SPIN PRE-EQUILIBRIUM HEAVY-ION REACTIONS [1]

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Abstract

We present a simple model showing an outline how to handle the spin-dependent initial exciton number for heavy-ion collisons at modest energies in a feasible way. It is based on the simple model suggested a dozen years ago by Cindro *et al.* We have added now the dependence on the angular momentum. As a result, one does not get any single pair of E and n_0, but rather a widely diversificated distribution over the excitation energies, spins and initial exciton numbers. A calculation is presented to illustrate the method and the influence of spin distributions on angle-integrated emission spectra.

1 Introduction

The pre-equilibrium models became rather popular tool to analyse and understand nuclear reactions at excitation energies ranging from several tens of MeV up to the GeV regions. A whole variety of the models has been developed (see, e.g., [1]). At the low-energy end, they usually employ some classification of the states according to their complexity, e.g. the exciton number, which develops during the course of a reaction. Higher energies prefer thermodynamical approaches and/or "classical-like" descriptions within the nuclear molecular dynamic family of the models.

A great variety of experimental data obtained at relatively low energies, say, below the pion threshold in the reactions induced by light projectiles and/or below the Fermi energy in the heavy-ion collisions, together with the ease of use of the phenomenological pre-equilibrium models, made rather popular the pre-equilibrium exciton model or its younger brother, the hybrid model. They both are essentially based on the notion of an exciton, i.e. the particle above or the hole below the Fermi level [2]. The initial stage of the reaction is the one which is responsible for the essential part of emission at the high-energy end of the spectrum. Here, the initial number of degrees of freedom — or, in the language of the exciton or hybrid models, the initial exciton number — plays a crucial role.

For a long time, there was not any reliable rule how to treate the pre-equilibrium initial stage of heavy-ion collisions. The original recipe of Blann [3], that the initial exciton number equals to the projectile mass number, did not work sufficiently well. The initial exciton number in the heavy-ion reactions, when treated as a parameter to yield a reasonable description of the observed data, shows some simple trends and under suitable scalling it

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becomes an universal function independent on the type of impacting ions, capable to be used for a wide range of (practically all possible) targets. These systematics ([4, 5]) can be used as a suitable starting points for pre-equilibrium heavy ion collision calculations, as was really done (e.g. [6]).

Though the universal behaviour of the initial states describing a wide range of various combinations of projectiles and targets was successfully used, its understanding came only with the tandem of papers of Cindro *et al.* [7], who recognized that one can reasonably estimate the initial number of degrees of freedom by calculating the overlaps of colliding nuclei in the momentum space [3]. For practical reasons, however, even the original (and more simple) approach of Cindro *et al.* [7, 8] has been to a good degree approximated by using some simple derived trends.

Both Cindro *et al.* [7, 8] and Ma Yugang *et al.* [9] gave only a single value for the initial number of degrees of freedom, without any indication of its spin dependence. The heavy-ion collisions are the type of reactions where the spin plays a very important role. It is the aim of the present paper to present a very simple model of the initial number of degrees of freedom in heavy-ion collisions capable to be used for calculations sensitive to the spin variables and to illustrate the basic features of the proposed scheme [4].

2 Basis of the model

The pre-equilibrium exciton model is governed by the set of master equations which describes both competing rocesses, namely the equilibration of the nucleus and the emission. In the most simple case, when we consider just one possible excitation energy and only the composite system prior to and including the first emission, this set reads [11, 12]

$$\begin{aligned}\frac{\mathrm{d}P(n,E,t)}{\mathrm{d}t} = \quad & P(n-2,E,t)\,\lambda^{+}(n-2,E) + P(n+2,E,t)\,\lambda^{-}(n+2,E) \\ & -P(n,E,t)\,[\lambda^{+}(n,E) + \lambda^{-}(n,E) + L(n,E)],\end{aligned} \tag{1}$$

where $P(n,E,t)$ is the occupation probability of finding the nucleus of the excitation energy E in an n-exciton state at time t, $\lambda^{\pm}(n,E)$'s are the transition rates from an n-exciton to $(n \pm 2)$ exciton state, and $L(n,E)$ is the emission rate (integrated over outgoing energies and summed over all possible ejectiles). The set of master equations is solved with the initial condition, which is usually written in the form

$$P(n,E,t=0) = \delta_{nn_0}. \tag{2}$$

The behaviour at long times, i.e. that after all excited nuclei decay,

$$P(n,E,t\to\infty) = 0 \qquad\qquad \text{for all } n, \tag{3}$$

[3] The simple approach has been soon improved by Ma Yugang *et al.* [9] using the Boltzmann-Uehling-Uhlenbeck dynamics.

[4] The very raw idea of this approach has been published recently [10]. However, that paper has not contained any calculations of heavy-ion reactions at all.

simply follows from the properties of the equations themselves. As an alternative to (2), one can use not a single value n_0, but also a distribution over some range of initial exciton numbers [13]. In such a case, the solution is simply an incoherent sum of properly weighted contributions arising from different initial exciton numbers. Solving the set of master equations (1) one obtains the time integrals $\tau(n, E)$

$$\tau(n, E) = \int_0^\infty P(n, E, t)\mathrm{d}t, \tag{4}$$

i.e. the time spent by the excited nucleus in an n-exciton state. The cross sections and related quantities are now straightforwardly expressed as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon_x} = \sigma_\mathrm{R} \sum_n \tau(n, E)\lambda_x^c(n, E, \varepsilon_x), \tag{5}$$

where $\lambda_x^c(n, E, \varepsilon_x)$ is the emission rate of the particle x with energy ε_x from the n-exciton state of the excitation energy E.

In the above scheme, the initial exciton number n_0, which is specific for each projectile-target combination and energy, plays the key role. Once we know reliably the initial exciton number, all the rest is governed by the master equations with the emission and decay rates which should *not* depend on the conditions of how the composite system has been created.

If one ignores the structure effects, a good overall rule says that for reactions induced by nucleons and α particles, the initial exciton number is equal or close to the mass number of the projectile [11, 14] and that all these initial excitons are of particle type [5]. The philosophy of the initial exciton number close to the mass number of the projectile has been suggested also for the heavy-ion induced reactions [3], however, with not much success.

Several years later, Korolija *et al.* [4, 16] and Běták [5] recognized that the initial exciton number needed to yield reasonable results of particle spectra and cross sections observed in heavy-ion collisions follows — if properly scaled — some trends which are universal for all impacting ions.

3 Spin-dependent formulation

While the spin-independent version of the pre-equilibrium model serves well for calculations of particle spectra, cross sections and many other quantities, there are — at least — two regions, where the spin effects play a very important role, namely the γ de-excitation to the discrete levels with given quantum numbers, and the heavy-ion reactioons, where the momenta introduced by the projectile to the system are *much* higher than was the case of the reactions induced by nucleons.

Here, one has to leave the simple spin-independent description and to apply the spin-dependent one. To do that, we have to add the spin variables into the set of master equations. The set of master equations coupling different energies, different spins and

[5] See also the discussion in [15].

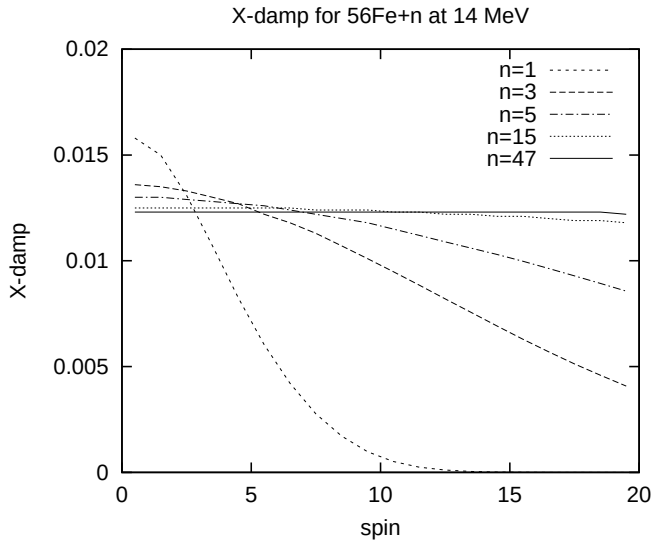

Figure 1: The $X_{nJ}^{\downarrow}$ functions *vs.* spin in the reaction of 14 MeV neutrons with ^{56}Fe, as obtained for different exciton numbers.

different nuclei is [17]

$$\begin{aligned}\frac{\mathrm{d}P(i,n,E,J,t)}{\mathrm{d}t} &= P(i,n-2,E,J,t)\,\lambda^{+}(i,n-2,E,J) \\ &+ P(i,n+2,E,J,t)\,\lambda^{-}(i,n+2,E,J) \\ &- P(i,n,E,J,t)\,\left[\lambda^{+}(i,n,E,J)+\lambda^{-}(i,n,E,J)+L(i,n,E,J)\right] \\ &+ \sum_{i',J',n',x}\int P(i',n',E',J',t)\,\lambda_x^c([i',n',E',J']\stackrel{\varepsilon}{\rightarrow}[i,n,E,J])\mathrm{d}\varepsilon, \quad (6)\end{aligned}$$

where i (or j) denotes the nuclei within the reaction chain and J and J' are the spins. The spin-dependent intranuclear transition rates

$$\lambda^{\pm}(E,J,n)=\frac{2\pi}{\hbar}|M|^2 {Y_n}^{\downarrow}{X_{nJ}}^{\downarrow} \quad (7)$$

have been derived by Obložinský [18]. Here, ${Y_n}^{\downarrow}$ is the energy part of the accessible final states, which remains the same as it was in the case of spin-independent calculations, and ${X_{nJ}}^{\downarrow}$ represents the angular momentum part (for the explicite form, see [18]).

Fig. 1 illustrates the spin dependence of the $X_{nJ}^{\downarrow}$ function for different exciton numbers in the ^{56}Fe+n reaction at 14 MeV. Here, $n=15$ is close to the equilibrium exciton number, and $n=47$ is the maximal exciton number occuring in this reaction, and it is far beyond the equilibrium.

There are no new factors in the nucleon emission rates. What is more complicated, however, are the γ emission ones. For this case, Obložinský derived the necessary formulae, later on generalized for other than E1 transitions [18]

$$\lambda_\gamma([E,J,n]\stackrel{\varepsilon_\gamma,\lambda}{\rightarrow}[U,S,m])=\frac{\varepsilon_\gamma^{2\lambda}\sigma_a(\varepsilon_\gamma)}{3\pi^2\hbar^3c^2}\cdot\frac{b_{mS}^{nJ}\omega(m,E-\epsilon_\gamma,S)}{\omega(n,E,J)} \quad . \quad (8)$$

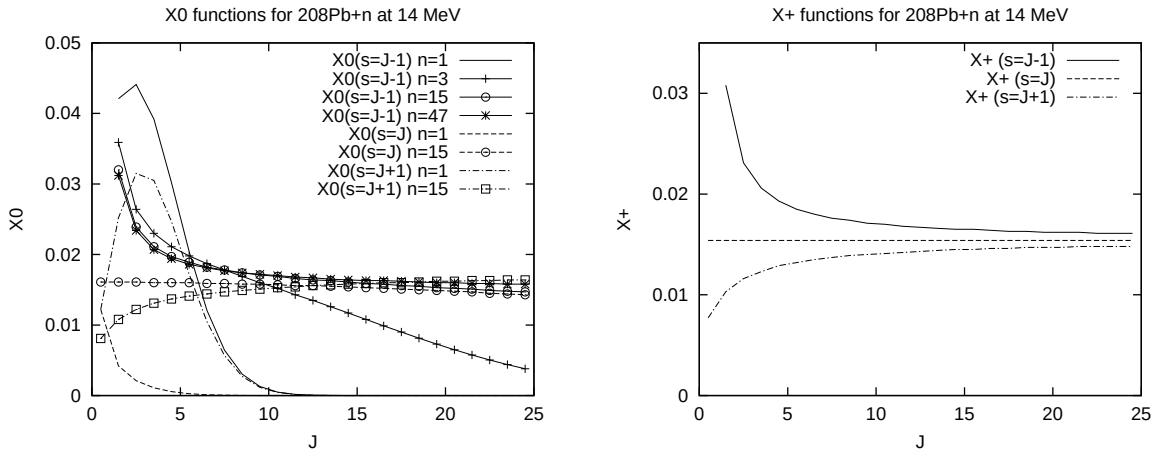

Figure 2: The x_{nS}^{nJ} $(X0)$ and $x_{nS}^{n+2,J}$ $(X+)$ functions in the reaction of 14 MeV neutrons with ^{208}Pb.

In (8), the b's are the branching ratios for the γ emission, which factorize similarly as the transition rates above. As we have two different processes responsible for the γ emission, $\Delta n = 0$ and $|\Delta n| = 2$, there are also two different branching ratios associated to these processes, each of them of different spin structure. The corresponding spin coupling terms are

$$x_{nS}^{nJ} = \frac{(2\lambda+1)(2J+1)}{R_n(S)} \sum_{j_1 j_2 j_3} (2j_1+1)(2j_2+1) R_1(j_1) R_1(j_2) R_{n-1}(j_3) \times \left(\begin{array}{ccc} j_2 & \lambda & j_1 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{array} \right)^2 \left\{ \begin{array}{ccc} j_2 & j_3 & S \\ J & \lambda & j_1 \end{array} \right\}^2 \tag{9}$$

and

$$x_{nS}^{n+2,J} = \frac{2J+1}{2S+1} \sum_{j_1 j_2} (2j_1+1)(2j_2+1) R_1(j_1) R_1(j_2) \times \left(\begin{array}{ccc} j_2 & j_1 & \lambda \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right)^2 \Delta(S\lambda J) \ . \tag{10}$$

For the notation of spins as well as for other details, see the original paper [18].

Fig. 2 illustrates the spin coupling terms for the γ emission in the case of ^{208}Pb+n at 14 MeV as functions of spin and for different exciton numbers.

The total spectrum is obtained by summing the contributions from the whole course of a reaction over all possible spins and integrating over all energies of all intermediate composite systems, i.e.

$$\frac{\mathrm{d}\sigma_x}{\mathrm{d}\varepsilon_x} = \sum\!\!\!\!\!\!\int_{E,J,J_c,n,i} \sigma(i, E_c, J_c, E, J) \tau(i, n, E, J) \lambda_x([i, n, E, J] \xrightarrow{\varepsilon_x} [\text{anything}]). \tag{11}$$

Here, $\sigma(i, E_c, J_c, E, J)$ represents the (total) population of states of nucleus i in the reaction chain which have the energy E and spin J, if the composite system has been created with

cross section $\sigma(E_c, J_c)$. To get $\sigma(i, E_c, J_c, E, J)$, one needs to follow all prior history of the system, i.e. to calculate all emissions (both of particles and γ's) which could lead to energy E and spin J. The symbol $\sum\!\!\!\!\!\!\int$ represents summation over the discrete and integration over the continuous variables.

4 Pre-Equilibrium Heavy-Ion Collisions

The interaction of heavy projectile with the target at incident energies roughly starting from the Coulomb barrier and extending close to the Fermi energy can be easily described using the proximity potential. The calculation using code TRAJEC [19] shows that there are essentially three stages of this collision: *i)* fast approach phase of nearly free incoming ions, followed by *ii)* intense slowing down due to the deep-inelastic collisions of heavy ions accompanied by friction, and finally *iii)* a long phase of co-existing excited nuclear system [20].

The model data can be successfully approximated by [7]

$$\frac{E}{n_0} = 4.6 + 0.54 \frac{E_{\rm cm} - V_{\rm C}}{A_{\rm proj}} \tag{12}$$

at higher and by

$$\frac{n_0}{A_{\rm proj}} = 0.09 + \left(0.38 - 0.08 \cdot \frac{A_{\rm targ} - A_{\rm proj}}{A_{\rm targ} + A_{\rm proj}}\right) \sqrt{\frac{E_{\rm cm} - V_{\rm C}}{A_{\rm proj}}} \tag{13}$$

at lower energies [8]. In these equations, $E_{\rm cm}$ is the center-of-mass energy, $V_{\rm C}$ is the Coulomb barrier and $A_{\rm proj}$ and $A_{\rm targ}$ are the mass numbers of the projectile and the target (the form of (13) implicitely assumes that $A_{\rm proj} \leq A_{\rm targ}$). The "switching point" from (12) to (13) is about 5 or 10 MeV per nucleon above the Coulomb barrier. Of course, in the very vicinity of the Coulomb barrier itself (or even slightly below it), all the approximations become very dangerous and one has no other choice than to evaluate properly the overlaps of nuclei as suggested by the model. For simplicity, the equations (12) and (13) are written for energies expressed in MeV (otherwise conversion coefficients are to be inserted). The model-derived initial exciton numbers have been successfully used in analyses of γ spectra from several heavy-ion reactions [21, 22].

The formulation of the problem, as it has been just presented, has some specific features when one applies it to the heavy-ion problems, where really very high values of angular momenta are involved. These features become important just at computer application: the spin-coupling coefficients entering the $X^{\downarrow}_{nJ}$, x^{nJ}_{nS} and $x^{n+2,J}_{nS}$ functions need for their calculations factorials of combinations of all entering momenta. This very soon leads to overflow problems and other inconveniences. A similar trouble, though less pronounced, is that one has to deal also with higher exciton numbers in heavy-ion collisions than in the reactions induced by light projectiles. Fortunately, all these spin-coupling functions tend (with increasing spin and/or increasing exciton number) either to negligibly small values, or — within a very good precision — to a constant (see also Figs. 1 and 2). We have benefitted from this behaviour and extrapolated the correctly calculated values at low

spins and low exciton numbers to higher values, and used these approximations thereafter [6]. As the influence of the extrapolated portions on the equilibration and the emission process is not a crucial one, such an approach can be reasonably justified.

5 Spin-Dependent Initial Configuration for Heavy Ions

In the preceding Sections (especially in the last one), we have all necessary ingredients prepared. What is to be done is the step from the spin-independent understanding of the initial configuration to the spin-dependent one.

We decompose the movement of the two colliding nuclei into their radial and tangential parts. The latter one is determined by the impact parameter, and it fully transforms into rotation of the double-nuclear system [7]. The rotational energy is

$$E_{\rm rot}(l) = \frac{l(l+1)\hbar^2}{2\mu R(l)^2}, \tag{14}$$

where μ is the reduced mass and $R(l)$ is the distance of the centers of nuclei at their contact. As has been shown [20], this distance can be approximated to a good degree by a constant independent of l. The energy $E_{\rm rot}(l)$ is the quantity which has to be subtracted from the incident energy. The energy diminished by the rotation

$$E'_{\rm cm}(l) = E_{\rm cm} - E_{\rm rot}(l) \tag{15}$$

is now the one which is responsible for approaching the nuclei in their radial co-ordinate and which has to be considered as a replacement of the original $E_{\rm cm}$ determining the overlap of nuclei and therefore the initial number of degrees of freedom, i.e. the initial exciton number n_0. Clearly, it is dependent on the angular momentum, so that we do not get any more a single value of the excitation energy E with one corresponding value of n_0, but rather a rather extensive table which attaches to each partial wave l its corresponding excitation energy $E(l)$ and the initial number of excitons $n_0(l)$, each of them entering the calculations with the corresponding cross section $\sigma(E_C, J_C)$. A possible future refinement of the approach may, in principle, yield not only single value of $n_0(l)$, but even a kind of their distribution. Current pre-equilibrium exciton-model codes based on the master equations, like DEGAS [17], can handle such distributions equally easily as a single value ascribed to a collision.

Fig. 3 brings the initial exciton numbers and the corresponding starting excitation energies E calculated for the ^{40}Ca+^{40}Ca reaction at 1000 MeV. Here, the excitation energy is uniquelley determined by the orbital angular momentum l, and the lowest excitation energies correspond to the highest angular momenta. Obviously, each of the initial points for the pre-equilibrium calculations enters with different partial cross section, which has

[6] Thanks to M. Herman for this idea.

[7] We have to keep in mind that the fusion of the two colliding nuclei occurs at times substantionally longer than is the period of rotation or the time characteristic for fast pre-equilibrium emission (see [5, 20]).

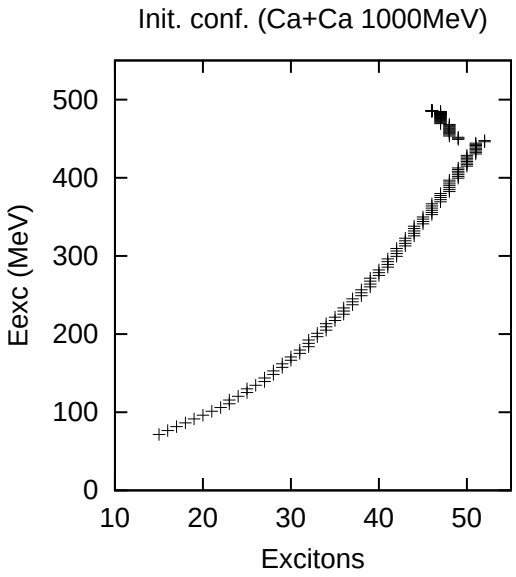

Figure 3: Initial configurations calculated for the ^{40}Ca+^{40}Ca reaction at 1000 MeV.

been evaluated — just to illustrate the possibilioties of the method — within the *"black sphere"* approximation.

6 Calculations

For the first attempt to use the spin-dependent pre-equilibrium heavy-ion calculations, we have chosen the reaction of ^{40}Ca+^{40}Ca at 1000 MeV, where Cardella *et al.* have measured hard γ emission [23]. In fact, we have already analyzed this reaction, but only using the spin-independent version of the pre-equilibrium exciton model [22] [8].

Obviously, the calculations are much more complex ones than in the case of low-energy nucleon- (or light-projectile-) induced reactions. We have to handle with a set of master equations, whose dimensions are about two orders of magnitude higher than in a classical case.

We have not aimed to reach full and satisfactory description of the data in our present paper, but just to show the feasibility of the suggested way and to illustrate, what are the manisfestations of the role of spin in heavy-ion reactions at modest energies. In addition, one has to keep in mind that what we use, is just an extremely simple model full of rough approximations (black sphere cross sections, extrapolation of spin coupling functions to higher exciton numbers and spins, etc.). That — together with significantly increased requirements put on the computer — has been the reason that we are comparing only the first chance γ emissions, and not the full γ spectra.

As is seen from Fig. 4, the influence of the proper account of angular momenta and spin couplings on the calculated γ spectra is not significant. It is somewhat more pronounced for the nucleon channels, but we do not have the experimental data counterpart to our

[8] Unfortunately, a numerical error in normalization of calculations presented in [22] appeared: all calculated spectra of [22] have to be multiplied by a factor of 0.713.

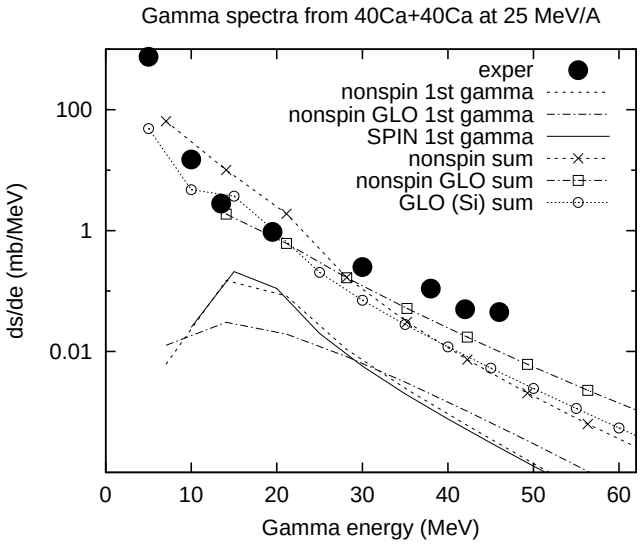

Figure 4: Gamma spectra from ^{40}Ca+^{40}Ca at 1000 MeV (experimental data of Cardella *et al.* [23].

calculations, we do not feel reasonable to open the discussion on suitability of spin couplings in this case now.

7 Conclusions

We have suggested a method how to estimate the spin-dependent initial exciton number for heavy-ion collisons at modest energies and we have succeeded to handle the enormously enlarged set of master equations of the pre-equilibrium exciton model with very complicated initial conditions and all necessary spin couplings. Our approach is based on the simple model suggested a decade ago by N. Cindro *et al.* [7], where we have introduced the dependence on the angular momentum. As a result, one does not get any single pair of E and n_0, but rather a widely diversificated distribution over the excitation energies, spins and initial exciton numbers. Current pre-equilibrium master-equation based exciton-model codes (e.g. [17]) can handle this situation with practically the same level of ease as was in the case for the spin-independent view on the heavy-ion collisions.

8 Acknowledgments

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