Analysis of previous microscopic calculations for the second 0^+ state in ^{12}C in terms of 3-alpha particle Bose-condensed state

Y. Funaki[1], A. Tohsaki[2], H. Horiuchi[1], P. Schuck[3], and G. Röpke[4]
[1] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*
[2] *Department of Fine Materials Engineering, Shinshu University, Ueda 386-8567, Japan*
[3] *Institut de Physique Nucléaire, 91406 Orsay Cedex, France*
[4] *FB Physik, Universität Rostock, D-18051 Rostock, Germany*
(Dated: June 20, 2003)

The wave function of the second 0^+ state of ^{12}C which was obtained long time ago by solving the microscopic 3α problem is shown to be almost completely equivalent to the wave function of the 3α condensed state which has been proposed recently by the present authors. This equivalence of the wave functions is shown to hold in two cases where different effective two-nucleon forces are adopted. This finding gives strong support for interpreting the second 0^+ state of ^{12}C which is the key state for the synthesis of ^{12}C in stars ('Hoyle' state), and which is one of the typical mysterious 0^+ states in light nuclei, as a gas-like structure of three α particles, Bose-condensed into an identical s-wave function.

The α clustering nature of the nucleus ^{12}C has been studied by many authors using various approaches [1]. Among these studies, solving the fully microscopic three-body problem of α clusters gives us the most important and reliable theoretical information of α clustering in ^{12}C within the assumption that no α cluster is distorted or broken except for the change of the size parameter of the α cluster's internal wave function. As representatives for the solution of the microscopic 3α problem where the antisymmetrization of nucleons is exactly treated, we here quote two works: one by Uegaki et al. [2] and the other by Kamimura et al. [3] both of which were published almost a quarter century ago. In these works, the ^{12}C levels are described by the wave function of the form $\mathcal{A}\{\chi(\boldsymbol{s},\boldsymbol{t})\phi_\alpha^3\}$ with $\mathcal{A}$ standing for the antisymmetrizer, $\phi_\alpha^3 \equiv \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)$ for the product of the internal wave functions of three α clusters, and $\boldsymbol{s}$ and $\boldsymbol{t}$ for the Jacobi coordinates of the center-of-mass motion of three α clusters. Here $\phi(\alpha_i)$ $(i=1,2,3)$ is the internal wave function of the α-cluster α_i having the form $\phi(\alpha_i) \propto \exp[-(1/8b^2)\sum_{m>n}^4(\boldsymbol{r}_{im}-\boldsymbol{r}_{in})^2]$. The wave function $\chi(\boldsymbol{s},\boldsymbol{t})$ of the relative motion of three α clusters is obtained by solving the eigen-energy problem of the full three-body equation of motion; $\langle\phi_\alpha^3|(H-E)|\mathcal{A}\{\chi(\boldsymbol{s},\boldsymbol{t})\phi_\alpha^3\}\rangle = 0$, where H is the microscopic Hamiltonian consisting of the kinetic energy, effective two-nucleon potential, and the Coulomb potential between protons. The difference between the works by Uegaki et al. and Kamimura et al. lies in the adopted effective two-nucleon forces, besides the differing techniques of solution.

Both calculations by Uegaki et al. and Kamimura et al. reproduced reasonably well the observed binding energy and r.m.s. radius of the ground 0^+ state which is the state with normal density, while they both predicted a very large r.m.s. radius for the second 0^+ state which is larger than the r.m.s. radius of the ground 0^+ state by about 1 fm, i.e. by over 30%. The observed 0_2^+ state lies slightly above the 3α breakup threshold and the energies of the calculated 0_2^+ state reproduced reasonably well the observed value although the value by Uegaki et al. is slightly higher than the 3α breakup threshold by about 1 MeV. The second 0^+ state of ^{12}C is well known as the key state for the synthesis of ^{12}C in stars (Hoyle state) and also as one of the typical mysterious 0^+ states in light nuclei which are very difficult to understand from the point of view of the shell model. For the understanding of the nature of the 0_2^+ state with dilute density, the analysis by Uegaki et al. of the Reduced Width Amplitude (RWA) function of the ^{8}Be-α breakup is very useful. The RWA function $y_L(\rho)$ which is defined as $y_L(\rho) = \sqrt{12!/8!4!}\langle[\Phi(^8\mathrm{Be},L)\phi(\alpha)Y_L(\hat{\rho})]_{J=0}|\mathcal{A}\{\chi(\boldsymbol{s},\boldsymbol{t})\phi_\alpha^3\}\rangle$ with ρ standing for the realtive coordinate between ^{8}Be and α, proved to have similar magnitude for all partial waves L (L= 0, 2, 4) for the ground 0^+ state but it turned out to be large only for L=0 for the 0_2^+ state. This result for the 0_2^+ state with dilute density implies that the 0_2^+ state has a gas-like structure of three α-particles which interact weakly among one another, predominantly in relative S waves. This understanding of the 0_2^+ state structure had been already presented by Horiuchi on the basis of the 3α OCM (orthogonality condition model) calculation [4], and is quite different from the picture of a 3α linear-chain structure [5] for this state. It should be mentioned here that both calculations by Uegaki et al. and Kamimura et al. reproduced well not only the energy but also other observed quantities related to the 0_2^+ state indicating that their wave fuctions of the 0_2^+ state are highly reliable. For example, the reduced α-decay widths of the 0_2^+ state calculated by Uegaki et al. and Kamimura et al. at the channel radius $a = 7$ fm are 0.39 and 0.56, respectively, while the observed value is 0.38. The calculated values of the monopole matrix element $M(0_2^+ \to 0_1^+)$ by Uegaki et al. and Kamimura et al are 6.6 fm^2 and 6.7 fm^2, respectively, while the observed value is 5.4 fm^2.

Recently, based on the investigations by Röpke, Schuck, and coauthors on the possibility of α-particle condensation in low-density nuclear matter [6], the present authors proposed a conjecture that near the $n\alpha$ threshold in self-conjugate $4n$ nuclei there exist excited states of dilute density which are composed of a weekly interacting gas of self-bound α particles and which can be considered as an $n\alpha$ condensed state [7]. This conjecture was backed by examining the structure of ^{12}C and ^{16}O using a new α-cluster wave function of the α-cluster condensate type. The new α-cluster wave function actually succeeded to place a level of dilute density (about one third of ground state density) in each system of ^{12}C and ^{16}O in the vicinity of the three respectively four α breakup threshold, without using any adjustable parameter. In the case of ^{12}C, this success of the new α-cluster wave function may seem rather natural because, as we explained above, we had already known that the microscopic 3α cluster models had predicted that the 0_2^+ in the vicinity of the 3α breakup threshold has a gas-like structure of 3α-particles which interact weakly with each other predominantly in relative S waves.

The new α-cluster wave function of the α-cluster condensate type used in Ref. [7] represents a condensation of α-clusters in a spherically symmetric state. The present authors extended the wave function so that it can describe the α-cluster condensate with spatial deformation [8]. They applied this new wave function to ^{8}Be and succeeded to reproduce not only the binding energy of the ground state but also the energy of the excited 2^+ state. In addition, they found that although the effect of the spatial deformation is not large, the introduction of the spatial deformation brought forth a 100 % overlap of the condensate wave function with the "exact" wave function given by the microscopic 2α cluster model which solves the 2α-cluster equation of motion, $\langle\phi_\alpha^2|(H-E)|\mathcal{A}\{\chi(\boldsymbol{r})\phi_\alpha^2\}\rangle = 0$. This fact forces us to modify our understanding of the ^{8}Be structure from the 2α 'dumb-bell' structure to the 2α dilute (gas-like) structure.

The purpose of this short note is to report on our study of ^{12}C using the extended 3α condensate wave function with spatial deformation and comparing the obtained results for ^{12}C with those of the "exact" 3α cluster model wave functions by Uegaki et al. and by Kamimura et al. The most remarkable result of this comparison is that the 0_2^+ wave functions by Uegaki et al. and by Kamimura et al. are almost completely equivalent to our condensate wave functions with slight spatial deformation which are obtained by using the same effective two-nucleon force as Uegaki et al. and Kamimura et al., respectively. This result implies that the "exact" 3α cluster model wave functions for the second 0^+ state of ^{12}C can definitely be interpreted as 3α-particle Bose-condensed state.

The wave function of the $n\alpha$-cluster condensate with spatial deformation was introduced in Ref. [8] and the detailed explanation of it is given there. So here we give a brief explanation which is necessary in this paper. The wave function has the form

$$\Phi_{n\alpha}(\beta_x,\beta_y,\beta_z) = \int d^3R_1\cdots d^3R_n \times \exp\Big\{-\sum_{i=1}^{n}\Big(\frac{R_{ix}^2}{\beta_x^2}+\frac{R_{iy}^2}{\beta_y^2}+\frac{R_{iz}^2}{\beta_z^2}\Big)\Big\}\Phi^{\mathrm{B}}(\boldsymbol{R}_1,\cdots,\boldsymbol{R}_n)$$
$$\propto \mathcal{A}\Big[\exp\Big\{-\sum_{i=1}^{n}\Big(\frac{2X_{ix}^2}{B_x^2}+\frac{2X_{iy}^2}{B_y^2}+\frac{2X_{iz}^2}{B_z^2}\Big)\Big\}\phi(\alpha_1)\cdots\phi(\alpha_n)\Big],$$

where $\boldsymbol{X}_i = (1/4)\sum_{n=1}^4 \boldsymbol{r}_{in}$ is the center-of-mass coordinate of the ith α-cluster α_i, $\phi(\alpha_i)$ is the same internal wave function of the α-cluster α_i as the previous microscopic 3α cluster model, $B_k^2 = b^2+2\beta_k^2$ $(k=x,y,z)$, and $\Phi^{\mathrm{B}}(\boldsymbol{R}_1,\cdots,\boldsymbol{R}_n)$ is Brink's α-cluster model wave function [9]. It is to be noted that $\Phi_{n\alpha}(\beta_x,\beta_y,\beta_z)$ expresses the state where n α-clusters occupy the same spatially deformed center-of-mass orbit $\exp[-(2/B_x^2)X_x^2-(2/B_y^2)X_y^2-(2/B_z^2)X_z^2]$, while the internal α-cluster wave functions stay spherical. $\Phi_{n\alpha}(\beta_x,\beta_y,\beta_z)$ can be written as a product of the total center-of-mass wave function and the internal wave function $\widehat{\Phi}_{n\alpha}(\beta_x,\beta_y,\beta_z)$ as

$$\Phi_{n\alpha}(\beta_x,\beta_y,\beta_z) \propto \exp\Big\{-\frac{2nX_{Gx}^2}{B_x^2}-\frac{2nX_{Gy}^2}{B_y^2}-\frac{2nX_{Gz}^2}{B_z^2}\Big\} \times\widehat{\Phi}_{n\alpha}(\beta_x,\beta_y,\beta_z),$$
$$\widehat{\Phi}_{n\alpha}(\beta_x,\beta_y,\beta_z) = \mathcal{A}\Big[\exp\Big\{-\sum_{i=1}^{n}\Big(\frac{2}{B_x^2}(X_{ix}-X_{Gx})^2 +\frac{2}{B_y^2}(X_{iy}-X_{Gy})^2+\frac{2}{B_z^2}(X_{iz}-X_{Gz})^2\Big)\Big\}\phi(\alpha_1)\cdots\phi(\alpha_n)\Big].$$

All the calculations are made not with $\Phi_{n\alpha}(\beta_x,\beta_y,\beta_z)$ but with $\widehat{\Phi}_{n\alpha}(\beta_x,\beta_y,\beta_z)$ which is an eigen state of total momentum with eigen value zero. In this paper we assume axial symmetry of the deformation around the intrinsic z-axis and put $\beta_x=\beta_y$. The α-condensed wave function with good angular momentum which is obtained by spin projection is then written as

$$\widehat{\Phi}_{n\alpha}^{J}(\beta_x\!=\!\beta_y,\beta_z)=\int d\cos\theta\, d_{M0}^J(\theta)\widehat{R}_y(\theta)\widehat{\Phi}_{n\alpha}(\beta_x\!=\!\beta_y,\beta_z),$$

where $\widehat{R}_y(\theta)$ is the rotation operator around the intrinsic y axis which rotates $\widehat{\Phi}_{n\alpha}$ by an angle θ, and $d_{M0}^J(\theta)$ is the small D-function.

As effective two-nucleon forces, we adopt the ones of Uegaki et al. and Kamimura et al. One is the Volkov force No.1 [10] with Majorana parameter $M=0.575$, used by Uegaki et al., and the other is the Volkov force No.2 [10] with Majorana parameter $M=0.59$, used by Kamimura et al. Hereafter, we refer to the former force as force I while the latter force is referred to as force II. We adopt the same values for the oscillator parameter b as Uegaki et al. and Kamimura et al., namely $b=1.41$ fm for force I while $b=1.35$ fm for force II.

In Fig.1 we give the contour map of the $J^\pi=0^+$ binding energy surface corresponding to the spin-projected state $\widehat{\Phi}_{3\alpha}^{J=0}(\beta_x=\beta_y,\beta_z)$ in the two parameter space, $\beta_x(=\beta_y)$ and β_z. The adopted effective force for this energy surface is force II. We see the energy minimum

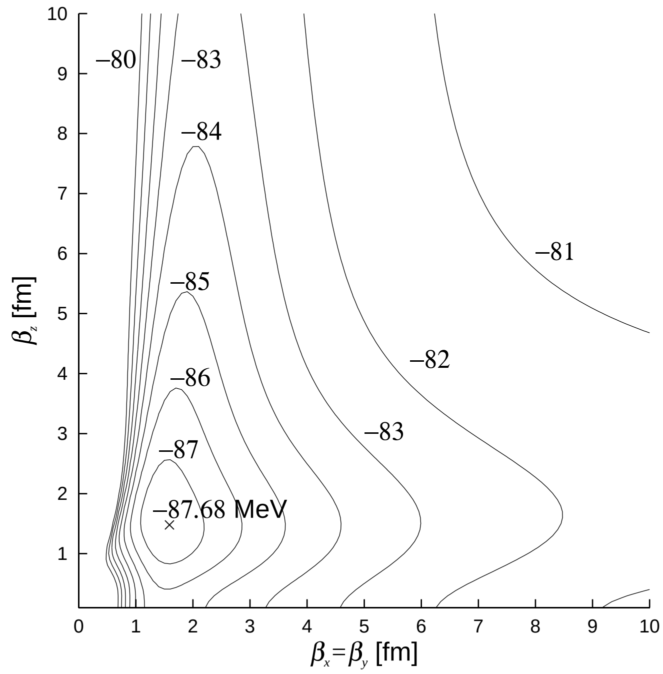

FIG. 1: Contour map of the energy surface of the 0^+ state in the two parameter space, $\beta_x(=\beta_y)$ and β_z. The adopted effective force is force II.

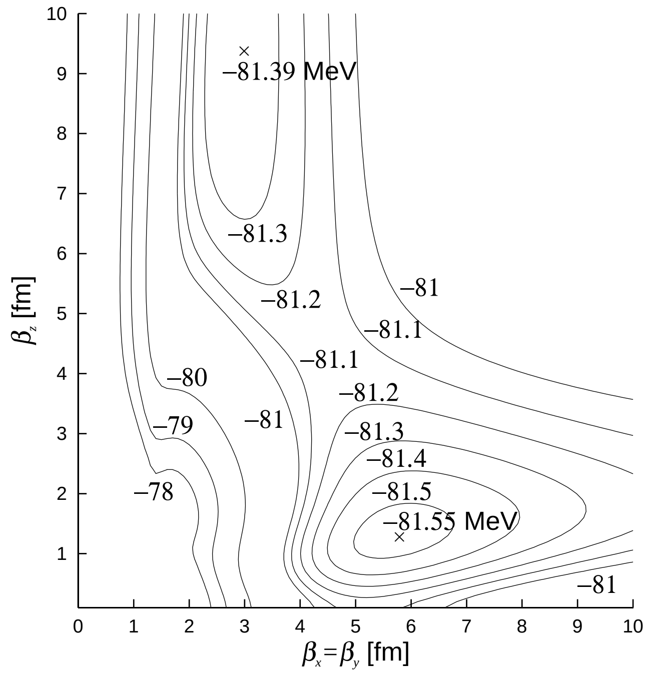

FIG. 2: Contour map of the energy surface corresponding to the 0^+ state orthogonalized to the state at the minimum energy point in Fig. 1. The adopted effective force is force II.

at $\beta_x(=\beta_y) = 1.5$ fm and $\beta_z = 1.5$ fm, which means that the minimum has a spherical shape. The minimum energy of -87.68 MeV is about 1.7 MeV higher than the binding energy of -89.4 MeV obtained by Kamimura et al. for the ground 0^+ state. The energy surface in the case of force I is similar to the energy surface of Fig.1. The minimum energy obtained by the use of force I is -86.09 MeV and it is about 1.8 MeV higher than the binding energy of -87.92 MeV obtained by Uegaki et al. for the ground 0^+ state.

In Fig.2 we give the contour map of the $J^\pi = 0^+$ binding energy surface corresponding to the state orthogonalized to the minimum energy state $\widehat{\Phi}^{J=0}_{3\alpha}(\text{min.})$ (the state at the minimum energy point in Fig.1). The orthogonalized state is denoted as $P_\perp \widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z)$ and is expressed as

$$\begin{aligned} & P_\perp \widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z) \\ & = (1 - |\widehat{\Phi}^{N,J=0}_{3\alpha}(\text{min.})\rangle\langle\widehat{\Phi}^{N,J=0}_{3\alpha}(\text{min.})|)\widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z), \\ & \widehat{\Phi}^{N,J=0}_{3\alpha}(\text{min.}) \equiv \widehat{\Phi}^{J=0}_{3\alpha}(\text{min.}) / \sqrt{\langle\widehat{\Phi}^{J=0}_{3\alpha}(\text{min.})|\widehat{\Phi}^{J=0}_{3\alpha}(\text{min.})\rangle} \end{aligned}$$

The adopted effective force for Fig.2 is force II. We see an energy minimum at $\beta_x(=\beta_y) = 5.7$ fm and $\beta_z = 1.3$ fm in the oblate region of the map and a second energy minimum at $\beta_x(=\beta_y) = 2.9$ fm and $\beta_z = 9.4$ fm in the prolate region of the map. The minimum energy value is -81.55 MeV and, what is very remarkable, this value is almost the same as the binding energy of -81.66 MeV obtained by Kamimura et al. for the second 0^+ state. The minimum energy of -81.55 MeV is close to the second minimum energy of -81.39 MeV, and there is a valley with an almost flat bottom connecting these two minima. An almost flat bottom of the valley means that the energy of the spherical configuration is only slightly higher than that of the deformed configuration, namely the energy gain due to the deformation is small. The energy surface by the orthogonalized state $P_\perp \widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z)$ in the case of the force I is similar to the energy surface of Fig.2, and the minimum energy of the orthogonalized state is -79.83 MeV. Here again it is very remarkable that this value is almost the same as the binding energy of -79.3 MeV obtained by Uegaki et al. for the second 0^+ state.

The fact that for each case of the two different effective forces a single orthogonalized state $P_\perp \widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z)$ yields almost the same energy as the "exact" energy of the 0^+_2 state obtained by solving a full three-body problem of the microscopic 3α-cluster model, strongly suggests that the 0^+_2 state wave function given by the microscopic 3α-cluster model is similar to the rather simple state $P_\perp \widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z)$ as long as the adopted effective two-nucleon force is reasonably realistic.

We also calculated the $J^\pi = 2^+$ energy surface corresponding to the spin-projected state $\widehat{\Phi}^{J=2}_{3\alpha}(\beta_x = \beta_y, \beta_z)$ for the forces I and II. The minimum energies for the forces I and II are obtained to be -83.61 MeV at $\beta_x(=\beta_y) = 1.30$ fm and $\beta_z = 0.35$ fm, and to be -84.65 MeV at $\beta_x(=\beta_y) = 1.50$ fm and $\beta_z = 0.35$ fm, respectively. These minimum energy values for forces I and II are both higher by about 2 MeV than the lowest 2^+ energies by

Uegaki et al. and by Kamimura et al., respectively, whose values are shown in Table I.

We also performed the GCM (generator coordinate method) calculation for $J^\pi = 0^+$ and 2^+ by superposing $\widehat{\Phi}^J_{3\alpha}(\beta_x = \beta_y, \beta_z)$ over various sets of (β_x, β_z);

$$\sum_{(\beta_x,\beta_z)} \langle \widehat{\Phi}^J_{3\alpha}(\beta'_x = \beta'_y, \beta'_z)|(H - E_k)|\widehat{\Phi}^J_{3\alpha}(\beta_x = \beta_y, \beta_z)\rangle \times f^J_k(\beta_x, \beta_z) = 0.$$

The adopted values of β_x are $\beta_x = (i - 0.5)$ fm with $i = 1 \sim 6$, and those of β_z is $\beta_z = (j - 0.5)$ fm with $j = 1 \sim 8$. Hence the total number of the adopted grid points (β_x, β_z) is 48. The calculated eigen energies of the 0^+_1, 0^+_2, and 2^+_1 states are given in Table I for the two forces I and II. We have checked the convergence of the calculation of the eigen energies by changing the sets of (β_x, β_z) for the GCM calculation. We see in Table I that all the GCM eigen energies of the 0^+_1, 0^+_2, and 2^+_1 states are almost the same as the energies of the microscopic 3α-cluster model in both cases of forces I and II. Since the eigen energies obtained by solving the full three-body problem of the microscopic 3α-cluster model are the "exact" energies, we can say by using the minimax theorem of the variational problem that this almost complete equivalence of our GCM energies with the exact energies means that our GCM wave functions of the 0^+_1, 0^+_2, and 2^+_1 states are almost equivalent respectively to the 0^+_1, 0^+_2, and 2^+_1 wave functions of the microscopic 3-α-cluster model in both cases of force I and II. In order to check further this almost complete equivalence, we give in Table II the comparison of the calculated r.m.s. radii and monopole matrix elements $M(0^+_2 \to 0^+_1)$ between our GCM and the microscopic 3α-cluster model. We see nice agreement of the calculated quantities between our GCM and the microscopic 3α-cluster model. In Table II, we see that the large r.m.s. radius of the 0^+_2 state is also predicted by our GCM as by the microscopic 3α-cluster model, but at the same time we see that the calculated value corresponding to our GCM is slightly larger than that of the 3α-cluster model. We think the reason is because our wave function of the 0^+_2 state which contains a large amount of the components of the 3α condensed wave functions $\widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z)$ with large β_x and/or β_z so as to yield large r.m.s. radius may have a longer tail than the former 3α-cluster model. This possibly longer tail behavior of the GCM 0^+_2 wave function may explain the slight underestimation of the monopole matrix element of the GCM versus the 3α-cluster model through the slightly enhanced mismatch between the 0^+_1 and 0^+_2 wave functions in the GCM case.

The fact that the second 0^+ wave function of the microscopic 3α-cluster model is almost completely equivalent to our GCM wave function of the second 0^+ state which has a very large r.m.s. radius or equivalently very dilute density is very important. Since our GCM wave function of the 0^+_2 state expresses the Bose-condensed state of 3α-clusters, as is clear from its large r.m.s. radius and from its functional form, we can say that the second 0^+ wave function of the microscopic 3α-cluster model obtained long time ago underlines the fact that the second 0^+ state of ^{12}C in the vicinity of the 3α breakup threshold has a gas-like structure of 3α clusters with "Bose-condensation".

Now we discuss the relation between our GCM wave function of the 0^+_2 state which we denote as $\Psi_{\rm GCM}(0^+_2)$ and the orthogonalized state $P_\perp \widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y, \beta_z)$ with minimum energy which we denote by $\Psi_\perp(0^+_2)$. Although the energy of $\Psi_\perp(0^+_2)$ is almost equivalent to that of $\Psi_{\rm GCM}(0^+_2)$ and also to that of the 0^+_2 wave function of the microscopic 3α cluster model, we cannot simply conclude that $\Psi_\perp(0^+_2)$ is almost equivalent to $\Psi_{\rm GCM}(0^+_2)$. It is because $\Psi_\perp(0^+_2)$ is not yet guaranteed to be orthogonal to the 0^+_1 wave function. The orthogonality of $\Psi_\perp(0^+_2)$ to $\widehat{\Phi}^{J=0}_{3\alpha}({\rm min.})$ which is the state at the minimum energy point of the energy surface is not the same as the orthogonality to the 0^+_1 wave function, and $\Psi_\perp(0^+_2)$ may contain some amount of the 0^+_1 wave function. We therefore calculated the squared overlap value of the two wave functions, $|\langle\Psi_\perp(0^+_2)|\Psi_{\rm GCM}(0^+_2)\rangle|^2$. The obtained values are 0.95 and 0.97 for forces I and II, respectively. These large overlap values mean that the GCM 0^+_2 wave functions are very similar to $\Psi_\perp(0^+_2)$ in both cases of force I and II and hence verify our former statement that the 0^+_2 wave function of the microscopic 3α cluster model is very similar to a simple state $\Psi_\perp(0^+_2)$ so long as the adopted two- nucleon force reasonably describes the physics.

We also studied the magnitude of the spherical condensate component contained in our GCM 0^+_2 wave functions. For this purpose, we first constructed the projection operator $P_{\rm sph}$ onto the functional space $\rm S_{sph}$ spanned by spherical condensate wave functions as $P_{\rm sph} = \sum_k |\Psi^k_{\rm sph}\rangle\langle\Psi^k_{\rm sph}|$, where $\Psi^k_{\rm sph}$ are the orthonormal basis functions of the space $\rm S_{sph}$. $\Psi^k_{\rm sph}$ are constructed as follows,

$$\sum_{\beta_x} \langle\widehat{\Phi}^{J=0}_{3\alpha}(\beta'_x = \beta'_y = \beta'_z)|\widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y = \beta_z)\rangle g^k(\beta_x) = \mu_k g^k(\beta'_x),$$

$$\sum_{\beta_x} g^{k'}(\beta_x) g^k(\beta_x) = \delta_{k'k},$$

$$\Psi^k_{\rm sph} = \frac{1}{\sqrt{\mu_k}} \sum_{\beta_x} g^k(\beta_x)\widehat{\Phi}^{J=0}_{3\alpha}(\beta_x = \beta_y = \beta_z).$$

The calculated values of $|\langle\Psi_{\rm GCM}(0^+_2)|P_{\rm sph}|\Psi_{\rm GCM}(0^+_2)\rangle|^2$ are 0.92 and 0.91 for forces I and II, respectively. Of course, we checked the convergence of the calculation by changing the number of the adopted components $\Psi^k_{\rm sph}$ in $P_{\rm sph}$. The large magnitudes of these values imply that $\Psi_{\rm GCM}(0^+_2)$ is mostly composed of the spherical condensate component by more than 91 %. At the same time we have to note that some amount (less than 9 %) of the deformed component which is orthogonal to the spherical component is necessary in order to have quantitatively good reproduction of the observed quantities.

TABLE I: Comparison of the minimum energy of the spin-projected energy surface, GCM eigen energy, and the energy given by the full 3α calculation. Comparison is made for the 0_1^+, 0_2^+, and 2_1^+ states for two cases of the effective two-nucleon force. The energy surface for the 0_2^+ state means that of the orthogonalized state $P_\perp \widehat{\Phi}_{3\alpha}^{J=0}(\beta_x = \beta_y, \beta_z)$. Energies are in MeV.

	Volkov No.1 $M = 0.575$, $E_{\rm th}(3\alpha) = -81.01$			Volkov No.2 $M = 0.59$, $E_{\rm th}(3\alpha) = -82.04$		
	$E_{\rm min}$ of energy surface	GCM eigen energy	full 3α calculation [2]	$E_{\rm min}$ of energy surface	GCM eigen energy	full 3α calculation [3]
0_1^+	-86.09	-87.81	-87.92	-87.68	-89.52	-89.4
0_2^+	-79.83	-79.97	-79.3	-81.55	-81.79	-81.7
2_1^+	-83.61	-85.34	-85.7	-84.65	-86.71	-86.7

TABLE II: Comparison of the r.m.s. radii $R_{\rm rms}$ and the monopole matrix element $M(0_2^+ \to 0_1^+)$ obtained by the GCM calculation with those by the full 3α calculation. Comparison is made for two cases of the effective two-nucleon force. $R_{\rm rms}$ are in fm, and $M(0_2^+ \to 0_1^+)$ are in fm^2.

	Volkov No.1 $M = 0.575$		Volkov No.2 $M = 0.59$	
	GCM calculation	full 3α calculation [2]	GCM calculation	full 3α calculation [3]
$R_{\rm rms}(0_1^+)$	2.40	2.53	2.40	2.40
$R_{\rm rms}(0_2^+)$	4.44	3.50	3.83	3.47
$R_{\rm rms}(2_1^+)$	2.38	2.50	2.38	2.38
$M(0_2^+ \to 0_1^+)$	5.36	6.6	6.45	6.7

We finally make a remark on the ground 0^+ wave function. Since this state has a normal radius and density, three α clusters overlap strongly with each other in this state, which is totally different from the situation of the 0_2^+ state where the mutual overlap of three, or even two α clusters is small. Therefore even though the 0_1^+ state is well represented by a superposition of our condensate wave functions (we recall that our wave function contains the Slater determinant as a limit case), it does not mean at all that the state has an α condensation character which is only valid for the gas-like state of α-clusters.

In summary, we have shown that the 0_2^+ wave function of ^{12}C which was obtained long time ago by solving the full three-body problem of the microscopic 3α cluster model is almost completely equivalent to the wave function of the 3α condensed state. This equivalence has been shown to hold for two different effective two-nucleon forces. This result gives us strong support to our opinion that the 0_2^+ state of ^{12}C has a gas-like structure of 3α clusters with "Bose-condensation". A more detailed report of the present problem will be given elsewhere.

[1] For example, Y. Fujiwara, H. Horiuchi, K. Ikeda, M. Kamimura, K. Kato, Y. Suzuki, and E. Uegaki, Prog. Theor. Phys. Suppl. No.68, 29 (1980).

[2] E. Uegaki, S. Okabe, Y. Abe, and H. Tanaka, Prog. Theor. Phys. **57**, 1262 (1977); E. Uegaki, Y. Abe, S. Okabe, and H. Tanaka, Prog. Theor. Phys. **59**, 1031 (1978); **62**, 1621 (1979).

[3] Y. Fukushima and M. Kamimura, *Proc. Int. Conf. on Nuclear Structure*, Tokyo, 1977, ed. T. Marumori (Suppl. of J. Phys. Soc. Japan, Vol.44, 1978), p.225; M. Kamimura, Nucl. Phys. A **351**, 456 (1981).

[4] H. Horiuchi, Prog. Theor. Phys. **51**, 1266 (1974); **53**, 447 (1975).

[5] H. Morinaga, Phys. Rev. **101**, 254 (1956); Phys. Lett. **21**, 78 (1966).

[6] G. Röpke, A. Schnell, P. Schuck, and P. Nozieres, Phys. Rev. Lett. **80**, 3177 (1998); M. Beyer, S. A. Sofianos, C. Kuhrts, G. Röpke, and P. Schuck, Phys. Lett. B **488**, 247 (2000).

[7] A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. Lett. **87**, 192501 (2001).

[8] Y. Funaki, H. Horiuchi, A. Tohsaki, P. Schuck, and G. Röpke, Prog. Theor. Phys. **108**, 297 (2002).

[9] D. M. Brink, *Proc. Int. School Phys. Enrico Fermi* **36** (Academic Press, New York and London, 1966), p.247.

[10] A. B. Volkov, Nucl. Phys. **74**, 33 (1965).