Multistep reactions at energies up to 120 MeV

P. Demetriou[a], **A. Marcinkowski**[b]

[a] Institut d'Astronomie et d'Astrophysique, Université Libre de Bruxelles
Campus de la Plaine, CP-226, 1050 Brussels, Belgium,
[b] The Andrzej Soltan Institute for Nuclear Studies,
00-681 Warsaw, Hoża 69, Poland

Abstract

The multistep direct reaction theory of Feshbach, Kerman and Koonin (FKK) [1] is applied to the analysis of nucleon scattering and charge-exchange reactions to the continuum for incident energies up to 120 MeV. The theory is modified to include non-DWBA matrix elements as well as both the coherent vibrations and incoherent particle-hole excitations [2]. It is shown that the distinction between bound and unbound final particle-hole states is important. Only the former are considered in the calculation of the one-step cross sections that are subsequently convoluted to give the FKK multistep cross sections. The results for proton scattering by iron at 62 MeV and for the (p,xn) reaction on zirconium at 45, 80 and 120 MeV are presented. It is shown that up to 7-step reactions can contribute at the highest energy.

1 Introduction

The multistep direct reaction theory of Feshbach, Kerman, Koonin [1], is based on the convolution of one-step cross sections leaving the residual nucleus in well-defined, bound final states. The one-step cross sections include the excitation of collective one-phonon vibrations as well as, incoherent particle-hole states, that satisfy the energy-weighted sum rules [2,3]. At energies above 40 MeV, the nucleus is likely to be excited to unbound particle-hole final states [4]. In such a case, the one-step reaction leads to two continuum particles in the final state and thus to two-particle emission, i.e. a knockout reaction (a,ab). The transition matrix involves a three-body final state that cannot be treated in the normal DWBA approximation for a (a,b) reaction. Furthermore, the knockout reaction can be followed by multistep scattering processes that are out of the scope of the FKK multistep reaction theory of [1]. However, multistep scattering in (a,ab) reactions has been treated rigorously by Ciangaru [5], who has applied similar statistical postulates as those adopted in the FKK theory. The resulting formalism is rather involved and has never been implemented in computational calculations. Instead, the approximate model of Chadwick et al [6], based on the FKK theory, has been widely used in practical applications where two-particle emission is involved. According to the model, the emission of one of the two particles is described by the FKK MSD cross section and the emission of the other one is described by the probability of escape given by transmission coefficients. This crude approach has been applied to incident energies up to 200 MeV. The resulting cross sections MSD2, for the emission of one of the two particles at each reaction stage M, proved to be able to account for the low energy part of the energy spectra and also for the rather symmetric angular distributions at high excitation energies [6].

In this paper we apply the 1SD cross section of [2], including a macroscopic term describing the collective vibrations and the microscopic terms describing incoherent particle-hole excitations. The density of the bound 1p1h final states of [7], with restrictions for particle-binding and finite hole-depth, is used to obtain the $1\mathrm{SD}_{bound}$ cross sections. The latter are folded in the convolution integral to give the MSD cross sections. The 1SD cross sections to both bound and unbound 1p1h final states are calculated with the density formula of [8], with finite hole-depth restrictions only. The 1SD cross sections are used to evaluate the 1SD2 cross sections according to the prescription of [6]. The distinction between protons and neutrons as leading continuum particles gives rise to different sequences of reaction stages contributing to the MSD cross sections. All the sequences are taken into account in the calculations. Good agreement between theory and experiment is obtained at incident energies of 45, 62, 80 and 120 MeV.

2 Calculations and comparison with experiment

The multistep cross section of the FKK theory is obtained by multiple folding of the $1\mathrm{SD}_{bound}$ cross sections [1,9],

$$(\frac{\mathrm{d}^2\sigma}{\mathrm{dEd}\Omega})_{MSD} = \int \frac{m_1 E_1}{(2\pi)^2\hbar^2}\mathrm{dE_1 d\Omega_1} \int \frac{\mathrm{m_2 E_2}}{(2\pi)^2\hbar^2}\mathrm{dE_2 d\Omega_2} \cdots \int \frac{\mathrm{m_{M-1} E_{M-1}}}{(2\pi)^2\hbar^2}\mathrm{dE_{M-1} d\Omega_{M-1}}$$
$$\times \left(\frac{\mathrm{d}^2\sigma}{\mathrm{dE_M d\Omega_M}}\right)_{\mathrm{1SD}} \cdots \left(\frac{\mathrm{d}^2\sigma}{\mathrm{dE_2 d\Omega_2}}\right)_{\mathrm{1SD}} \left(\frac{\mathrm{d}^2\sigma}{\mathrm{dE_1 d\Omega_1}}\right)_{\mathrm{1SD}} . \quad (1)$$

In Eq. (1), M-1 of the successive double-differential $1\mathrm{SD}_{bound}$ cross sections include matrix elements with biorthogonally conjugate distorted waves $\hat{\chi}^+$ and χ^+, which give rise to the enhanced non-DWBA matrix elements in the intermediate stages of the MSD reaction [10]. The non-DWBA matrix elements can be expressed in terms of normal DWBA matrix elements by using the relation $< \hat{\chi}^+| = \mathrm{S}^{-1} < \chi^-|$. Since for each transfer of orbital angular momentum or multipolarity L , a number of partial waves ℓ_{M-1}, ℓ_{M-2} contribute to the incoming and outgoing distorted waves, respectively, each one of the incoming waves in the outgoing channel is enhanced by the inverse of the corresponding elastic scattering matrix $\mathrm{S}^{-1}_{\ell_{M-1}}$. In the statistical theory of FKK, the enhanced transition matrix elements are averaged over energy [11]. In the current approach, the effect of energy averaging is approximated by replacing the enhancing factors $\mathrm{S}^{-1}_{\ell_{M-1}}$ acting on the partial waves ℓ_{M-1} for a specific L, by an average enhancing factor $< \mathrm{S}_L >^{-1}$. The elastic scattering matrix elements are expressed in terms of the partial wave transmission coefficients T_L of the optical potential, S_L^2=1-T_L. The energy averaging of $< S_L >^{-1}$ removes any singularities that may arise at the energies of the single-particle resonances when the T_L approaches unity. The enhanced DWBA matrix elements in the intermediate stages of the MSD reaction, Eq. (1), give rise to enhanced intermediate $1SD_{bound}$ cross sections. In particular, each term of the $1SD_{bound}$ cross section corresponding to a specific angular momentum transfer L, is enhanced by a factor $< S_L >^{-2}$. Both the collective term as well as the terms describing the incoherent excitations in $1SD_{bound}$ are enhanced.

The macroscopic DWBA cross sections include the one-phonon 1^-, 2^+, 3^-, 4^+ states, as well as the giant dipole, quadrupole and the low-energy part of the giant octupole resonances, for the even-even core nuclei of interest. The experimental deformation parameters are used where available, while for the giant resonances the latter are determined by depleting the energy-weighted sum rules by the known strengths of the one-phonon states. The weak coupling model is used for odd-mass nuclei. In the case of charge-exchange reactions, the isovector cross section are calculated in an approximate way described in detail in [4,12]. The microscopic DWBA cross sections are calculated with a two-particle form factor, with a real effective Yukawa interaction of strength $V_{\pi\pi} = V_{\nu\nu} = 12.7$ MeV and $V_{\pi\nu} = V_{\nu\pi} = 43.1$ MeV [13]. The energy dependence of the strength is included according to [14]. The DWBA cross sections are averaged over final, shell model particle-hole states. The distorted waves are obtained from the phenomenological optical model potentials of [15,16] for neutrons, and [17,18] for protons. The same potentials are used to evaluate the enhancing factors $< S_L >^{-2}$. Both the microscopic and macroscopic DWBA cross sections are calculated with the DWUCK-4 code [19].

The $1SD_{unbound}$ cross section is obtained by subtracting $1SD_{bound}$ from 1SD, i.e. $1SD_{unbound}$ = 1SD-$1SD_{bound}$. $1SD_{unbound}$ corresponds to that part of the 1SD cross section that, at energies above 40 MeV gives rise to two-particle emission, since it involves an unbound particle in the final 1p1h state that can escape the nucleus. At energies below 40 MeV, it is safe to assume that the unbound particle undergoes damping tansitions, before ending in the final bound particle-hole state. The 1SD cross sections are used as input for the calculation of the spectrum of the second of the two particles emitted, 1SD2, according to the prescription of [6]. It is worth noting that, in the case of the (p,n) reaction, the second neutron can be emitted only after a (p,p′) reaction that excites both neutron and proton particle-hole pairs in unbound states.

By distinguishing between a proton or neutron leading particle in the continuum, a number of different sequences of M-1 collisions can occur at the M-th stage of the reaction. In the case of a (p,n) reaction, the two-step contribution will include a (p,n′)(n′,n) and a (p,p′)(p′,n) term. All sequences are lumped together to give the MSD cross sections which are compared with experiment in Figures 1 to 4. The cut-off of the $1SD_{bound}$ cross section at the excitation energy of 40 MeV, is a direct result of including only bound final 1p1h states. The $1SD_{bound}$ cross sections satisfy the EWSR's independent of the incident energy. With increasing incident energy, the excitation of unbound 1p1h states becomes more likely and the $1SD_{unbound}$ increases. With the current approximate method of [6], only two-particle emission from the first stage of the reaction 1SD2 can be calculated, since two-particle emission from higher stages requires the use of MSD cross sections to unbound final states. These contributions can only be estimated in the framework of a more rigorous theory of multistep two-particle preequilibrium emission [5].

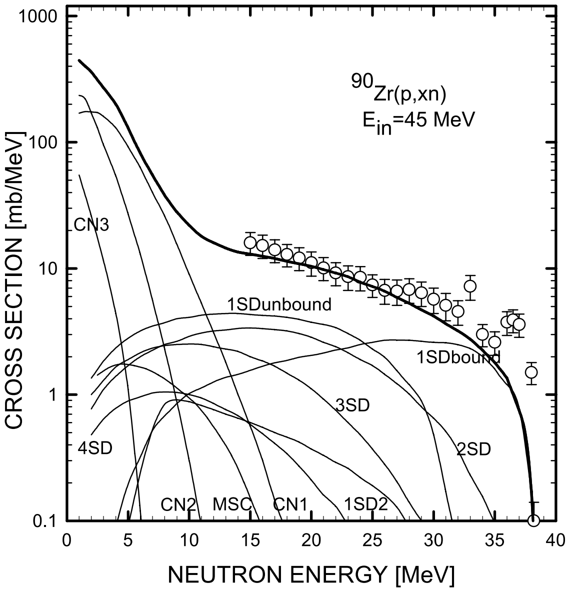

Fig. 1. Comparison of the MSD cross sections calculated with the non-DWBA matrix elements with the inclusive spectrum of neutrons emitted from the ^{90}Zr(p,xn)^{90}Nb reaction at incident energy of 45 MeV [20]. The thick line is a sum of all contributions. CN1-CN3 denote primary, secondary and tertiary neutrons evaporated from the compound nucleus. MSC shows the pre-equilibrium compound emission. The 1SD cross section is split into the $1SD_{bound}$ and $1SD_{unbound}$ components. The experimental peaks are due to the isobaric analog and the low-energy $s_{1/2}$-, $p_{5/2}$-hole states. 1SD2 shows direct secondary neutron emission that follows primary proton emission.

The compound nucleus emission is calculated according to the theory of Hauser and Feshbach. Preequilibrium compound emission is described by the multistep compound reaction theory of Feshbach, Kerman, Koonin [1], including the gradual absorption of the incident flux [21–23].

3 Conclusions

The multistep direct theory of Feshbach, Kerman, Koonin [1], using non-normal DWBA matrix elements, is used to analyse charge-exchange and proton scattering at energies ranging from 45 MeV to 120 MeV. One-step cross sections are distinguished in $1SD_{bound}$, exciting the nucleus to bound final 1p1h states and 1SD, exciting the nucleus to both bound and unbound final 1p1h states. The former cross sections satisfy the EWSR's and are inserted in the convolution integral of FKK to give the multistep cross sections. With increasing incident energy, it becomes highly probable that the particle in the unbound

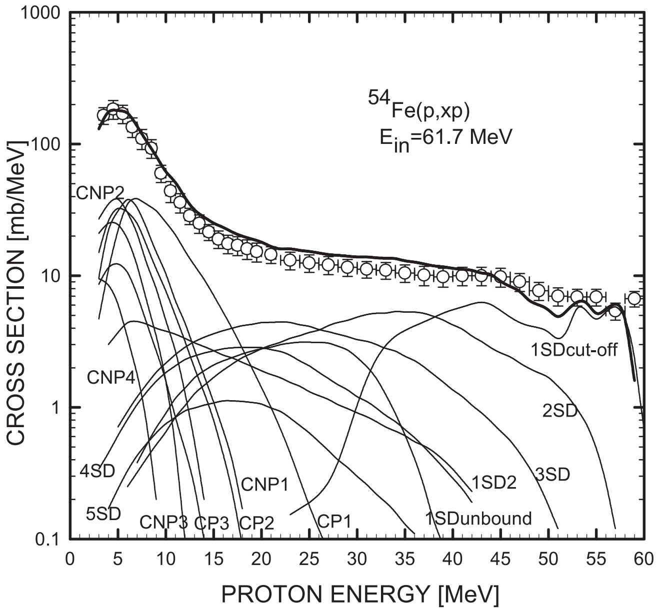

Fig. 2. Same as in Fig. 2 but for the inclusive spectrum of protons emitted from the ^{54}Fe(p,xp)^{54}Fe reaction at incident energy of 61.7 MeV [24]. The 1SD$_{cut-off}$ cross section approximates the bound-particle restriction in the final 1p1h state density (see Ref. [4]).

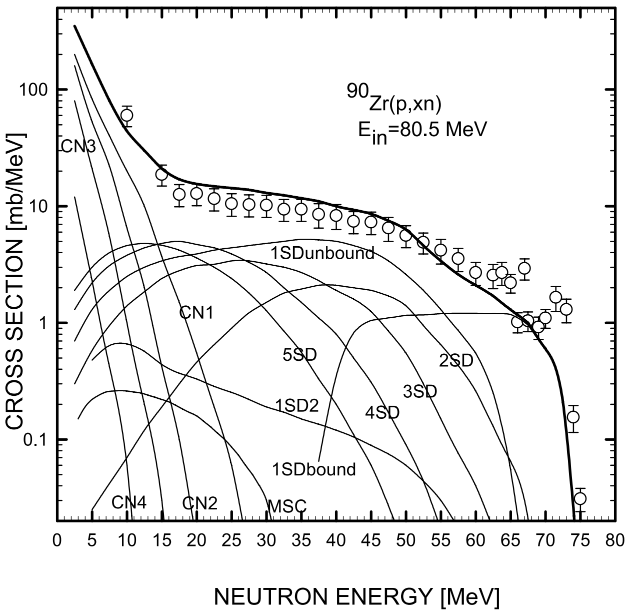

Fig. 3. The same as in Fig. 2 but for the incident energy of 80.5 MeV [25]. Up to 4 neutrons evaporate successively from the compound nucleus CN1-CN4.

final 1p1h state is also emitted along with the leading continuum particle. Such a direct knockout resulting in the emission of two particles, together with the possible multistep scattering involved, is described by the theory of Ciangaru [5].

So far, the 1p1h states restricted for particle binding have been used in calculations of multistep compound reactions, describing successive composite bound states embedded in

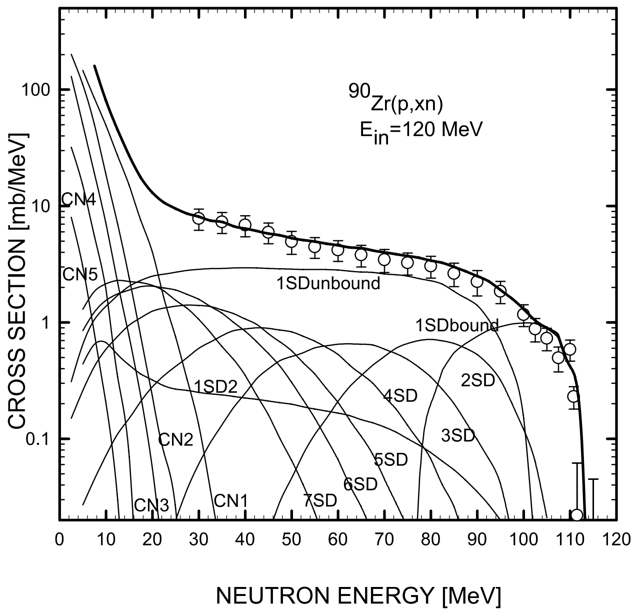

Fig. 4. The same as in Fig. 3 but for the incident energy of 120 MeV [26]. The MSD cross sections appear convergent only at the 7SD reaction stage.

the continuum before their decay into the exit channels [23]. In [12], it was shown that the restricted 1p1h-state density applies also to the description of the final bound-particle-hole states in the intermediate MSD reaction stages.

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