Short-Range Correlations and One-Nucleon Removal Reactions on Open *s*-*d* Shell Nuclei

M.K. Gaidarov[a], K.A. Pavlova[a], A.N. Antonov[a], C. Giusti[b], S.E. Massen[c], Ch.C. Moustakidis[c], K. Spasova[d]

[a] *Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria*
[b] *Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy*
[c] *Department of Theoretical Physics, Aristotle University of Thessaloniki, GR-54006 Thessaloniki, Greece*
[d] *Department of Theoretical and Applied Physics, "Bishop K. Preslavski" University, Shumen 9712, Bulgaria*

Single-particle overlap functions and spectroscopic factors are calculated on the basis of Jastrow-type one-body density matrices of open-shell nuclei constructed by using a factor cluster expansion. The calculations use the relationship between the overlap functions corresponding to bound states of the $(A-1)$-particle system and the one-body density matrix for the ground state of the A-particle system. In this work we extend our previous analyses of reactions on closed-shell nuclei by using the resulting overlap functions for the description of the cross sections of (p,d) reactions on the open *s*-*d* shell nuclei ^{24}Mg, ^{28}Si and ^{32}S and of ^{32}S$(e,e'p)$ reaction. The relative role of both shell structure and short-range correlations incorporated in the correlation approach on the spectroscopic factors and the reaction cross sections is pointed out.

1 Introduction

While collective phenomena have been known since the early times of nuclear physics, effects produced by short-range correlations (SRC) are more difficult to be experimentally singled out. The experimental and theoretical works in the past decade have provided us with a much clearer picture on the consequences of these correlations. There are clear signatures of the presence of correlation effects on some quantities related to the behaviour of the single nucleon in nuclear medium. For instance, the nucleon-nucleon (NN) correlations are responsible for the reduction of the spectroscopic strengths of the nuclear hole states [1–3]. The theoretical understanding of this fact requires the knowledge of the removal spectral function which can be represented in a natural way by both overlap functions and single-nucleon spectroscopic factors [4]. They are directly related to observable quantities such as (p,d), $(e,e'p)$ and (γ,p) reaction cross sections. The comparison between the theoretically calculated and measured cross sections could serve as a test for the proper account of correlations within the theoretical approach considered.

Recently, a general procedure has been adopted [5] to extract the bound-state overlap functions and the associated spectroscopic factors and separation energies on the base of the ground-state one-body density matrix (OBDM). The latter can be expressed in terms of the overlap functions $\phi_\alpha(\mathbf{r})$, which describe the residual nucleus as a hole state in the target, in the form

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \sum_\alpha \phi^*_\alpha(\boldsymbol{r})\phi_\alpha(\boldsymbol{r}'). \tag{1}$$

In the case of a target nucleus with $J^\pi = 0^+$ each of the bound-state overlap functions is characterized by the set of quantum numbers $\alpha \equiv nlj$, with n being the number of the state with a given multipolarity l and a total angular momentum j. Here we would like to give only the expression for the lowest $n = n_0$ neutron lj-bound-state overlap function which is determined by the asymptotic behaviour of the associated partial radial contribution of the one-body density matrix $\rho_{lj}(r, r')$ $(r' = a \to \infty)$

$$\phi_{n_0lj}(r) = \frac{\rho_{lj}(r, a)}{C_{n_0lj}\ \exp(-k_{n_0lj}\, a)/a}, \tag{2}$$

where the constants C_{n_0lj} and k_{n_0lj} are completely determined by $\rho_{lj}(a, a)$. In this way the separation energy

$$\epsilon_{n_0lj} \equiv E^{A-1}_{n_0lj}\ -\ E^A_0 = \frac{\hbar^2\ k^2_{n_0lj}}{2m_{\rm n}} \tag{3}$$

and the spectroscopic factor $S_{n_0lj} = \langle \phi_{n_0lj} | \phi_{n_0lj} \rangle$ can be determined as well. For protons some mathematical complications arise due to an additional long-range part of the interaction originating from the Coulomb force, but the general conclusions of the consideration remain the same. The applicability of this theoretical scheme has been demonstrated in Refs. [6–10] by means of realistic one-body density matrices of ^{16}O and ^{40}Ca constructed within different correlation methods. The inclusion of short-range and tensor correlations has caused a depletion of the levels below Fermi level which is reflected in the substantial reduction of the spectroscopic factors for the quasihole states.

There is no systematic study of the one-body density matrix (and related quantities) which includes both closed- and open-shell nuclei. For that reason, in [11] expressions for the OBDM's $\rho(\mathbf{r}, \mathbf{r}')$ and momentum distributions which could be used for both closed- and open-shell nuclei have been obtained. In [12] analytical expressions for the charge form factors and densities of the *s-p* and *s-d* shell nuclei have been also derived. The $\rho(\mathbf{r}, \mathbf{r}')$ was constructed using the factor cluster expansion of Clark and co-workers [13–15] and Jastrow correlation function which incorporates SRC for closed-shell nuclei. This approach was extrapolated to the case of $N{=}Z$ open-shell nuclei in Ref. [11]. The expressions for $\rho(\mathbf{r}, \mathbf{r}')$ are functionals of the spherical harmonic oscillator (HO) orbitals and depend on the HO parameter and the correlation parameter. The values of the parameters which have been used for the closed-shell nuclei ^{4}He, ^{16}O and ^{40}Ca were determined in [12] by a fit of the theoretical charge form factor, derived with the same cluster expansion, to the experimental one. For the open-shell nuclei ^{12}C, ^{24}Mg, ^{28}Si and ^{32}S new values of these parameters have been obtained to give a better fit to the experimental data [11]. Moreover, these nuclei were treated as $1d$ shell nuclei and as $1d$-$2s$ shell nuclei. In the latter case the A-dependence of the high-momentum components of the momentum distributions becomes quite small.

In order to reveal better the nuclear structure properties of the open-shell nuclei and to draw a parallel with the closed-shell ones, it is desirable to test the resulting overlap functions in calculations of one-nucleon pickup and knockout reactions, which is the aim of the present paper. We analyze (p, d) reactions on ^{24}Mg [16], ^{28}Si [17] and ^{32}S [16] and the ^{32}S$(e, e'p)$ reaction [18]. Such an investigation allows us to examine the relationship between the OBDM and the associated overlap functions in that region of nuclei and also to estimate the role of SRC incorporated in the correlation approach used in the reaction cross section calculations.

The paper is organized as follows. A short description of the correlated OBDM of the target nucleus is given in Sec. II. The results of the calculations are presented and discussed in Sec. III. The summary of the present work is given in Sec. IV.

2 Correlated one-body density matrix

In the present work we start from a Jastrow-type trial many-particle wave function

$$\Psi(\mathbf{r}_1,...,\mathbf{r}_A) = \mathcal{F}\Phi = \prod_{i<j}^{A} f(r_{ij})\Phi(\mathbf{r}_1,...,\mathbf{r}_A), \tag{4}$$

where $\mathcal{F}$ is a model operator which introduces SRC, Φ is an uncorrelated (Slater determinant) wave function built up from single-particle wave functions which correspond to the occupied states, and $f(r_{ij})$ is the state-independent correlation function which was chosen to have the form

$$f(r_{ij}) = 1 - \exp[-\beta(\mathbf{r}_i - \mathbf{r}_j)^2]. \tag{5}$$

The correlation function goes to 1 for large values of $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and to 0 for $r_{ij} \to 0$. Apparently, the effect of SRC introduced by the function $f(r_{ij})$ becomes larger when the correlation parameter β becomes smaller and vice versa.

For our aims we represent the one-body density matrix $\rho(\mathbf{r},\mathbf{r}')$ by the form

$$\rho(\mathbf{r},\mathbf{r}') = \frac{\langle\Psi|\mathbf{O}_{\mathbf{r}\mathbf{r}'}|\Psi'\rangle}{\langle\Psi|\Psi\rangle} = N\langle\Psi|\mathbf{O}_{\mathbf{r}\mathbf{r}'}|\Psi'\rangle = N\langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle, \tag{6}$$

where $\Psi' = \Psi(\mathbf{r}'_1,\mathbf{r}'_2,...,\mathbf{r}'_A)$, N is the normalization factor and the integration is carried out over the vectors $\mathbf{r}_1$ to $\mathbf{r}_A$ and $\mathbf{r}'_1$ to $\mathbf{r}'_A$. The one-body "density operator" $\mathbf{O}_{\mathbf{r}\mathbf{r}'}$ has the form

$$\mathbf{O}_{\mathbf{r}\mathbf{r}'} = \sum_{i=1}^{A} \delta(\mathbf{r}_i - \mathbf{r})\delta(\mathbf{r}'_i - \mathbf{r}') \prod_{j\neq i}^{A} \delta(\mathbf{r}_j - \mathbf{r}'_j). \tag{7}$$

In order to evaluate the correlated one-body density matrix $\rho_{cor}(\mathbf{r},\mathbf{r}')$ we consider firstly the generalized integral

$$I(\alpha) = \langle\Psi|\exp\left[\alpha I(0)\mathbf{O}_{\mathbf{r}\mathbf{r}'}\right]|\Psi'\rangle, \tag{8}$$

corresponding to $\mathbf{O}_{\mathbf{r}\mathbf{r}'}$ (given by (7)), from which we have

$$\langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle = \left[\frac{\partial \ln I(\alpha)}{\partial\alpha}\right]_{\alpha=0}. \tag{9}$$

For the cluster analysis of Eq. (9), after considering the sub-product integrals [13–15] and their factor cluster decomposition following the procedure of Ristig, Ter Low, and Clark, one can obtain an expression for the $\rho_{cor}(\mathbf{r},\mathbf{r}')$ [11]

$$\rho_{cor}(\mathbf{r},\mathbf{r}') \simeq N[\langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_1 - O_{22}(\mathbf{r},\mathbf{r}',\mathrm{g}_1) - O_{22}(\mathbf{r},\mathbf{r}',\mathrm{g}_2) + O_{22}(\mathbf{r},\mathbf{r}',\mathrm{g}_3)]. \tag{10}$$

Expansion (10) of the present work has one- and two-body terms and the normalization of the wave function is preserved by the normalization factor N. The expressions of $\langle\mathbf{O}_{\mathbf{r}\mathbf{r}'}\rangle_1$ and the three two-body terms $O_{22}(\mathbf{r},\mathbf{r}',\mathrm{g}_l)$ ($l = 1,2,3$) depend on the single-particle (SP) radial wave functions and so they are suitable to be used for analytical calculations with HO orbitals. These expressions were derived for the closed-shell nuclei with $N = Z$, where η_{nl} is 0 or 1. For the open-shell nuclei (with $N = Z$) we use the same expressions as in Refs. [11] and [12] where now $0 \leq \eta_{nl} \leq 1$. In this way the systematic study of important quantities such as bound-state overlap functions and spectroscopic factors [6–9] can be extended to open-shell nuclei within the factor cluster expansion from Refs. [11,13–15].

3 Results of calculations and discussion

The one-body density matrices (10) constructed by using of factor cluster expansion have been applied to calculate neutron and proton overlap functions including NN correlations and related to possible $1p$-, $1d$- and $2s$-quasihole states in the ^{24}Mg, ^{28}Si and ^{32}S nuclei. We note that in our work the extracted overlap functions are not separated in respect to the spin-orbit partners $j = l \pm 1/2$. Although the OBDM's do not allow to obtain different results for $d_{3/2}$ and $d_{5/2}$ quasihole states, it is useful in some calculations of reaction cross sections for transitions to these states to test the overlap function corresponding to the $1d$ bound state.

The values of the spectroscopic factors (SF) and of the separation energies for neutrons and protons deduced from the calculations are listed in Table 1. The separation energies (3) derived from the procedure are in acceptable agreement with the corresponding empirical ones. As a common feature, a substantial reduction of the spectroscopic factors of the states which are below the Fermi level (of the independent-particle picture) is observed for all nuclei considered due to the short-range NN correlations and their particular open-shell structure. For instance, the values of the SF for the states with l=0 and l=2 are much smaller than the values obtained for the l=1 state. The particular features of the nuclear structure in the $2s$-$1d$ region (in which the $1p$ state is not a valence one) are responsible for the larger reduction of the spectroscopic factors of the levels, though partly occupied, in these open s-d shell nuclei in comparison with the "more occupied" $1p$ quasihole state. It can be also seen from Table 1 that the trend of the calculated SF for proton bound states follows that one corresponding to the neutron overlap functions.

Table 1: Neutron and proton spectroscopic factors (SF) and separation energies (ϵ_n and ϵ_p in MeV) calculated on the basis of the one-body density matrices [11] for ^{24}Mg, ^{28}Si, ^{32}S and ^{40}Ca. Comparison is made with the experimental data ϵ_n^{exp} and ϵ_p^{exp} for the separation energies [16–20].

		neutrons			protons		
Nucleus	nl	ϵ_n	ϵ_n^{exp}	SF	ϵ_p	ϵ_p^{exp}	SF
^{24}Mg	$1p$	18.85	19.29	0.9474	14.13	14.33	0.9798
	$1d$	16.94	16.53	0.4026	11.02	11.69	0.4586
	$2s$	18.37	18.89	0.4706	13.63	14.08	0.5414
^{28}Si	$1p$	21.34	21.35	0.8528	15.21	15.64	0.9788
	$1d$	17.29	17.18	0.5071	11.49	11.59	0.6265
	$2s$	17.32	17.96	0.3958	12.09	12.43	0.4675
^{32}S	$1p$	22.54	22.80	0.7454	15.68	16.29	0.8856
	$1d$	16.98	17.33	0.5682	9.82	10.13	0.6636
	$2s$	14.75	15.09	0.4712	8.53	8.86	0.5648
^{40}Ca	$1d$	15.45	15.64	0.7691	8.34	8.33	0.6729
	$2s$	17.86	18.19	0.8001	10.33	10.94	0.8051

We would like to note the larger reduction of the spectroscopic factors corresponding to quasihole states of the open-shell nuclei considered in comparison with the spectroscopic factors for the states in closed-shell ^{16}O [7, 10] and ^{40}Ca [10] nuclei when SRC are taken into account. To discuss this point, firstly we give in Table 1 the spectroscopic factors for the $1d$ and $2s$ quasihole states of the closed-shell ^{40}Ca deduced from the one-body density matrix calculations within the same Jastrow approach [11] used in the present work. For comparison, the SRC accounted for within the low-order approximation to the Jastrow correlation method [10] produce spectroscopic

factors of 0.892 for the $1d$ state and 0.956 for the $2s$ in ^{40}Ca nucleus. It turns out that the method from [11] leads to a suppression of the spectroscopic factors for the closed-shell ^{40}Ca nucleus. As a common feature, the spectroscopic factors for the s-d open-shell nuclei are smaller than those for the closed-shell nuclei.

Second, in order to check our theoretically calculated spectroscopic factors it is useful to compare them with some single-particle occupation probabilities available for the considered $1d$-$2s$ shell nuclei. The Hartree-Fock calculations performed in [21] for the (p,p') scattering analyses treating protons and neutrons equivalently yield values of 0.479 and 0.682 respectively for the $1d_{5/2}$ fractional occupancies of ^{24}Mg and ^{28}Si. The $2s_{1/2}$ proton occupancy of 0.67(9) derived for ^{32}S was reported in Ref. [22]. As can be seen from Table 1 our calculated spectroscopic factors for these states are smaller than the resulting occupancies thus satisfying the general property $S_{nlj} \leq N_{nlj}^{max}$, i.e. in each lj subspace the spectroscopic factor S_{nlj} is smaller than the largest natural occupation number N_{nlj}^{max} [5].

The distorted wave born approximation (DWBA) calculations of the pickup reactions on the considered open-shell nuclei were performed using the DWUCK4 code [23] assuming zero-range approximation for the p-n interaction inside the deuteron. The same approach has been adopted in previous analyses of (p,d) reactions on ^{16}O [6, 7] and ^{40}Ca [6, 8]. The values of the optical potential parameters have been taken in each case to be the same as in the corresponding standard DWBA calculations.

Fig. 1 shows the differential cross sections for the transitions to the ground $3/2^+$ state and to the excited $5/2^+$ [at excitation energy E_x=0.45 MeV] and $1/2^-$ [at E_x=2.76 MeV] states in ^{23}Mg nucleus. A comparison with the experimental data from [16] is also made. In each panel, the results obtained with overlap function, with bound-state wave function following the standard DWBA procedure and with uncorrelated shell-model (SM) wave function are plotted. As can be seen our calculations using the overlap function for the transition to the ground $3/2^+$ state agree fairly well with the experimental angular distribution for the same transition reproducing the amplitude of the first maximum and qualitatively the shape of the differential cross section. *We emphasize that this result is obtained without any additional normalization since the calculated overlap function for* $3/2^+$ *state already includes the spectroscopic factor of 0.4* (see Table 1). The standard DWBA curves and those corresponding to the uncorrelated case need an adjustment by a fitting parameter, i.e., the phenomenological spectroscopic factor. For instance, the values of the phenomenological spectroscopic factors deduced from the traditional DWBA calculations for the transitions to $5/2^+$ and $3/2^+$ in ^{23}Mg (2.50 and 0.25, respectively) are very close to the empirical values obtained in [16], although a clear physical interpretation could not be done. Moreover, at larger angles our calculations lead to better agreement with the experimental data comparing with the standard DWBA analysis. In the latter the overlap function is replaced by a SP wave function corresponding to a given mean-field potential. In such calculations the NN correlations are included approximately by adjusting the mean-field potential parameter values. Another reason for the observed discrepancies in [16] is that a spherical potential has been used for generating the bound neutron state in DWBA instead of a more realistic deformed one. We would like to mention here *the principal role of the overlap function for the good description of the differential cross section* for the transition to the ground $3/2^+$ state in ^{23}Mg. It is obtained on the basis of a correlated OBDM and allows one to account for the short-range correlations in the case of pickup reaction. The results of the calculations carried out additionally for the transitions to the excited states are less satisfactory for angles larger than 10 degrees. One of the reasons for this is the unrealistic (HO) asymptotic behaviour of the Jastrow-type OBDM [11] which has been used to calculate the overlap functions following the method described in Section II. Concerning the similarity of the shape of the theoretical curves (though different by one order of magnitude) in the upper and middle panels of Fig. 1 we note that for the ground state the $1d_{3/2}$ overlap function is needed while for the first excited state the $1d_{5/2}$ overlap

function is needed. The usage of a common $1d$ overlap function in our work obviously gives a larger deviation from the data for the latter case. On the other hand, a large fragmentation of the single-particle strengths is a common feature of the studied $2s$-$1d$ nuclei. For example, pickup from the $1p_{1/2}$ an 0 $1p_{3/2}$ sub-shells in $2s$-$1d$ nuclei has been an intriguing topic from experimental point of view [16] indicating several $1p$ states observed in ^{23}Mg.

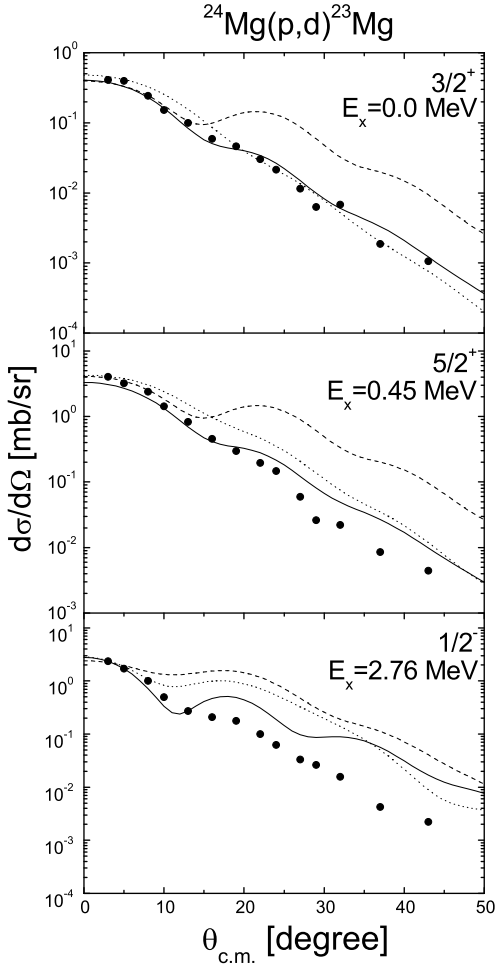

Figure 1: Differential cross-section for the ^{24}Mg(p,d) reaction at incident proton energy E_p=185 MeV to the $3/2^+$ ground and to the $5/2^+$ and $1/2^-$ excited states in ^{23}Mg. The neutron overlap functions are derived from the OBDM (solid line). The calculated standard DWBA curve (dotted line) and the uncorrelated SM result (dashed line) are also presented. The experimental data [16] are given by the full circles.

An example of electron-induced proton knockout from ^{32}S for the transition to the ground $2s_{1/2}$ state of ^{31}P is given in Fig. 2. Calculations have been done with the code DWEEPY [24], which is based on the nonrelativistic distorted wave impulse approximation (DWIA) description of the nucleon knockout process and includes final-state interactions and Coulomb distortion of the electron waves [25]. In the figure the result obtained with the proton overlap function for the $2s$ state of ^{32}S and the optical potential from [26] is compared with the NIKHEF data from [18]. A reasonable agreement with the experimental data for the reduced cross section is obtained. In the analysis of [18] the calculations are performed within the same DWIA framework and with

the same optical potential, but a phenomenological single-particle wave function is used with a radius adjusted to the data. *We emphasize that in the present work the overlap function is theoretically calculated on the basis of the Jastrow-type OBDM of* ^{32}S *and does not contain any free parameters.* It can be seen from Fig. 2 that our spectroscopic factor of 0.5648 gives a good agreement with the size of the experimental cross section and, in addition, it is in accordance with the integrated strength for the valence $2s_{1/2}$ shell in ^{32}S [18] which amounts to 65(7)% of the SP strength obtained using the shell-model bound-state function. The result for the $^{32}\mathrm{S}(e,e'p)$ cross section obtained with the harmonic-oscillator bound-state wave function is also illustrated in Fig. 2. It has been computed with the same oscillator parameter value b=2 fm for the $2s_{1/2}$ ground-state wave function as in the original calculations of the OBDM without SRC [11]. In order to perform a consistent comparison with the result when considering theoretically calculated overlap function, we have applied the same spectroscopic factor of 0.5648. In this case, the size of the reduced cross section is also reproduced, but the HO wave function gives much worse description of the experimental data. Thus, the comparison made in Fig. 2 shows the important role of the SRC accounted for in our approach for the correct description of knockout reactions.

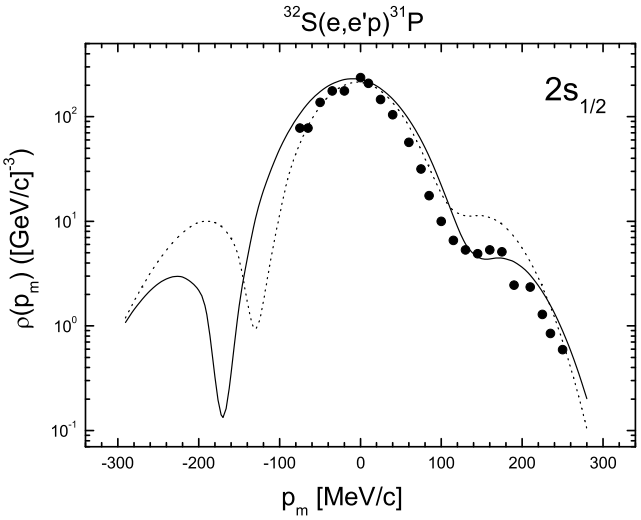

Figure 2: Reduced cross section of the $^{32}\mathrm{S}(e,e'p)$ reaction as a function of the missing momentum p_{m} for the transition to the $1/2^+$ ground state of ^{31}P. The proton overlap function is derived from the OBDM (solid line). The result with the uncorrelated (HO) wave function is given by dotted line. The experimental data (full circles) are taken from Ref. [18].

4 Conclusions

The results of the present work can be summarized as follows:
i) Single-particle overlap functions, spectroscopic factors and separation energies are calculated from the Jastrow-type one-body density matrices, which were derived using factor cluster expansion, for the ground state of the open-shell ^{24}Mg, ^{28}Si and ^{32}S nuclei.
ii) Taking into account both short-range correlations and specific nuclear structure it is found that the deduced spectroscopic factors S_{nlj} of the hole states of open-shell nuclei in our particular Jastrow approach are substantially smaller than those of the closed-shell ones satisfying at the same time the general relation with the natural occupation probabilities N_{nlj}, namely

$S_{nlj} \leq N_{nlj}^{max}$.
iii) The absolute values of the differential cross sections of (p,d) reactions on ^{24}Mg, ^{28}Si and ^{32}S as well as of ^{32}S$(e,e'p)$ reduced cross section are calculated by using the theoretically obtained overlap functions which already contain NN correlations. The acceptable agreement with the experimental data shows that this method is applicable also to the case of open-shell nuclei. The description of the cross sections might be improved including the deformation effects in the even-even Z=N nuclei in the $2s$-$1d$ region.
iv) Using more realistic OBDM's with a correct asymptotic behaviour one can expect that the resulting overlap functions will be able to describe more accurately the experimental cross sections of the one-nucleon removal reactions on open s-d shell nuclei. In this case more definite conclusions about the role of the SRC in open- and closed-shell nuclei can be drawn.

The authors are grateful to Dr. L. Lapikás for providing us the experimental data from [18] and for the fruitful discussions with him and Prof. K. Amos. This work was partly supported by the Bulgarian National Science Foundation under the Contracts Nrs.Φ–809 and Φ–905.

References

[1] A.N. Antonov, P.E. Hodgson and I.Zh. Petkov, *Nucleon Momentum and Density Distributions in Nuclei* (Clarendon Press, Oxford, 1988).

[2] A.N. Antonov, P.E. Hodgson and I.Zh. Petkov, *Nucleon Correlations in Nuclei* (Springer–Verlag, Berlin, 1993).

[3] L. Lapikás, Nucl. Phys. **A553**, 297c (1993).

[4] A.N. Antonov, M.V. Stoitsov, M.K. Gaidarov, S.S. Dimitrova, and P.E. Hodgson, J. Phys. **G21**, 1333 (1995).

[5] D. Van Neck, M. Waroquier, and K. Heyde, Phys. Lett. B **314**, 255 (1993).

[6] S.S. Dimitrova, M.K. Gaidarov, A.N. Antonov, M.V. Stoitsov, P.E. Hodgson, V.K. Lukyanov, E.V. Zemlyanaya, and G.Z. Krumova, J. Phys. G **23**, 1685 (1997).

[7] M.K. Gaidarov, K.A. Pavlova, S.S. Dimitrova, M.V. Stoitsov, A.N. Antonov, D. Van Neck, and H. Müther, Phys. Rev. C **60**, 024312 (1999).

[8] M.V. Ivanov, M.K. Gaidarov, A.N. Antonov, and C. Giusti, Phys. Rev. C **64**, 014605 (2001).

[9] M.K. Gaidarov, K.A. Pavlova, A.N. Antonov, M.V. Stoitsov, S.S. Dimitrova, M.V. Ivanov, and C. Giusti, Phys. Rev. C **61**, 014306 (2000).

[10] M.V. Stoitsov, S.S. Dimitrova, and A.N. Antonov, Phys. Rev. C **53**, 1254 (1996).

[11] Ch.C. Moustakidis and S.E. Massen, Phys. Rev. C **62**, 034318 (2000).

[12] S.E. Massen and Ch.C. Moustakidis, Phys. Rev. C **60**, 024005 (1999).

[13] J.W. Clark and M.L. Ristig, Nuovo Cim. LXXA **3**, 313 (1970).

[14] M.L. Ristig, W.J. Ter Low, and J.W. Clark, Phys. Rev. C **3**, 1504 (1971).

[15] J.W. Clark, Prog. Part. Nucl. Phys. **2**, 89 (1979).

[16] J. Källne and B. Fagerström, Physica Scripta **11**, 79 (1975).

[17] O. Sundberg and J. Källne, Arkiv Fysik **39**, 323 (1969).

[18] J. Wesseling, C.W. de Jager, L. Lapikás, H. de Vries, M.N. Harakeh, N. Kalantar-Nayestanaki, L.W. Fagg, R.A. Lindgren, and D. Van Neck, Nucl. Phys. **A547**, 519 (1992).

[19] P.M. Endt and C. van der Leun, Nucl. Phys. **A105**, 1 (1967).

[20] C. Mahaux and R. Sartor, Nucl. Phys. **A528**, 253 (1991).

[21] K. Amos, J. Morton, I. Morrison, and R. Smith, Aust. J. Phys. **31**, 1 (1978).

[22] J. Wesseling, C.W. de Jager, L. Lapikás, H. de Vries, L.W. Fagg, M.N. Harakeh, N. Kalantar-Nayestanaki, R.A. Lindgren, E. Moya De Guerra, and P. Sarriguren, Phys. Rev. C **55**, 2773 (1997).

[23] K. Langanke, J.A. Maruhn, and S.E. Koonin, *Computational Nucl. Phys. 2: Nucl. Reactions*, (Springer-Verlag, Berlin-Heidelberg-New York), 88 (1993).

[24] C. Giusti and F.D. Pacati, Nucl. Phys. **A473**, 717 (1987); Nucl. Phys. **A485**, 461 (1988).

[25] S. Boffi, C. Giusti, F.D. Pacati, and M. Radici, *Electromagnetic Response of Atomic Nuclei, Oxford Studies in Nuclear Physics* (Clarendon Press, Oxford, 1996).

[26] P. Schwandt, H.O. Meyer, W.W. Jacobs, A.D. Bacher, S.E. Vigdor, M.D. Kaitchuck, and T.R. Donoghue, Phys. Rev. C **26**, 55 (1982).