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g_A in Nuclei *)

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Abstract

Unlike the vector current, j_μ, the axial current j^5_μ is not conserved. In a nuclear medium or nucleus relativistic invariance is lost and space and time components become disconnected. Experimentally, $\bar{N} g_A \gamma\gamma^5 N \sim \bar{N} g_A \sigma N$ appears to decrease toward the chiral limit of $g_A = 1$, whereas the time component $g_A \bar{N}\gamma^0\gamma^5 N \sim g_A \bar{N}\sigma \cdot \frac{\mathbf{p}+\mathbf{p}'}{2M} N$ increases about 50% in light nuclei and 100% in heavy ones.

We will review work that has been done and then discuss our own work, which focuses on the time component of the axial current matrix element,

$$\langle N(p_2)|\vec{A}_\mu|N(p_1)\rangle = \bar{u}(p_2)\frac{\vec{\tau}}{2}\left[\gamma_\mu\gamma_5 g_A + q_\mu\gamma_5 h_A(q^2)\right]u(p_1)\,. \tag{1}$$

The space component, which scales as $\bar{N}\sigma N$, is measured in allowed Gamow-Teller beta decays. The decrease of g_A from 1.26 by about 10% can, arguably, be ascribed to nuclear structure effects. Considerable attention has been given to the role of the $\Delta(1232)$ and chiral invariance restoration [1], as well as to short-range correlations [2]. A recent calculation by Carter and Prakash [3] uses most of these effects, but omits the Δ while including the chiral mean field, σ. They find a medium-dependent g_A^* which is approximately 1.1 at normal nuclear densities; they also apply their theory to neutron star matter and find nearly identical results.

The time component of the axial vector current can be obtained from first-forbidden beta decays [5], e.g. $0^- \to 0^+$, or in allowed Gamow-Teller

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transitions (e.g. $1^+ \to 0^+$) with polarized nuclei [6]. The angular asymmetry for mirror nuclei is only sensitive to $g_A^* \bar{N}\gamma^0\gamma^5 N$.

Already in the 1970's Kubodera, Delorme, and Rho [7] showed that nuclear exchange currents are responsible for, at least, the major part of the increase of g_A^*. Why are exchange currents so important in the time component and not for the space part? The reason is that the pion adds an extra γ^5 (see Fig. 1) which makes the space component effectively $\bar{N}\boldsymbol{\gamma}\gamma^5\gamma^5 N = \bar{N}\boldsymbol{\gamma} N \sim \bar{N}\mathcal{O}\left(\frac{\mathbf{p}}{M}\right)N$, whereas the time component becomes $\bar{N}\gamma^0\gamma^5\gamma^5 N = \bar{N}\gamma^0 N \sim \bar{N}\mathcal{O}(1)N$. Thus pion exchange currents are more important for the time component. By contrast, the time component of the axial current cannot reach the $\Delta(1232)$ whereas the space component can do so.

For the time component of the axial vector current, pion exchange currents have been examined by several authors [5, 7, 8, 9]; some have also included shorter range exchanges such as two pions, ρ, and A_1. Some aspects of the exclusion principle have been included in pion exchange calculations, e.g., via use of the a shell model [5, 9]. The authors find, approximately,

$$g_A^* = g_A(1+\delta)\,, \tag{2}$$

for various nuclei,

$$\begin{array}{ccc} A \sim 16 & \delta_{exp} \approx 0.64 & \delta_{th} \approx 0.5 - 0.6\,, \\ A \sim 96 & \delta_{exp} \approx 0.75 & \delta_{th} \approx 0.6\,, \\ A \sim 200 & \delta_{exp} \approx 1 & \delta_{th} \approx 0.8\,. \end{array}$$

However, no one has included the effect of a pion loop on a single nucleon, as in Figs. 2, where the same enhancement occurs as in a pion exchange between two nucleons and the exclusion principle is also important. This is the effect we have examined.

We start with the chiral Lagrangian of Carter and Prakash [3, 4], of which the relevant parts are

$$\begin{aligned} L_1 &= -\bar{N}g\left[(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5) + \frac{1}{2}g'\gamma^\mu\vec{\tau}\cdot\vec{a}_\mu\gamma_5\right]N\,, \\ L_2 &= \frac{1}{2}\frac{D}{\sigma_0^2}\bar{N}\left[\gamma^\mu\vec{\tau}\cdot[\vec{\pi}\times\Delta_\mu\vec{\pi} + \gamma_5(\sigma\Delta_\mu\,\vec{\pi} - \vec{\pi}\Delta_\mu\sigma)]\right]N\,, \\ \Delta_\mu\sigma &= \partial_\mu\sigma + g'\vec{a}_\mu\cdot\vec{\pi}\,, \\ \Delta_\mu\vec{\pi} &\approx \partial_\mu\vec{\pi} - g'\sigma\vec{a}_\mu\,. \end{aligned} \tag{3}$$

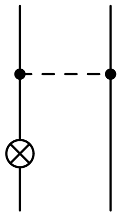

Figure 1: Pion exchange (dashed line) between nucleons (solid lines). The large crossed circle is the bare axial current coupling.

where g' is the coupling to the chiral vector mesons, ρ and A_1.

We treat the σ field in mean field theory. The diagrams shown in Fig. 2 contribute. Consider Fig 2a. To start with the nucleon has a momentum k_F. Since we need the propagators in the presence of the nucleus, they have to be above the Fermi sea (e.g. $|\mathbf{p}| > k_F$), and we have

$$\frac{1}{\not p - M + i\epsilon} \rightarrow \frac{(\not p + M)\Theta(|\mathbf{p}| - k_F)}{2E_p(p_0 - E_p + i\epsilon)} . \tag{4}$$

Here $E_p = \sqrt{\mathbf{p}^2 + M^2}$.

The effect of the in-medium sigma field is to reduce the nucleon mass, $M \rightarrow M^* \sim 0.8M$. We need to compute integrals of the form $\int_{k_F}^{\infty} d^3pF(p, M^*)$, but only want the change of g_A from the free nucleon value. We thus need

$$\int_0^{\infty} d^3p[F(p, M^*) - F(p, M)] - \int_0^{k_F} d^3pF(p, M^*) \equiv (\mathcal{I}^*_{\infty} - \mathcal{I}_{\infty}) - \mathcal{I}^*_{k_F} , \tag{5}$$

where we have defined $\mathcal{I}^*_k = \int_0^k d^3pF(p, M^*)$ for integrands $F(p, M^*)$ corresponding to Figs. 2. The last integral is always finite, but not so the first ones. We thus assume that the pion coupling to the nucleon has a dipole form factor

$$f(k, \Lambda) = \left(\frac{\Lambda^2}{\Lambda^2 + k^2}\right)^2 , \tag{6}$$

with the cutoff $\Lambda = 0.9$ or 1.1 GeV.

The constant D was fixed in Ref. [4] to fit $g_A = 1.26$ for the free nucleon,

$$g_A \approx \left(1 + D\frac{\bar{\sigma}^2}{\sigma_0^2}\right) , \tag{7}$$

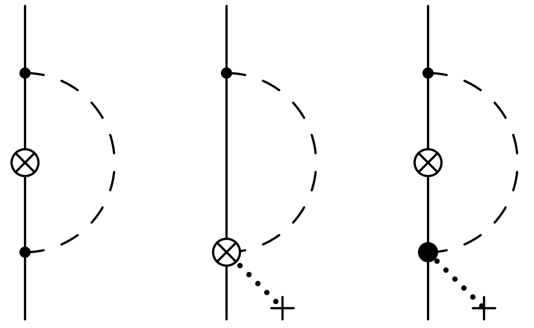

Figure 2: Leading contributions to the axial coupling in medium. Solid, dashed, and dotted lines respectively denote nucleons, pions, and sigma mesons (which terminate in the chiral mean field). The large crossed circles are bare axial current couplings.

where σ_0 is the vacuum expectation value of the σ field, $\sigma_0 = 102$ MeV, and $\bar{\sigma}$ is the mean field result at finite density. There are corrections due to heavier meson fields, which we have omitted here. The reduction of the space part of the axial vector coupling in a medium occurs due to to the decrease of $\bar{\sigma}^2$. This reduction occurs for the time component as well.

In our work there is a need to readjust the value of the constant D due to the inclusion of higher order diagrams. This reduces D by about 10%. Results are shown in the table below. We find that the contribution of Fig. 2b is large and negative (i.e., $\delta < 0$). Furthermore the difference between the contribution of the M^* and M terms is negative for Fig. 2a and almost cancels the contribution due the integral up to k_F. The contribution of Fig. 2c is positive but small, and the two loop diagrams give a negligible contribution.

		Λ= 1.1 GeV	Λ = 0.9 GeV
Fig. 2a	$\mathcal{I}^*_{k_F}$	+.21	+.19
	$\mathcal{I}^*_\infty - \mathcal{I}_\infty$	-.26	-.21
Fig. 2b	$\mathcal{I}^*_{k_F}$	-.18	-.17
	$\mathcal{I}^*_\infty - \mathcal{I}_\infty$	-.03	-.02
Fig. 2c	$\mathcal{I}^*_{k_F}$	+.01	+.004
	$\mathcal{I}^*_\infty - \mathcal{I}_\infty$	+.08	+.054

In sum, $g_A^* = g_A(1+\delta)$, with $\delta = -.13(-.11)$ for $\Lambda = 1.1(0.9)$ GeV.

In conclusion, the omitted effect is indeed of the order of magnitude required (10-15%), *but* of the wrong sign! It makes the discrepancy between theory and experiment worse.

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