Theoretical approaches to nuclear reactions with exotic beams.

Angela Bonaccorso *
Istituto Nazionale di Fisica Nucleare, Sez. di Pisa,
and Dipartimento di Fisica, Università di Pisa, 56127 Pisa.

July 22, 2003

Abstract

In this talk I present some theoretical approaches to nuclear reactions with exotic beams. In particular their potential application to the analysis of data from experiments which could be performed at the forthcoming Italian INFN facilities with radioactive beams, namely at the Laboratorio Nazionale del Sud and at the Laboratorio Nazionale di Legnaro. First I will discuss nuclear and Coulomb breakup experiments which involve heavy exotic beams and intermediate incident energies, thus being suited for the LNL-SPES proposed facility. Then I will discuss transfer to the continuum reactions aiming at performing spectroscopy in the continuum of light unbound nuclei like ^{5}He, or ^{10}Li or ^{13}Be. Such reactions are best matched if the incident beam energy is very low, as it will be at the LNS-EXCYT facility.

1 Introduction

Nuclei far from the stability valley are often called "exotic" because they exhibit properties rather different from those of nuclei in the rest of the nuclear chart [1]. Most of them are neutron rich and unstable against β-decay. It is interesting to study them because they give information on the structure of matter under extreme conditions and allow to test nuclear models otherwise based only on properties of stable nuclei.

From the theoretical point of view one of the most interesting characteristic of some medium-light unstable nuclei such as ^{6}He, ^{11}Li, ^{14}Be is the fact that they are build on a core which is an unbound nucleus. The dominant feature is the appearance of intruder states in the single particle level scheme and the strong coupling between deformed cores and valence particles. The exact location and structure of the low energy resonances of those unbound nuclei will provide a keystone in the understanding of the nuclear interaction. In fact it has been seen that the effective potentials necessary to describe the data already existing

*In collaboration with D. M. Brink, N. Vinh Mau, J. Margueron, G. Blanchon, C. Papineau, C. Bertulani show some very unusual properties and a strong state dependence in particular in the spin-orbit and spin-isospin parts of the interaction. This is a very interesting characteristic in agreement with more fundamental findings based on "exact" calculations of light nuclei [2].

New techniques have been developed to study reactions initiated by these nuclei, which combine and unify the traditional treatment of bound and continuum scattering states. In this respective reaction theories like the transfer to the continuum [3, 4, 5] can be very useful.

Research on exotic nuclei has concentrated on studying elastic scattering [6] for the determination of the important reaction channels and breakup for spectroscopic properties like the determination of single particle state energies, angular momenta and spectroscopic factors [7, 8, 9, 10].

2 Reaction models for structure studies

One of the most suited measurement for an exotic projectile is the single-neutron removal cross section, in which only the projectile residue, namely the core with one less nucleon, is observed in the final state. This information together with the calculated cross sections [7, 8, 9, 10] has been used to extract single particle spectroscopic factors as in traditional transfer reactions. For halo nuclei interacting with light targets the interaction responsible for the reaction is the neutron-target complex potential. Besides the integrated removal cross section, denoted by σ_{-n}, the differential momentum distribution $d^3\sigma/dk^3$ is also measured. A particularly useful cross section is $d\sigma/dk_z$, the removal cross section differential in longitudinal momentum. It has been used to determine the angular momentum and spin of the neutron initial state [9] in a way similar to that proposed in [5, 11]. If the final state neutron can also be measured, the corresponding coincident cross section $A_p \rightarrow (A_p - 1) + n$ is called the diffractive (or elastic) breakup cross section if the interaction responsible for the removal is the neutron-target nuclear potential [12, 13]. In the case of heavy targets the coincident cross section contains also the contribution from Coulomb breakup due to the core-target Coulomb potential which acts as an effective force on the neutron. This observable is very useful to disentangle the reaction mechanism [14]. The difference between the removal and coincident cross sections is called the stripping (or absorption) cross section.

All theoretical methods used so far rely on a basic approximation to describe the collision with only the three-body variables of nucleon coordinate, projectile coordinate, and target coordinate. Thus the dynamics is controlled by the three potentials describing nucleon-core, nucleon-target, and core-target interactions. In most cases the projectile-target relative motion is treated semiclassically by using a trajectory of the center of the projectile relative to the center of the target $\mathbf{R}(t) = \mathbf{d} + \mathbf{v}t$ with constant velocity v in the z direction and impact parameter $\mathbf{d}$ in the xy plane.

2.1 Nuclear-Coulomb elastic breakup and nuclear absorption.

A full description of the treatment of the scattering equation for a projectile which decays by single neutron breakup following its interaction with the target, can be found in [4, 14]. There it was shown that within the semiclassical approach for the projectile-target relative motion, the amplitude for a transition from a nucleon bound state ψ_i in the projectile to a final continuum state ψ_f is best calculated in time dependent perturbation theory.

For light targets the recoil effect due to the projectile-target Coulomb potential can be neglected and the interaction responsible for the reaction is mainly the neutron-target nuclear potential. In the case of heavy targets the dominant reaction mechanism is Coulomb breakup. The Coulomb force does not act directly on the neutron but it affects it only indirectly by causing the recoil of the charged core. Therefore the neutron is subject to an effective force which gives rise to an effective dipole Coulomb potential $V_{eff}(\mathbf{r}, \mathbf{R}(t))$. In ref.[14] it was shown that the combined effect of the nuclear and Coulomb interactions to all orders can be taken into account by using the potential $V = V_{nt} + V_{eff}$ sum of the neutron-target optical potential and the Coulomb dipole potential. If for the neutron final continuum wave function we take a distorted wave of the eikonal-type, then the amplitude becomes :

$$A_{fi}\left(\mathbf{k},\mathbf{d}\right) = \frac{1}{i\hbar}\int d^3\mathbf{r}\int dt e^{-i\mathbf{k}\cdot\mathbf{r}+i\omega t} e^{\left(\frac{1}{i\hbar}\int_t^\infty V(\mathbf{r},t')dt'\right)} V\left(\mathbf{r},t\right)\phi_{l_i m_i}\left(\mathbf{r}\right) \tag{1}$$

where $\omega = \left(\varepsilon_f{}' - \varepsilon_0\right)/\hbar$ and ε_0 is the neutron initial bound state energy while $\varepsilon_f{}'$ is the neutron-core final continuum energy. Eq.(1) is appropriate to calculate the coincidence cross sections $A_p \rightarrow (A_p - 1) + n$ discussed in the previous section. Finally the differential probability with respect to the neutron energy and angles can be written as $\frac{d^3P_{nc}(d)}{d\varepsilon_f{}' \sin\theta d\theta d\phi} = \frac{1}{8\pi^3}\frac{mk}{\hbar^2}\frac{1}{2l_i+1}\Sigma_{m_i}|A_{fi}|^2$. where A_{fi} is given by Eq.(1) and we have averaged over the neutron initial state.

The effects associated with the core-target interaction will be included by multiplying the above probability by the probability for the core to be left in its ground state, defined in terms of a core-target S-matrix function of d, the core-target distance of closest approach. A simple parameterization [5] is $P_{ct}(d) = |S_{ct}|^2 = e^{(-\ln 2 exp[(R_s - d)/a])}$, where the strong absorption radius $R_s \approx 1.4(A_p^{1/3} + A_t^{1/3})fm$ is defined as the distance of closest approach for a trajectory that is 50% absorbed from the elastic channel and $a = 0.6fm$ is a diffusness parameter.

Thus the double differential cross section is

$$\frac{d^2\sigma}{d\varepsilon_f{}' d\Omega} = C^2 S\int_0^\infty d\mathbf{d}\frac{d^2P_{nc}(\mathbf{k},d)}{d\varepsilon_f{}'\Omega}P_{ct}(d), \tag{2}$$

(see Eq. (2.3) of [5]) and C^2S is the spectroscopic factor for the initial single particle orbital.

Inclusive cross sections in which only the core with $(A_p - 1)$ nucleons is detected need to take into account also the absorption of the neutron by the imaginary part of the n-target optical potential. For such reactions the Coulomb recoil effect can be neglected but the distorted eikonal-type wave function used in Eq.(1) is not accurate enough, in particular if the final continuum states are single particle resonances in the target plus one neutron nucleus. Then a distorted final neutron wave function, calculated by an optical model will be used. Also since the neutron is not detected one integrates over the neutron angles. Thus, according to [4] the final neutron probability energy spectrum with respect to the target reads

$$\frac{dP}{d\varepsilon_f} = \frac{1}{2}\Sigma_{j_f}(|1 - \langle S_{j_f}\rangle|^2 + 1 - |\langle S_{j_f}\rangle|^2)(2j_f + 1)(1 + F_{l_f,l_i,j_f,j_i})\left[\frac{1}{mv^2}\right]\frac{1}{k_f}|C_i|^2\frac{e^{-2\eta d}}{2\eta d}M_{l_f l_i}. \tag{3}$$

where S_{j_f} is the neutron-target optical model S-matrix, F_{l_f,l_i,j_f,j_i} is an l to j coupling factor, η is the transverse component of the neutron momentum which is conserved in the neutron transition, d is the core-target impact parameter, C_i is the initial state asymptotic normalization constant and $M_{l_f l_i}$ is a factor depending on the angular parts of the initial and final wave functions, v is the relative motion velocity at the distance of closest approach.

We are going to discuss now a series of experiments and corresponding theoretical calculations aimed at extracting spectroscopic information on one-neutron and two-neutron halo nuclei and to determine properties of neutron-exotic nucleus interactions.

2.2 Higher order effects.

A very comprehensive study of one neutron breakup mechanism, cross section and momentum distributions can be found in [12, 13, 15, 16, 17].

On the other hand a more recent work [14] has improved the previous knowledge of the breakup reaction, by studying the Coulomb-nuclear interference effects according to Eq.(1). New calculations that we have recently performed by using Eq.(1) to study higher order effects including the nuclear and the Coulomb dipole potential have shown that the sudden limit ($\omega = 0$) of Eq.(1) and can be used to calculate the nuclear and Coulomb force effects to all orders. We called the corresponding amplitude $A^{all-ord}_{sudd}$. Then we studied the first order approximation for the Coulomb term in which $e^{\left(\frac{1}{i\hbar}\int_t^{\infty} V_{eff}(\mathbf{r},t')dt'\right)} = 1$ but the ωt term was keept (this is the standard first order perturbation theory amplitude A^{1-ord}) and finally the sudden approximation restricted to first order giving A^{1-ord}_{sudd}. We have shown that for $R_s < d < 20fm$ the results obtained with $A^{all-ord}_{sudd}$ are equal to those obtained with A^{1-ord}_{sudd}, starting from about $\theta = 10deg$ thus showing that at small angles the higher order terms give some contribution. On the other hand for $d > 20fm$ we found that higher order effects are always negligible since using $A^{all-ord}_{sudd}$ or A^{1-ord}_{sudd} does not give any difference. Then we concluded that time dependent perturbation theory is valid and appropriate apart from the small impact parameter, small angle region. There we took into account higher order terms in the sudden approximation because the second order term calculated with full time dependence or in the sudden limit, give the same results. The all order calculations were finally performed according to an amplitude defined as $A^{'bbm'} = A^{all-ord}_{sudd} + A^{1-ord} - A^{1-ord}_{sudd}$, valid at all core-target impact parameters and not giving rise to divergences in the final integral over impact parameters in Eq.(2).

More recently we have extended the method, for the Coulomb potential only, to include the first and second order terms in the quadrupole interaction [18] and we have studied the breakup of a charged halo. Because the quadrupole term is important at small distances we used the sudden limit and found that for impact parameters larger than 30 fm the only non negligible contribution is given by the dipole first order term. Furthermore we found no interference between first order and second order terms and between dipole and quadrupole. However since the quadrupole is important at small impact parameters where the nuclear interaction is also very important, it is clear that through the coupling with the nuclear the overall interference pattern could be slightly different. In Fig.(1) we show the impact parameter dependence of the breakup probability due to the dipole and quadrupole first and second order terms. The calculation was done for a proton bound in a s-state with

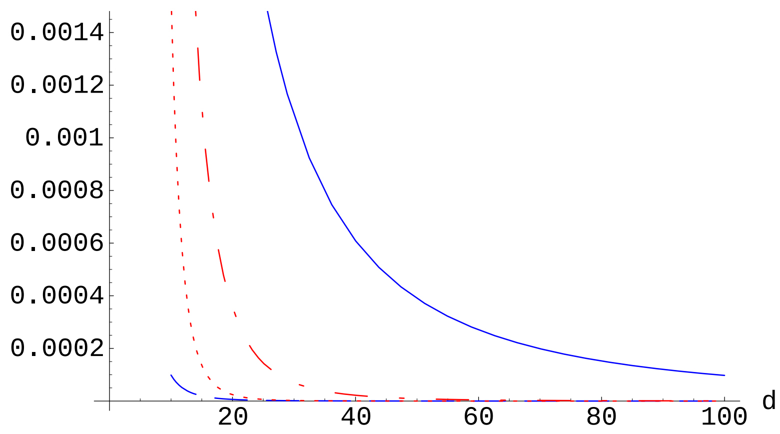

Figure 1: Breakup probabilities as a function of the core-target impact parameter d (fm) for various Coulomb potential multipolarities. The dipole first (full line) and second order (dashed) terms, quadrupole first (dotdashed) and second order (dotted).

an effective energy of $\varepsilon_i = -3MeV$ to a core nucleus with A=7 and Z=4 impinging at 80A.MeV on a ^{208}Pb target. We used Hankel functions for the initial state according to the effective binding energy model of [19]. The study of the difference between proton and neutron halo is relevant for our community in view of a ^{17}F breakup experiment proposed at the INFN-Legnaro National Laboratory.

3 Unbound nuclei studied via transfer.

We discuss now the results of a possible reaction aiming at clarifying the structure ^{11}Li which has been a challenge for long time [20]-[30]. ^{11}Li and ^{6}He are two-neutron halo nuclei. They are special because their corresponding A-1 systems are unbound and it is thanks to the pairing force acting between the two neutrons that they become bound. ^{11}Li has been very difficult to study from the experimental point of view because the ground state of ^{10}Li is unbound [24]-[30], and one of the available states for the valence neutron is a $2s_{1/2}$ virtual state. Recently a pickup experiment $d(^{11}Be,^{3}He)^{10}Li$ [30] has definitely confirmed the earlier hypothesis that the ground state of ^{10}Li is the 2s virtual state and that the $1p_{1/2}$ orbit gives an excited state. Three body models of ^{11}Li need as a fundamental ingredient the n-core (n-^{9}Li) interaction, which in turn determines the energies of the low energy unbound states in ^{10}Li. Following ref.[22] the two neutron hamiltonian is $H_{2n} = h_1 + h_2 + V_{nn}$. V_{nn} is the zero-range paring interaction. The single neutron hamiltonian is $h = t + V_{cn}$ where t is the kinetic energy and $V_{cn} = V_{WS} + \delta V$ is the neutron-core interaction. It is given by the usual Woods-Saxon potential plus spin-orbit plus a correction δV which originates from particle-vibration couplings. They are important for low energy states but can be neglected at higher energies. If Bohr and Mottelson collective model is used for the transition amplitudes between zero and one phonon states, then $\delta V(r) = 16\alpha e^{2(r-R)/a}/(1+e^{(r-R)/a})^4$ where $R \approx r_0 A^{1/3}$. According to [22] the best parameters for the n-^{9}Li Woods-Saxon and spin-orbit potential were $V_0 = -39.83MeV$, $V_{so} = 7.07MeV$, $r_0 = 1.27fm$, $a = 0.75fm$.

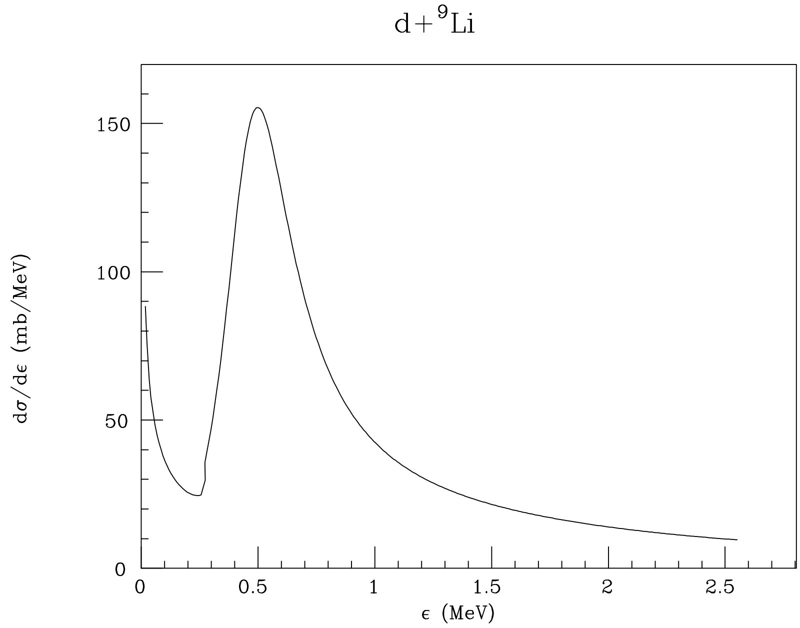

Figure 2: Neutron-^{9}Li relative energy spectra for transfer to the s and p continuum states in ^{10}Li.

The corresponding energies obtained for the 2s and $1p_{1/2}$ state are given in Table 1, together with the values of the strength α of the correction potential δV.

Table 1: Energy and width of unbound p-state, scattering length of the s-state and strength parameter for the δV potential.

		Γ(MeV)	a_s(fm)	$\alpha(MeV)$
$\epsilon_{2s_{1/2}}(MeV)$			-85	-12.
			85.9	-12.5
$\epsilon_{1p_{1/2}}(MeV)$	0.595	0.48		3.3

It would be therefore extremely interesting and important if an experiment could determine the energies of the two unbound ^{10}Li states such that the interaction parameter could be deduced. Two $^9Li(d,p)^{10}Li$ experiments have recently been performed. One at MSU at 20 A.MeV [32] and the other at the CERN REX-ISOLDE facility at 2 A.MeV[33]. For such transfer to the continuum reactions the theory underlined in section 2.2 is very accurate. We present in the following some of our results. A complete account of our calculations would be presented elsewhere [31]. In order to study the sensitivity of the results on the target and on the energies assumed for the s and p states, we have calculated the reaction $^9Li(X, X-1)^{10}Li$ at 2 A.MeV for three targets d, ^{9}Be, ^{13}C. The ^{13}C target has been chosen because in such a case the neutron transfer to the 2s state in ^{9}Li would be a spin-flip transition which as it is well known are enhanced at low incident energy. For the other two cases the transfer to the 2s state is a non spin-flip transition which is hindered. We show in Fig.(2) the neutron energy spectrum relative to ^{9}Li obtained with the interaction given above and the single particle energies of Table 1. In the case of the 2s virtual state we have studied the behavior of the scattering length obtained as $a_s = -\lim_{k\to 0} \frac{tan\delta_0}{k}$. By changing the potential trough an increase in the value of the parameter α the scattering length changes sign from negative to positive values when $\alpha \approx -12MeV$. Then $a_s = -85fm$ and the s-state is slightly unbound, while if $\alpha = -12.5MeV$ it is barely bound and $a_s = 85.9fm$. In both cases the cross section would increase toward zero energy because of the kinematical factor $1/k_f$ in Eq.(3).

The fact that a peak appears or does not appear for the s-state in the transfer spectrum, depends on the relative behavior of the two terms $|1-\langle S\rangle|^2$ and $1/k_f$ in Eq.(3). The $|1-\langle S\rangle|^2$ term has always a maximum whose value is equal to 4 if the phase shift goes trough $\pi/2$, while the $1/k_f$-term has a divergent-like behavior as the energy approaches zero. A peak in the cross section can appear in some cases even if the scattering length is positive. Therefore it appears that a study of the line shape for the s-state alone would not allow to determine the characteristics of the state and therefore the interaction which determines it. However since the potential has to change smoothly from the s to the p-state and since the experimental spectra would contain a background due to the presence of the l=0,1,2 states in the low energy region a global fit of the cross section energy dependence should help understanding the behavior of the interaction. This point is illustrated in Fig.(2) where we show the cross section energy dependence calculated according to Eq.(3). We have changed smoothly the potential such that it would give the s and p-states of Table 1. The S-matrix has been calculated at energy steps of 0.02 MeV with the state dependent potential and in each energy interval the sum of the contributions of the l=0,1,2 partial waves has been taken. The line shape for the inclusive spectrum thus comes out automatically from the calculation and no hypothesis on the shape of the resonance is made. Furthermore a "physical" background is naturally included. For the p-state we have seen that the peak in the transfer cross section is at the resonance energy for all targets and even if the beam energy is changed. A very important point is that the ratio of the cross section at the peak of the p-state and at threshold depends on the deformation assumed for the potential via the different values of α. Therefore we can conclude that if a transfer to the continuum experiment could measure with sufficient accuracy (energy resolution) the line-shapes or energy distribution functions in ^{10}Li our theory would be able to fix unambiguously the energy of the p-state and the relevant characteristics of the s-state. Those in turn could be used to test microscopic models of the n-^{9}Li interaction. A similar reaction has been proposed at the LNS using the new large acceptance spectrometer MAGNEX [34] with the ^8Li beam to study the structure of the unstable ^9Li and to determine the n-^8Li interaction.

4 Conclusions and future challenges

It is clear from what we have discussed in this paper that physics with radioactive beams is an extremely fascinating field in which the interplay between the understanding of the nuclear structure and that of the reaction mechanism is very strong and an enormous number of progress has been made in the last few years. There are however a number of improvements both experimental as well as theoretical that need to be pursued. We have shown that there are experiments which could be performed at the forthcoming facilities at Italian Nuclear Physics Laboratories for which the theoretician's community is ready to give the appropriate support.

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