}{\partial z} \approx iK^{(b)}_c(z)\phi^{\rm C}_c(b,z), \tag{25}$$

can be made, where the local wave number $K^{(b)}_c(z)$ is obtained by

$$\frac{\hbar^2}{2\mu}\left(K^{(b)}_c(z)\right)^2 = E - \frac{Z_{\rm c}Z_{\rm A}e^2}{R}. \tag{26}$$

(iii) We neglect the weak dependence of F on channel c. Then one obtains

$$\frac{i\hbar^2}{\mu}K^{(b)}_c(z)\frac{d}{dz}\psi^{(b)}_c(z) = \sum_{c'}\mathcal{F}^{(b)}_{cc'}(z)\,\psi^{(b)}_{c'}(z) e^{i\left(K^{(b)}_{c'} - K^{(b)}_c\right)z} \tag{27}$$

just in the same way as above. The resultant scattering amplitude is given by

$$f^{\mathrm{E}}_{i\ell m, i_0\ell_0 m_0} = \frac{2\pi}{iK_0}\sum_L H_L \frac{K^{(b_L^{\mathrm{mid}})}_{i\ell m}}{K_0}\sqrt{\frac{2L+1}{4\pi}}\, i^{(m-m_0)} Y_{L\, m-m_0}(\hat{\mathbf{K}}')\left(\mathcal{S}^{(b_L^{\mathrm{mid}})}_{i\ell m, i_0\ell_0 m_0} - \delta_{ii_0}\delta_{\ell\ell_0}\delta_{mm_0}\right), \tag{28}$$

where H_L is defined by

$$e^{-\pi\eta}\Gamma(1+i\eta_0)^2 F(-i\eta_0, 1, iK_0^{(b_L^{\mathrm{mid}})}(R-z))F(-i\eta_0, 1, iK_0^{(b_L^{\mathrm{mid}})}(R+z)), \quad z\to\infty. \tag{29}$$

3 Results and discussion

In order to see the validity of the hybrid calculation with CDCC and E-CDCC, we analyze ^{58}Ni(^{8}B,^{7}Be+p)^{58}Ni at 240 MeV. Parameters of the modelspace are as follows. The numbers of bin-states of ^{8}B are 16, 8 and 8 for s-, p- and d-states, respectively. The maximum excitation energy of ^{8}B is 10 MeV and $r_{\max}$ ($R_{\max}$) is 100 fm (500 fm). $L_{\max}$ is taken to be 1000 for saving computation time, which is somewhat small compared with the expected value of $L_{\max}$ that gives perfect convergence, i.e., $L_{\max}\sim 4000$. However, the validity of the hybrid calculation can definitely be clarified in the modelspace above. As for ^{8}B wave functions, the single-particle model by Kim *et al.* [17] was adopted. For nuclear interaction between ^{7}Be (p) and ^{58}Ni we used the parameter set of Cook et al. [18] (Becchetti and Greenlees [19]); we neglected the spin-dependent part for simplicity. Additionally, we replaced Eq. (10) by $(\hbar^2/2\mu)K_c^2 = E - \epsilon_{i,\ell}$ and neglected c-dependence of K in the calculation of f^{E}, which was found to have no effects on numerical results. We stress that this is not an adiabatic approximation since K_c is explicitly treated in solving the E-CDCC equation (27).

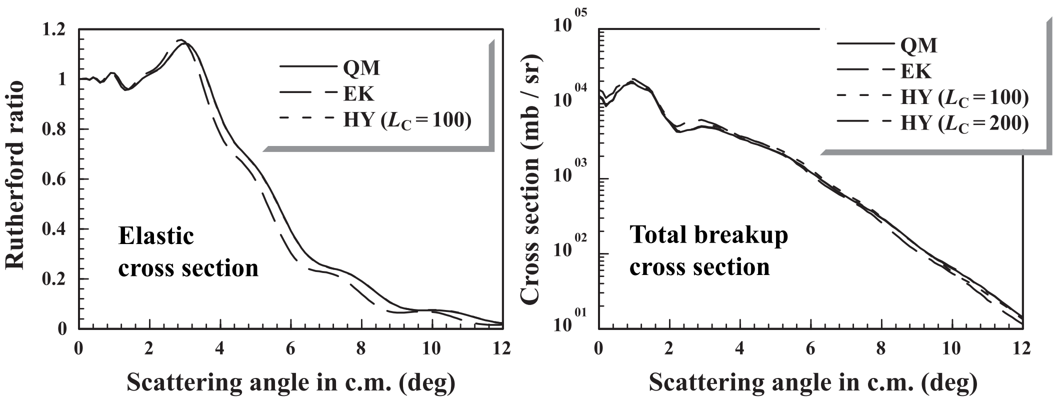

Figure 2: Angular distribution of the elastic (left panel) and total breakup (right panel) cross sections for ^{58}Ni(^{8}B,^{7}Be+p)^{58}Ni at 240 MeV. The solid, dashed and dotted lines show, respectively, the results of quantum-mechanical (QM), eikonal (EK) and hybrid (HY) calculation with $L_{\mathrm{C}} = 100$. The dash-dotted line in the right panel represents HY calcualtion with $L_{\mathrm{C}} = 200$.

In the left and right panels in Fig. 2 we show the elastic cross section (Rutherford ratio) and total breakup cross section as a function of scattering angle in center-of mass (c.m.) frame, respectively. The solid, dashed and dotted lines represent the results of quantum-mechanical (QM), eikonal (EK) and hybrid (HY) calculation, where the L_C are taken to be 1000, 0 and 100. In the right panel the result of HY calculation with $L_C = 200$ is also shown by the dash-dotted line. The agreement between QM and HY calculation with appropriate value of L_C, namely 100 (200) for elastic (breakup) cross section, is excellent; the error is only less than 1%. One also sees fairy large difference between EK and QM calculation. Since our EK calculation contains no corrections to the straight-line approximation, this does not directly show the fail of EK approximation. However, it seems quite difficult for EK calculation to obtain "perfect" agreement with the result of QM one. On the contrary, the HY calculation turned out to be applicable to analyses of ^{8}B dissociation where very high accuracy is required.

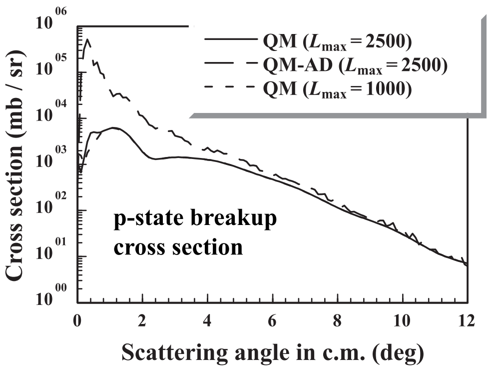

Figure 3: The p-states breakup cross sections by QM calculation with (dashed line) and without (solid line) adiabatic (AD) approximation. The maximum number of L is taken to be 2500. For comparison non-adiabatic QM result with $L_{\rm max} = 1000$ is shown by the dotted line.

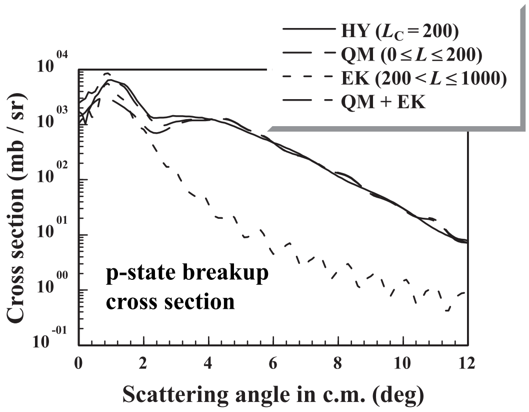

Figure 4: The p-state breakup cross sections by QM calculation with $0 \leq L \leq 200$ (dashed line) and by EK calculation with $200 < L \leq 1000$ (dotted line). The dash-dotted line is the incoherent sum of the dashed and dotted lines and the solid line is the HY result with $L_C = 200$, namely, the coherent sum of the two.

We show in Fig. 3 the p-state breakup cross sections by QM calculation with (dashed line) and without (solid line) adiabatic (AD) approximation. Since the convergence of AD calculation is rather slow compared with non-AD one, we take $L_{\rm max} = 2500$ to make fair comparison. One sees that AD calculation much overestimates the breakup cross section, which shows the failure of AD approximation for this reaction. In Fig. 3 the non-AD QM result is also shown by the dotted line. The difference between the solid and dotted lines is finite but quite small, which indicates the QM calculation with $L_{\rm max} = 2500$ works well, i.e., the large difference between AD and non-AD results is indeed due to the use of AD approximation.

In Fig. 4 the p-state breakup cross sections by QM calculation with $0 \leq L \leq 200$

and EK calculation with $200 < L \leq 1000$ are shown by the dashed and dotted lines, respectively. The dash-dotted line is the incoherent sum of the two, which deviates from the HY result shown by the solid line. This result shows that the essence of our HY calculation is the construction of the hybrid scattering amplitude not the hybrid cross section.

4 Summary and Conclusions

In the present paper we propose a new method to treat breakup process accurately and efficiently, that is, the hybrid (HY) calculation with the Continuum-Discretized Coupled-Channels method (CDCC) and the Eikonal-CDCC method (E-CDCC). E-CDCC describes the center-of-mass (c.m.) motion between the projectile and the target nucleus by straight-line (eikonal approximation) and treats the excitation of the projectile explicitly, by constructing discretized-continuum states as in CDCC, i.e., non-adiabatic and non-perturbative calculation can be done. When Coulomb distortion appears, we make a similar approximation to the eikonal one; we use Coulomb wave functions instead of plane waves. The resultant scattering amplitude by E-CDCC has a similar form to the quantum-mechanical (QM) one obtained by CDCC, which allows one to construct HY amplitude f^{H} in an intuitive way; f^{H} is given by the sum of the partial amplitude calculated by CDCC for smaller angular momentum L and that by E-CDCC for larger L, i.e., larger impact parameter b.

We analyzed ^{58}Ni(^{8}B,^{7}Be+p)^{58}Ni at 240 MeV by QM, full-eikonal (EK) and HY calculation. It was found that the elastic and total breakup cross sections obtained by QM calculation are "perfectly" reproduced by the HY calculation, namely, the error is only less than 1%. Constructing HY cross section, not HY amplitude, turned out to fail to reproduce the corresponding QM result, which shows the importance of the interference between the two L-regions above.

In conclusion, the hybrid calculation using CDCC and E-CDCC allows one very accurate analyses for breakup processes. The accuracy of the model is enough to be applied to ^{8}B dissociation relating to the astrophysical factor $S_{17}(0)$, the aim of which is determining $S_{17}(0)$ with less than 5% errors. E-CDCC drastically reduces computaion time and eliminates many problems concerned with huge angular momentum in solving Coupled-Channels (CC) equations. Thus, the hybrid calculation opened the door to the systematic analyses of ^{8}B dissociation measured at RIKEN, MSU and GSI. Extracted $S_{17}(0)$ by using the ANC method will be reported in near future.

Acknowledgement

The authors wish to thank M. Kawai, T. Motobayashi and T. Kajino for fruitful discussions and encouragement. We are indebted to the aid of JAERI and RCNP, Osaka University for computation. This work has been supported in part by the Grants-in-Aid for Scientific Research of the Ministry of Education, Science, Sports, and Culture of Japan (Grant Nos. 14540271 and 12047233).

References

[1] J. N. Bahcall *et al.*, Astrophys. J. **555**, 990 (2001) and references therein.

[2] J. N. Bahcall *et al.*, JHEP **0108**, 014 (2001) [arXiv:hep-ph/0106258]; JHEP **0302**, 009 (2003) [arXiv:hep-ph/0212147].

[3] A. Azhari et al., Phys. Rev. C **60**, 055803 (1999); Phys. Rev. Lett. **82**, 3960 (1999).

[4] L. Trache et al., Phys. Rev. Lett. **87**, 271102 (2001).

[5] K. Ogata *et al.*, Phys. Rev. C **67**, R011602 (2003).

[6] H. M. Xu *et al.*, Phys. Rev. Lett. **73**, 2027 (1994).

[7] T. Motobayashi *et al.*, Phys. Rev. Lett. **73**, 2680 (1994); T. Kikuchi *et al.*, Eur. Phys. J. A **3**, 209 (1998).

[8] N. Iwasa *et al.*, Phys. Rev. Lett. **83**, 2910 (1999).

[9] B. Davids *et al.*, Phys. Rev. Lett. **86**, 2750 (2001); Phys. Rev. C **63**, 065806 (2001).

[10] M. Kamimura *et al.*, Prog. Theor. Phys. Suppl. **89** (1986); N. Austern *et al.*, Phys. Rep. **154**, 125 (1987).

[11] N. Yamashita, Master thesis, Kyushu University, 2003.

[12] J. von Schwarzenberg *et al.*, Phys. Rev. C **53**, 2598 (1996); J. Kolata *et al.*, Phys. Rev. C **63**, 024616 (2001).

[13] H. Esbensen and G. F. Bertsch, Phys. Rev. C **59**, 3240 (1999).

[14] R. A. D. Piyadasa *et al.*, Phys. Rev. C **60**, 044611 (1999).

[15] N. Austern *et al.*, Phys. Rev. Lett. **63**, 2649(1989); N. Austern *et al.*, Phys. Rev. C **53**, 314 (1996).

[16] M. Kawai, private communication (2003).

[17] K. H. Kim *et al.*, Phys. Rev. C **35**, 363 (1987).

[18] J. Cook, Nucl. Phys. **A388**, 153 (1982).

[19] F. D. Becchetti and G. W. Greenlees, Phys. Rev. **182**, 1190 (1969).