TWO-NUCLEON OVERLAP FUNCTIONS IN THE JASTROW CORRELATION METHOD AND CROSS SECTIONS OF THE ^{16}O$(e, e'NN)$ REACTIONS

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The two-nucleon overlap functions corresponding to transitions from the ground state of ^{16}O to states of the residual nuclei are calculated in the framework of the Jastrow correlation method using the relationship between the two-body density matrix and the two-particle overlap functions. The latter are applied to calculate cross sections of the direct knockout reactions ^{16}O$(e, e'NN)$. The main contributions of the removal of NN pairs from ^{16}O are considered in the calculations using the Jastrow two-nucleon overlap functions which include short-range correlations. The results are compared with the cross sections calculated with different theoretical treatments of the two-nucleon overlap functions.

1 Introduction

The electromagnetically induced two-nucleon knockout has been devised as the most direct tool to study the short-range correlations (SRC) in a nucleus [1–4]. Two nucleons can be naturally ejected by two-body currents, which effectively take into account the influence of subnuclear degrees of freedom like mesons and isobars. Direct insight into SRC can be obtained from the process where the real or virtual photon hits, through a one-body current, either nucleon of a correlated pair and both nucleons are then ejected from the nucleus.

Various theoretical models for cross section calculations have been developed in recent years in order to explore the effects of ground-state NN correlations on $(e, e'NN)$ [5–10] and (γ, NN) [11–17] knockout reactions. It appears from these studies that the most promising tool for investigating SRC in nuclei is represented by the $(e, e'pp)$ reaction, where the effect of the two-body currents is less dominant as compared to the $(e, e'pn)$ and (γ, NN) processes. Measurements of the exclusive ^{16}O$(e, e'pp)^{14}$C reaction performed at NIKHEF in Amsterdam [18–20] and MAMI in Mainz [21,22] have confirmed, in comparison with the theoretical results, the validity of the direct knockout mechanism for transitions to low-lying states of the residual nucleus and have given clear evidence of SRC for the transition to the ground state of ^{14}C.

One of the main ingredients in the transition matrix elements of exclusive two-nucleon knockout reactions is the two-nucleon overlap function (TOF). The TOF contains information on nuclear structure and correlations and allows one to write the cross section in terms of the two-hole spectral function [2]. The TOF's and their properties are widely reviewed, e.g., in [23].

In [8] the TOF's for the $^{16}\mathrm{O}(e, e'pp)^{14}\mathrm{C}$ reaction are given by the product of a coupled and fully antisymmetrized pair function of the shell model and a Jastrow-type correlation function which incorporates SRC. A more sophisticated treatment is used in [9], where the TOF's are obtained from an explicit calculation of the two-proton spectral function of $^{16}\mathrm{O}$ [24], which includes, with some approximations but consistently, both SRC and long-range correlations (LRC).

A different method to calculate the TOF's has been suggested in [25] using the established general relationships connecting TOF's with the ground state two-body density matrix (TDM). The procedure is based on the asymptotic properties of the TOF's in coordinate space, when the distance between two of the particles and the center of mass of the remaining core becomes very large. This procedure can be considered as an extension of the method suggested in [26], where the relationship between the one-body density matrix and the one-nucleon overlap function is established. The latter has been applied [27–35] to calculate the one-nucleon overlap functions, spectroscopic factors and to make consistent calculations of the cross sections of different one-nucleon removal reactions, such as (p, d), $(e, e'p)$, and (γ, p) [27, 30–35] on $^{16}\mathrm{O}$ [31, 32, 34] and $^{40}\mathrm{Ca}$ [33, 34], (p, d) on $^{24}\mathrm{Mg}$, $^{28}\mathrm{Si}$ and $^{32}\mathrm{S}$ [35], as well as $(e, e'p)$ on $^{32}\mathrm{S}$ [35] within various correlation methods.

The first aim of the present work is to apply the procedure suggested in [25] to calculate TOF's for $^{16}\mathrm{O}$ using the TDM calculated in [36] with the Jastrow correlation method (JCM), which incorporates the nucleon-nucleon SRC. As a second aim, the resulting two-proton overlap functions are used to calculate the cross section of the $^{16}\mathrm{O}(e, e'pp)$ reaction for the transition to the 0^+ ground and the 1^+ excited states of $^{14}\mathrm{C}$. The cross sections are calculated on the basis of the theoretical approach developed in [5, 8, 9].

2 Two-body density matrix and overlap functions

In this Section we present shortly the definitions and some properties of the TDM and related quantities in the overlap function representation. The method to extract the TOF's from the TDM [25] used in this work is also given.

The TDM is defined in coordinate space as:

$$\rho^{(2)}(x_1, x_2; x'_1, x'_2) = \langle \Psi^{(A)} | a^\dagger(x_1) a^\dagger(x_2) a(x'_2) a(x'_1) | \Psi^{(A)} \rangle, \tag{1}$$

where $|\Psi^{(A)}\rangle$ is the antisymmetric A-fermion ground state wave function normalized to unity and $a^\dagger(x)$, $a(x)$ are creation and annihilation operators at position x. The coordinate x includes the spatial coordinate $\mathbf{r}$ and spin and isospin variables. The TDM $\rho^{(2)}$ is trace-normalized to the number of pairs of particles:

$$\mathrm{Tr}\ \rho^{(2)} = \frac{1}{2}\int \rho^{(2)}(x_1,x_2)dx_1dx_2 = \frac{A(A-1)}{2}. \qquad (2)$$

Of direct physical interest is the decomposition of the TDM in terms of the overlap functions between the A-particle ground state and the eigenstates of the $(A-2)$-particle systems, since TOF's can be probed in exclusive knockout reactions.

The TOF's are defined as the overlap between the ground state of the target nucleus $\Psi^{(A)}$ and a specific state $\Psi_\alpha^{(C)}$ of the residual nucleus ($C = A-2$) [23]:

$$\Phi_\alpha(x_1,x_2) = \langle \Psi_\alpha^{(C)} | a(x_1)a(x_2) | \Psi^{(A)} \rangle. \qquad (3)$$

Inserting a complete set of $(A-2)$ eigenstates $|\alpha(A-2)\rangle$ into Eq. (1) one gets

$$\rho^{(2)}(x_1,x_2;x_1',x_2') = \sum_\alpha \Phi_\alpha^*(x_1,x_2)\Phi_\alpha(x_1',x_2'). \qquad (4)$$

The norm of the two-body overlap functions defines the spectroscopic factor

$$S_\alpha^{(2)} = \langle \Phi_\alpha | \Phi_\alpha \rangle. \qquad (5)$$

A procedure for obtaining the TOF's on the basis of the TDM suggested in [25] uses the particular asymptotic properties of the TOF's.

In the case when two like nucleons (neutrons or protons) unbound to the rest of the system are simultaneously transferred, the following hyperspherical type of asymptotics is valid for the two-body overlap functions [23, 37, 38]

$$\Phi(r,R) \longrightarrow N \exp\left\{ -\sqrt{\frac{4m|E|}{\hbar^2}\left(R^2+\frac{1}{4}r^2\right)} \right\} \left(R^2+\frac{1}{4}r^2\right)^{-5/2}, \qquad (6)$$

where r and R are the magnitudes of the relative and center-of-mass (c.m.) coordinates, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, respectively, m is the nucleon mass and $E = E^{(A)} - E^{(C)}$ is the two-nucleon separation energy.

For a target nucleus with $J^\pi_{tar.} = 0^+$ and for large $r' = a$ and $R' = b$ a single term with ν_0 (corresponding to the smallest two-nucleon separation energy) of the radial part of the TOF $\Phi_{\nu_0 JSLlL_R}(r,R)$ can be expressed in terms of the TDM as

$$\begin{aligned}\Phi_{\nu_0 JSLlL_R}(r,R) =& \frac{\rho^{(2)}_{JSLlL_R}(r,R;a,b)}{\Phi_{\nu_0 JSLlL_R}(a,b)} = \\ =& \frac{\rho^{(2)}_{JSLlL_R}(r,R;a,b)}{N\exp\left\{-k\sqrt{\left(b^2+\frac{1}{4}a^2\right)}\right\}\left(b^2+\frac{1}{4}a^2\right)^{-5/2}},\end{aligned} \qquad (7)$$

Figure 1: The $^1S_0$ two-proton overlap functions for the nucleus ^{16}O leading to the 0^+ ground state of ^{14}C extracted from the JCM (left) and uncorrelated (right) two-body density matrices.

where $k = (4m|E|/\hbar^2)^{1/2}$ is constrained by the experimental values of the two-nucleon separation energy E. The relationship obtained in Eq. (7) makes it possible to extract TOF's with quantum numbers $JSLlL_R$ from a given TDM. The coefficient N and the constant k can be determined from the asymptotics of $\rho^{(2)}_{JSLlL_R}(r, R; r, R)$.

3 Results

3.1 The two-proton overlap functions

The procedure described briefly in Section II has been applied to calculate the two-proton overlap functions in the ^{16}O nucleus for the transition to the 0^+ ground and the 1^+ excited states of ^{14}C. The TDM obtained in [36] in the framework of the low-order approximation (LOA) of the Jastrow correlation method has been used [39].

The TOF's for the $^1S_0$ and $^3P_1$ states are obtained in the JCM. The result for $^1S_0$ state is presented in Fig. 1. It is compared with the uncorrelated TOF's obtained applying the same procedure to the uncorrelated TDM. The notation for the partial waves in our case is $^{2S+1}l_L$.

The spectroscopic factors corresponding to the $^1S_0$ and $^3P_1$ overlap functions are 0.958 and 0.957, respectively.

As a next step, we derive the total TOF $\Phi_{\nu JM}(x, X)$ in terms of a sum over all possible partial components, i.e.

$$\Phi_{\nu JM}(x, X) = \sum_{LSlL_R} \Phi_{\nu JSLlL_R}(r, R) A^{JM}_{SLlL_R}(\sigma_1, \sigma_2; \hat{r}, \hat{R}). \qquad (8)$$

We integrate the squared modulus of the total TOF in Eq. (8) over the angles and sum over the spin variables. The result can be written in the form (for the smallest value of $\nu = \nu_0$):

$$\overline{|\Phi_{JM}(x, X)|^2} \equiv |\widetilde{\Phi}_{JM}(r, R)|^2 = \sum_{LSlL_R} \rho^{(2)}_{JLSlL_R}(r, R), \qquad (9)$$

where the bar denotes the integration over the angles and summation over the spin variables, and $\widetilde{\Phi}_{JM}(r, R)$ is the radial part of the total TOF obtained after the integration and summation. Using the asymptotics of $\widetilde{\Phi}_{JM}(r, R)$ at

Figure 2: The differential cross section of the ^{16}O$(e, e'pp)$ reaction as a function of the recoil momentum $p_{\rm B}$ for the transition to the 0^+ ground state of ^{14}C in the super-parallel kinematics with $E_0 = 855$ MeV, $\omega = 215$ MeV and $q = 316$ MeV/c. The curves are obtained with different treatments of the TOF: $^1S_0$ (dashed line) and $^3P_1$ (dotted line) as "independent" TOF's in the JCM in the left panel and as partial components in the right panel.

$r \longrightarrow a$, $R \longrightarrow b$ one can write:

$$\widetilde{\Phi}_{JM}(r, R) = \frac{\sum\limits_{LSlL_R} \rho^{(2)}_{JSLlL_R}(r, R; a, b)}{N \exp\left\{-k\sqrt{\left(b^2 + \frac{1}{4}a^2\right)}\right\}\left(b^2 + \frac{1}{4}a^2\right)^{-5/2}} \quad . \tag{10}$$

The results for the $^1S_0$ and $^3P_1$ partial components have a similar behaviour as previous ones, the main difference is that they are somewhat reduced in magnitude. The spectroscopic factor corresponding to the total TOF is equal to unity in the uncorrelated case and 0.965 in the Jastrow case.

The Jastrow TDM (including only SRC) is not "rich" enough to be able to explain realistically transitions to all the excited states of ^{14}C. Therefore, only the transition to the 1^+ state is considered in the present paper as an example of the applicability of the method.

In the case of the transition to the 1^+ excited state of ^{14}C pp-pairs in the states $^3P_{0,1,2}$ give main contributions to the process. The value of the spectroscopic factor e.g. for $^3P_1$ is 0.967 in the Jastrow case and unity in the uncorrelated one.

3.2 The ^{16}O$(e, e'pp)^{14}$C reaction

The TOF's obtained from the TDM within the Jastrow correlation method have been used to calculate the cross section of the ^{16}O$(e, e'pp)^{14}$C knockout reaction in one-photon exchange approximation [2, 5].

As an example, the differential cross section calculated for the transition to the 0^+ ground state of ^{14}C is shown in Fig. 2 for the kinematical setting considered in the experiment performed at MAMI [21, 22].

The results are compared with the cross sections already shown in [9], where the TOF is taken from a calculation of the two-proton spectral function (SF) [24], where a two-step procedure has been adopted to include both SRC and LRC.

In the figure are also shown for a comparison the results obtained with a simpler approach, where the two-nucleon wave function is given by the product of the pair function of the shell model and of a Jastrow type central and state independent correlation function (SM+CORR).

Figure 3: The differential cross section of the ^{16}O$(e, e'pp)$ reaction as a function of the recoil momentum p_{B} for the transition to the 1^+ excited state of ^{14}C in the same kinematics as in Fig. 2.

SRC are quite strong and even dominant for the $^1S_0$ state and much weaker for the $^3P_1$ state. The role of the isobar current is strongly reduced for $^1S_0$ pp knockout, since there the magnetic dipole $NN \leftrightarrow N\Delta$ transition is suppressed [17, 40]. As a consequence, in the figures the $^1S_0$ results are dominated by the one-body current and thus by SRC, while the Δ current gives the main contribution to $^3P_1$ pp knockout.

It can be seen from Fig. 2 that the cross section calculated with the Jastrow TOF for the $^1S_0$ state is close to the SF and also to the SM+CORR results at low values of p_{B}, up to $\sim 150 - 200$ MeV/c. For $p_{\mathrm{B}} \geq 200$ MeV/c $^3P_1$ knockout becomes dominant with all the different treatments of the TOF. The results with the $^3P_1$ TOF from the Jastrow TDM is however much larger than the SF result and also larger than the SM+CORR cross section.

The cross section calculated with the total TOF, obtained from the combination of the $^1S_0$ and $^3P_1$ partial components, are shown in the right panel of Figs. 2. In both kinematical settings the $^1S_0$ component dominates at low values of p_{B}, while the $^3P_1$ component produces a strong enhancement at high momenta.

Although obtained from a calculation of the TDM within the JCM where only SRC are included, the TOF used in our calculations are able to reproduce the main qualitative features which were found in previous theoretical investigations. This means that the procedure suggested in [25] to calculate the TOF's from the TDM can be applied and exploited in the study of two-nucleon knockout reactions.

The differential cross section calculated for the transition to the 1^+ state of ^{14}C, at 11.3 MeV excitation energy, is shown in Fig. 3 in the same kinematical setting already considered for the 0^+ state in Fig. 2. With respect to the other results, the Jastrow TOF produces in the super-parallel kinematics a strong enhancement at high momenta, which makes the shape of the cross sections larger and flatter than that with the SF and SM+CORR TOF's.

4 Conclusions

The results of the present work can be summarized as follows:

i) The two-nucleon overlap functions (and their norms, the spectroscopic factors) corresponding to the knockout of two protons from the ground state of ^{16}O and the transition to the ground and 1^+ excited states of

^{14}C are calculated using the recently established relationship [25] between the TOF's and the TDM. In the calculations the TDM obtained within the JCM [36] is used. Though only SRC are accounted for in the Jastrow TDM, the results can be considered as a first attempt to use an approach which fulfils the general necessity the TOF's to be extracted from theoretically calculated TDM's corresponding to realistic wave functions of the nuclear states.

ii) The TOF's extracted from the Jastrow TDM are included in the theoretical approach of [5,8,9] to calculate the cross section of the $^{16}\mathrm{O}(e,e'pp)^{14}\mathrm{C}$ knockout reaction. Numerical results in different kinematics are compared with the cross sections calculated, within the same theoretical model for the reaction mechanism, with different treatments of the TOF, in particular with the more refined approach of [9, 24], where the TOF's are obtained from a calculation of the two-proton spectral function of ^{16}O where both SRC and LRC are included. The cross sections calculated in the present work, where the TOF's are extracted from the Jastrow TDM, confirm the dominant contribution of $^1S_0$ pp knockout at low values of recoil momentum, up to $\simeq 150 - 200$ MeV/c. The $^3P_1$ contribution is mainly responsible for the high-momentum part of the cross section at $p_\mathrm{B} \geq 200$ MeV/c.

iii) Our method is applied in the present work only to the 0^+ ground and the 1^+ excited states of ^{14}C. The main aim was to check the practical application of all steps of the method to a given state of the residual nucleus. Therefore, the results obtained for the $^{16}\mathrm{O}(e,e'pp)^{14}\mathrm{C}$ reaction, which are able to reproduce the main qualitative features of the cross sections calculated with different treatments of the TOF's, can serve as an indication of the reliability of the method, that can be applied to a wider range of situations and, as an alternative to an explicit calculation of the two-hole spectral function, to more refined approaches of the TDM [4, 41–43].

Acknowledgments

One of the authors (D. N. K.) would like to thank the Pavia Section of the INFN for the warm hospitality and for providing the necessary fellowship. The work was partly supported by the Bulgarian National Science Foundation under Contracts Φ-809 and Φ-905.

References

[1] K. Gottfried, Nucl. Phys. **5**, 557 (1958); Ann. of Phys. **21**, 63 (1963).

[2] S. Boffi, C. Giusti, F. D. Pacati, and M. Radici, *Electromagnetic Response of Atomic Nuclei*, Oxford Studies in Nuclear Physics (Clarendon Press, Oxford, 1996).

[3] A.N. Antonov, P.E. Hodgson and I.Zh. Petkov, *Nucleon Correlations in Nuclei* (Springer–Verlag, Berlin, 1993).

[4] H. Müther and A. Polls, Prog. Part. Nucl. Phys. **45**, 243 (2000).

[5] C. Giusti and F. D. Pacati, Nucl. Phys. **A535**, 573 (1991).

[6] J. Ryckebusch, M. Vanderhaeghen, K. Heyde, and M. Waroquier, Phys. Lett. B **350**, 1 (1995).

[7] J. Ryckebusch, V. Van der Sluys, K. Heyde, H. Holvoet, W. Van Nespen, M. Waroquier, and M. Vanderhaeghen, Nucl. Phys. **A624**, 581 (1997).

[8] C. Giusti and F. D. Pacati, Nucl. Phys. **A615**, 373 (1997).

[9] C. Giusti, F. D. Pacati, K. Allaart, W. J. W. Geurts, W. H. Dickhoff, and H. Müther, Phys. Rev. C **57**, 1691 (1998).

[10] C. Giusti, H. Müther, F. D. Pacati, and M. Stauf, Phys. Rev. C **60**, 054608 (1999).

[11] C. Giusti, M. Radici, and F. D. Pacati, Nucl. Phys. **A546**, 607 (1992).

[12] S. Boffi, C. Giusti, M. Radici, and F. D. Pacati, Nucl. Phys. **A564**, 473 (1993).

[13] J. Ryckebusch, M. Vanderhaeghen, L. Machenil, and M. Waroquier, Nucl. Phys. **A568**, 828 (1994).

[14] J. Ryckebusch, L. Machenil, M. Vanderhaeghen, V. Van der Sluys and M. Waroquier, Phys. Rev. **C49**, 2704 (1994).

[15] J. Ryckebusch, Phys. Lett. B **383**, 1 (1996).

[16] P. Wilhelm, H. Arenhövel, C. Giusti, and F. D. Pacati, Z. Phys. A **359**, 467 (1997).

[17] C. Giusti and F. D. Pacati, Nucl. Phys. **A641**, 297 (1998).

[18] C. J. Onderwater *et al.*, Phys. Rev. Lett. **78**, 4893 (1997).

[19] C. J. Onderwater *et al.*, Phys. Rev. Lett. **81**, 2213 (1998).

[20] R. Starink *et al.*, Phys. Lett. B **474**, 33 (2000).

[21] G. Rosner, in *Proceedings of 10^{th} Mini-Conference on Studies of Few-Body Systems with High Duty-Factor Electron Beams*, NIKHEF, Amsterdam, 1999, p.95.

[22] G. Rosner, Prog. Part. Nucl. Phys. **44**, 99 (2000).

[23] J. M. Bang, F. A. Gareev, W. T. Pinkston, and J. S. Vaagen, Phys. Rep. **125(6)**, 253 (1985).

[24] W. J. W. Geurts, K. Allaart, W. H. Dickhoff, and H. Müther, Phys. Rev. C **54**, 1144 (1996).

[25] A. N. Antonov, S. S. Dimitrova, M. V. Stoitsov, D. Van Neck, and P. Jeleva, Phys. Rev. C **59**, 722 (1999).

[26] D. Van Neck, M. Waroquier, and K. Heyde, Phys. Lett. B **314**, 255 (1993).

[27] M. V. Stoitsov, A. N. Antonov, and S. S. Dimitrova, Phys. Rev. C **53**, 1254 (1996).

[28] D. Van Neck, A. E. L. Dieperink, and M. Waroquier, Phys. Rev. C **53**, 2231 (1996); Z. Phys. A **355**, 107 (1996).

[29] D. Van Neck, L. Van Daele, Y. Dewulf, and M. Waroquier, Phys. Rev. C **56**, 1398 (1997).

[30] S. S. Dimitrova, M. K. Gaidarov, A. N. Antonov, M. V. Stoitsov, P. E. Hodgson, V. K. Lukyanov, E. V. Zemlyanaya, and G. Z. Krumova, J. Phys. G **23**, 1685 (1997).

[31] M. K. Gaidarov, K. A. Pavlova, S. S. Dimitrova, M. V. Stoitsov, A. N. Antonov, D. Van Neck, and H. Muether, Phys. Rev. C **60** 024312 (1999).

[32] M. K. Gaidarov, K. A. Pavlova, A. N. Antonov, M. V. Stoitsov, S. S. Dimitrova, M. V. Ivanov, and C. Giusti, Phys. Rev. C **61**, 014306 (2000).

[33] M. V. Ivanov, M. K. Gaidarov, A. N. Antonov, and C. Giusti, Phys. Rev. C **64**, 014605 (2001).

[34] M. V. Ivanov, M. K. Gaidarov, A. N. Antonov, and C. Giusti, Nucl. Phys. **A699**, 336 (2002).

[35] M. K. Gaidarov, K. A. Pavlova, A. N. Antonov, C. Giusti, S. E. Massen, Ch. C. Moustakidis, and K. Spasova, Phys. Rev. C **66**, 064308 (2002).

[36] S. S. Dimitrova, D. N. Kadrev, A. N. Antonov, and M. V. Stoitsov, Eur. Phys. J. A **7**, 335 (2000).

[37] S. P. Merkuriev, Sov. J. Nucl. Phys. **19**, 447 (1974).

[38] J. Bang, Physica Scripta **22**, 324 (1980).

[39] R. Jastrow, Phys. Rev. **98**, 1479 (1955).

[40] P. Wilhelm, J. A. Niskanen and H. Arenhövel, Nucl. Phys. **A597**, 613 (1996).

[41] S. C. Pieper, R. B. Wiringa, and V. R. Pandharipande, Phys. Rev. C **46,** 1741 (1992).

[42] J. H. Heisenberg and B. Mihaila, Phys. Rev. C **59**, 1440 (1999).

[43] A. Fabrocini, F. Arias de Saavedra, and G. Co', Phys. Rev. C **61**, 044302 (2000).