Structure and properties of nuclear matter in compact stars

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- Low-density nuclear matter in the crust of neutron stars or core of supernovae
- High-density matter in the core of neutron stars

〈 Low–density nuclear matter 〉

Low-density nuclear matter → Inhomogeneous structures.
There are two ways of understanding inhomogeneous matter.

1. From inhomogeneous to uniform

Compression of low-density matter.
→ crystal of atoms in degenerate electrons.
→ crystal of nuclei in degenerate neutrons.
→ uniform nuclear matter.

2. From uniform to inhomogeneous

Instability of uniform matter below the saturation density.
→ phase transition & mixed phase (clustering).

1. From inhomogeneous to uniform

First, electrons degenerate.

→ electron energy depends on the density.

→ Y_e and Y_p decrease. Baryon energy is not directly dependent on the density.

As Y_p decreases ,neutrons begin to drip. (🟥)

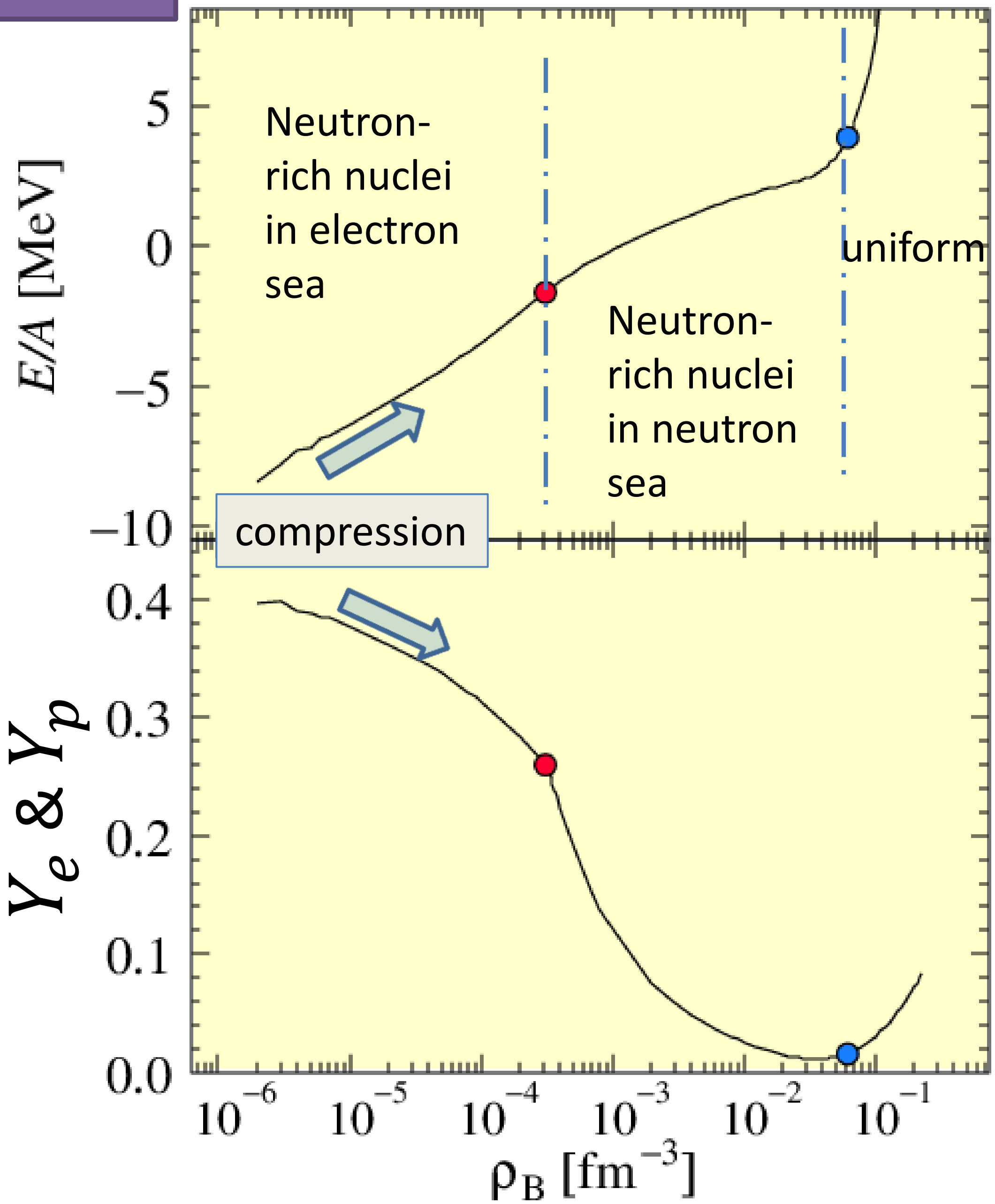
→ Neutrons have found large space to escape.

→ Y_e and Y_p decrease rapidly.

As neutrons degenerate, increase of Y_n (decrease of Y_p) is suppressed.

→ at last, Y_p increases and protons degenerate. (🟦)

→ Uniform matter.

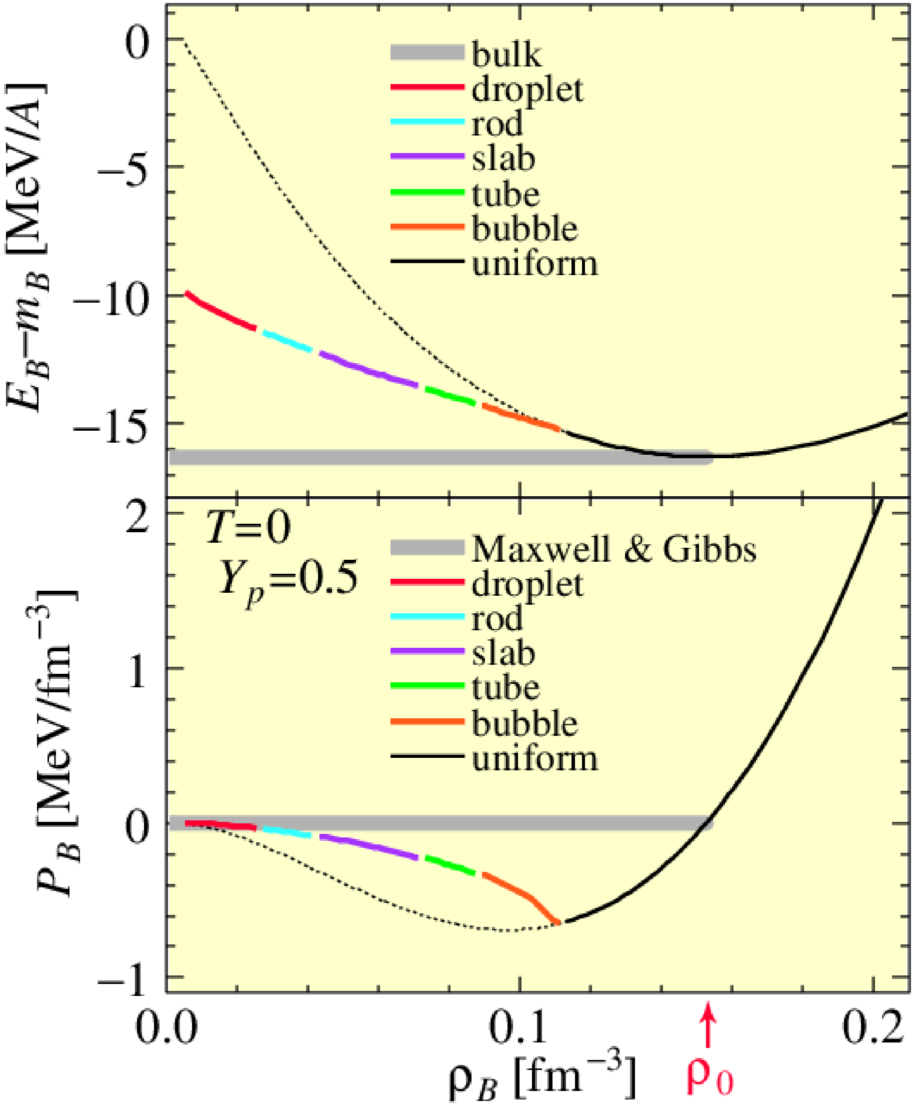


2. From uniform to inhomogeneous

Baryon partial pressure has a density region of negative gradient (Note that the total pressure and its gradient are positive due to electron pressure.)

For symmetric matter at *T*=0,
Maxwell constr leads:
negative pressure (at $\rho_B < \rho_0$) is not favored.
→ mixed phase (clustering)

However, at some density just below ρ_0, uniform matter is favored due to **finite size effects** (surface and Coulomb).



RMF + Thomas-Fermi model

Lagrangian

$$L = L_N + L_M + L_e,$$

$$L_N = \bar{\Psi}\left[i\gamma^\mu \partial_\mu - m_N^* - g_{\omega N}\gamma^\mu \omega_\mu - g_{\rho N}\gamma^\mu \vec{\tau}\vec{b}_\mu - e\frac{1+\tau_3}{2}\gamma^\mu V_\mu \right]\Psi$$

$$L_M = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{R}_\mu\vec{R}^\mu,$$

$$L_e = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \bar{\Psi}_e\left[i\gamma^\mu\partial_\mu - m_e + e\gamma^\mu V_\mu \right]\Psi_e, \qquad (F_{\mu\nu} \equiv \partial_\mu F_\nu - \partial_\nu F_\mu)$$

$$m_N^* = m_N - g_{\sigma N}\sigma, \qquad U(\sigma) = \frac{1}{3}bm_N(g_{\sigma N}\sigma)^3 + \frac{1}{4}c(g_{\sigma N}\sigma)^4$$

Nucleons interact with each other via coupling with σ, ω, ρ mesons. Simple but feasible!

From $\partial_\mu\left[\partial L/\partial(\partial_\mu\phi)\right] - \partial L/\partial\phi = 0,$

$(\phi = \sigma, \omega_\mu, R_\mu, V_\mu, \Psi),$

$$-\nabla^2\sigma(\mathbf{r}) + m_\sigma^2\sigma(\mathbf{r}) = g_{\sigma N}(\rho_n^{(s)}(\mathbf{r}) + \rho_p^{(s)}(\mathbf{r})) - \frac{dU}{d\sigma}(\mathbf{r}),$$

$$-\nabla^2\omega_0(\mathbf{r}) + m_\omega^2\omega_0(\mathbf{r}) = g_{\omega N}(\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})),$$

$$-\nabla^2 R_0(\mathbf{r}) + m_\rho^2 R_0(\mathbf{r}) = g_{\rho N}(\rho_p(\mathbf{r}) - \rho_n(\mathbf{r})),$$

$$\nabla^2 V_C(\mathbf{r}) = 4\pi e^2\rho_{\mathrm{ch}}(\mathbf{r}),$$

For Fermions, we employ Thomas-Fermi approx. with finite T

$$f_{i=n,p}(\mathbf{r};\mathbf{p},\mu_i) = \left(1 + \exp\left[\left(\sqrt{p^2 + m_i^*(\mathbf{r})^2} - \sqrt{p_{Fi}(\mathbf{r})^2 + m_i^*(\mathbf{r})^2}\right)\Big/T\right]\right)^{-1},$$

$$f_e(\mathbf{r};\mathbf{p},\mu_e) = \left(1 + \exp\left[\left(p - (\mu_e - V_C(\mathbf{r}))\right)/T\right]\right)^{-1},$$

$$\rho_{i=p,n,e,\nu}(\mathbf{r}) = 2\int_0^\infty \frac{d^3p}{(2\pi)^3} f_i(\mathbf{r};\mathbf{p},\mu_i),$$

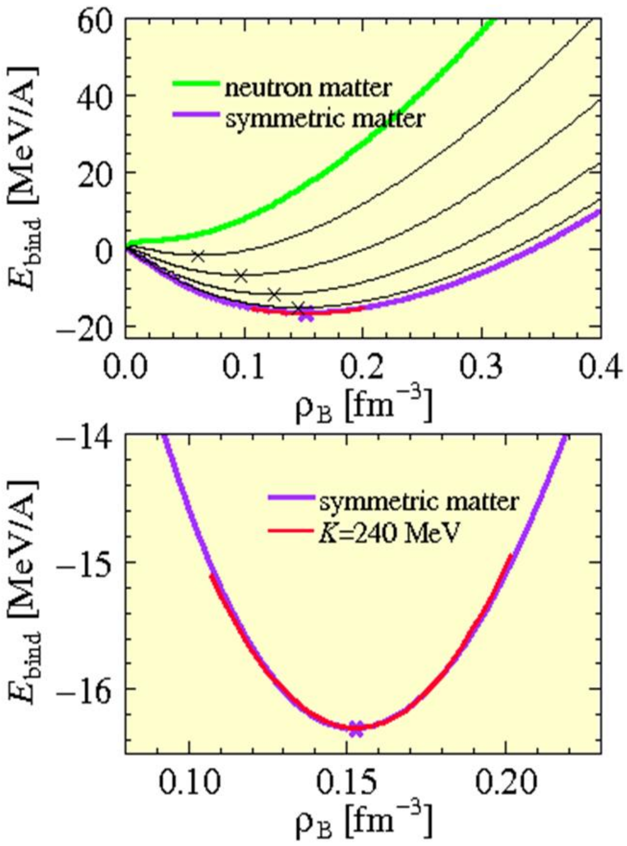
$$\mu_n = \sqrt{p_{Fn}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) - g_{\rho N}R_0(\mathbf{r}), \qquad \mu_n = \mu_p + \mu_e,$$

$$\mu_p = \sqrt{p_{Fp}(\mathbf{r})^2 + m_N^*(\mathbf{r})^2} + g_{\omega N}\omega_0(\mathbf{r}) + g_{\rho N}R_0(\mathbf{r}) - V_C(\mathbf{r}),$$

$$\int_V d^3r\left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r})\right] = \mathrm{const}, \qquad \int_V d^3r\,\rho_p(\mathbf{r}) = \int_V d^3r\,\rho_e(\mathbf{r}),$$

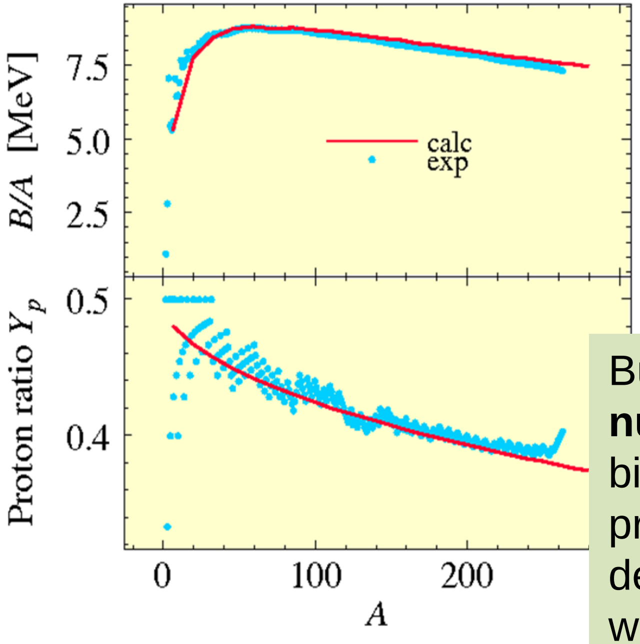
Choice of parameters

Matter properties

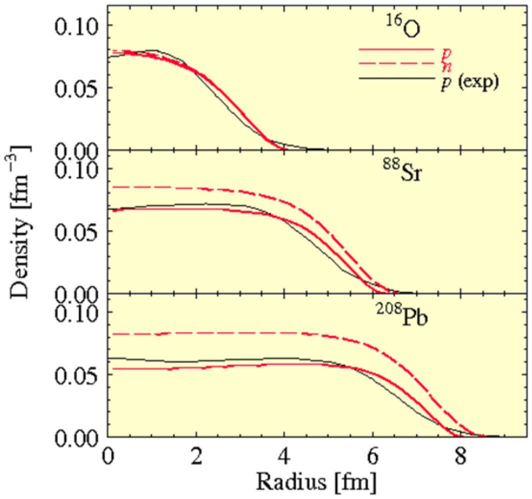


Saturation property of symmetric nuclear matter : minimum energy $E/A \approx -16$ MeV at $\rho_B \approx 0.16\ \mathrm{fm}^{-3}$.

Properties of nuclei

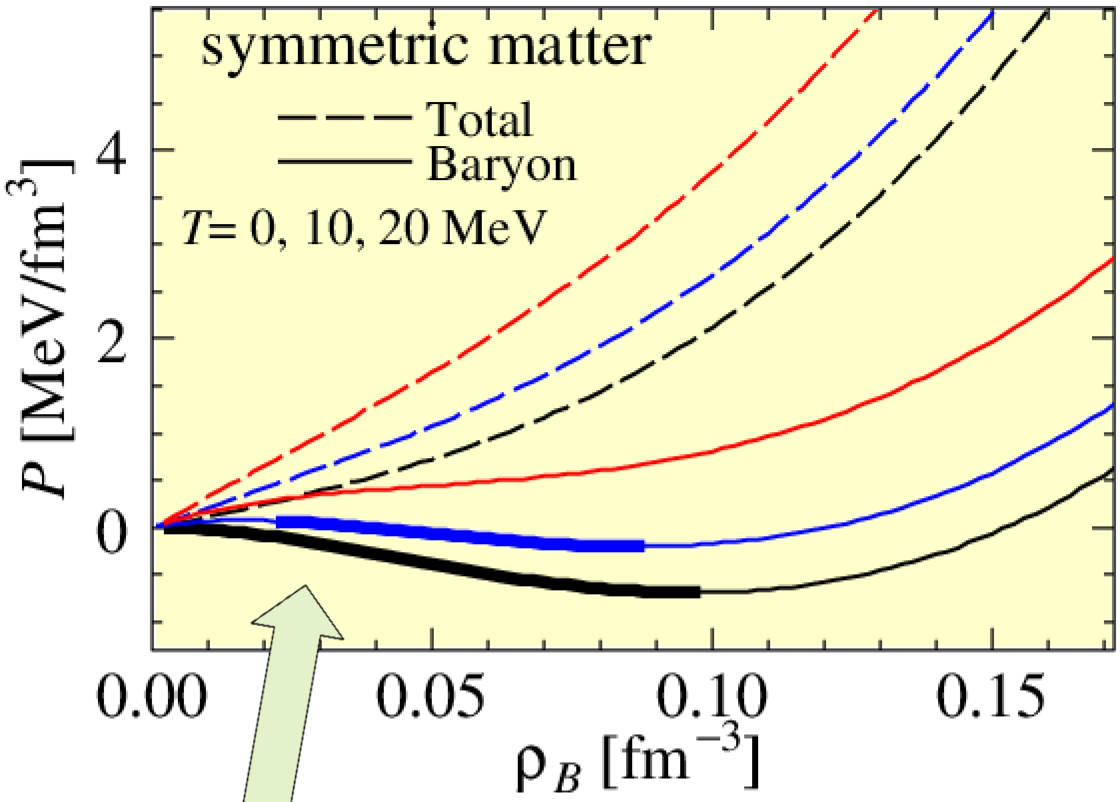


Bulk **properties of nuclei,** such as binding energies, proton fractions, and density profiles, are well reproduced.



Pressure of uniform nuclear matter

- Total pressure is positive. Monotonically increases with density and temperature.



- Baryon pressure exhibits van der Waals type behavior

  → mechanically unstable region ($dP/d\rho < 0$).

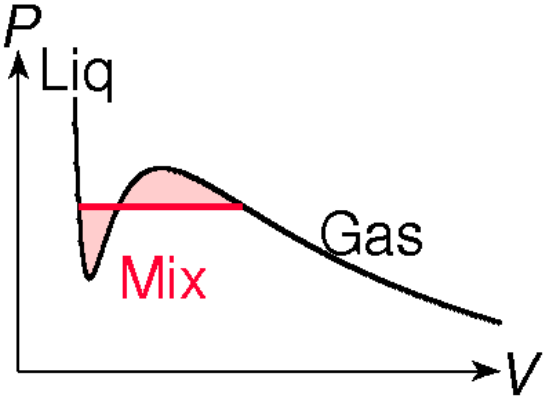
→ First order phase transition.

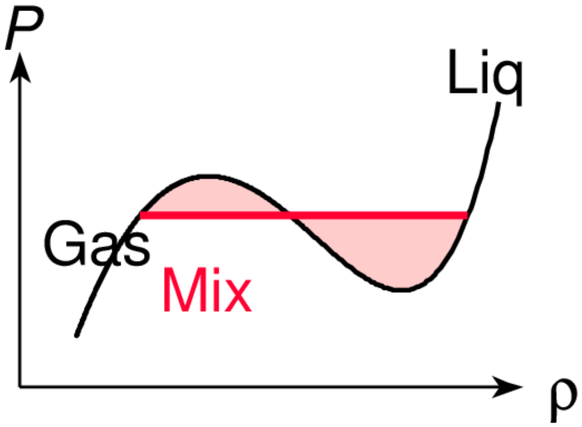
EOS of mixed phase

- Single component ***congruent***

(e.g. water)

Maxwell construction **satisfies** the Gibbs cond. $T^{I}=T^{II}$, $P^{I}=P^{II}$, $\mu^{I}=\mu^{II}$.





- Many components ***non-congruent***

(e.g. water+ethanol)

Gibbs cond. $T^{I}=T^{II}$, $P_i^{I}=P_i^{II}$, $\mu_i^{I}=\mu_i^{II}$.

No Maxwell construction *!*

- Many charged components

(nuclear matter)

Gibbs cond. $T^{I}=T^{II}$, $\mu_i^{I}=\mu_i^{II}$.

No Maxwell construction *!*

No constant *pressure !*

$$\frac{dP_i}{dr} = -\frac{\partial U_i\left(\rho_i; r\right)}{\partial r}$$

EOS of mixed phase

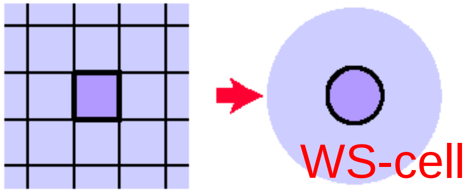
What is necessary?

Satisfying the Gibbs conditions, we have to look for inhomogeneous density distribution of nucleons and electrons, which minimize the free-energy density.

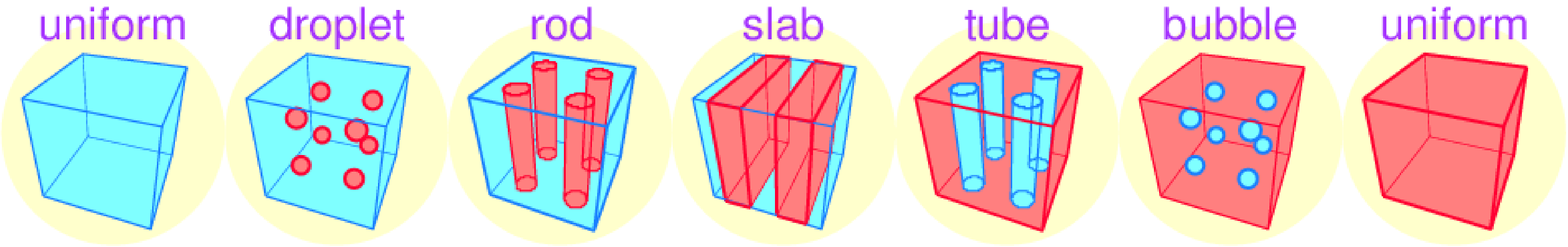
Numerical calculation of mixed-phase structure

- Assume regularity in structure: divide whole space into equivalent and neutral cells with a geometrical symmetry (3D: sphere, 2D : cylinder, 1D: plate).

  → Wigner-Seitz approx.



- Give a baryon density ρ_B and a geometry (Unif/Dropl/Rod/...).
- Solve the field equations numerically. Optimize the cell size (choose free-energy-minimum).
- Choose the free-energy-minimum geometry among 7 cases (Unif (I), Droplet, Rod, Slab, Tube, Bubble, Unif (II)),

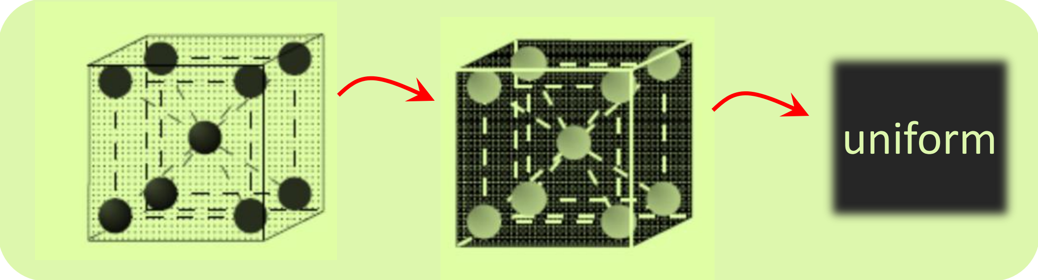


some of which are called “**pasta**” structures.

Nuclear "pasta" structures

- Baym, Bethe, Pethick, 1971

"Nuclei inside-out"



- Ravenhall et al 1983 & Hashimoto et al 1984

Concept of "pasta" structures. Minimizing free-energy of the inhomogeneous structure, i.e., achieving the balance between **surface tension** and the **Coulomb repulsion**
→ nuclear **pasta**

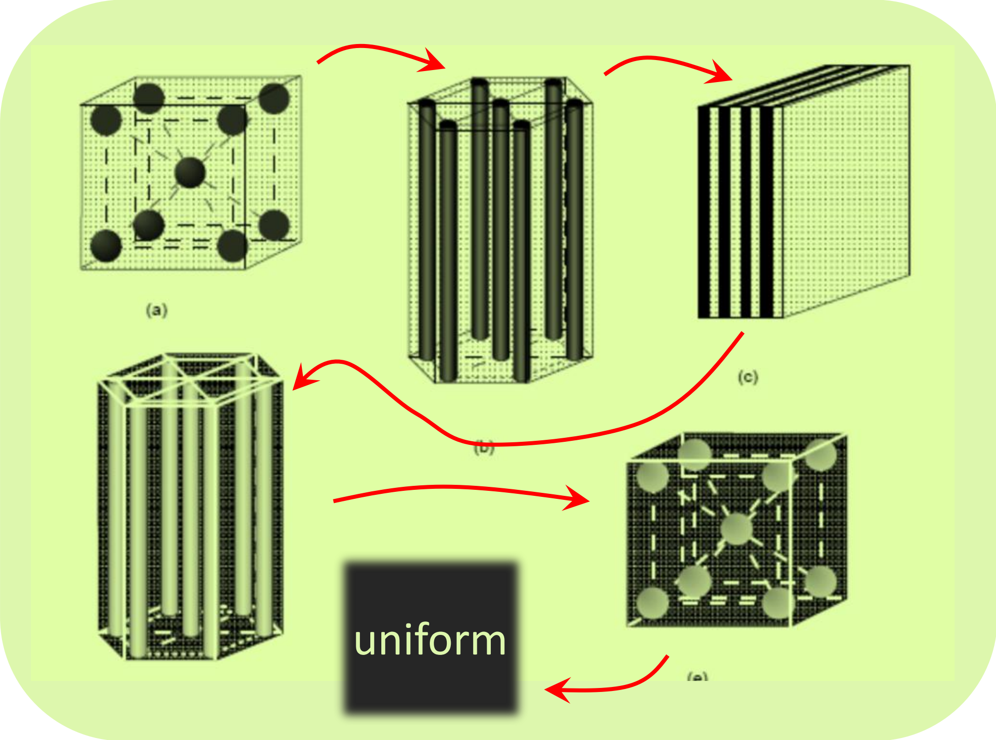
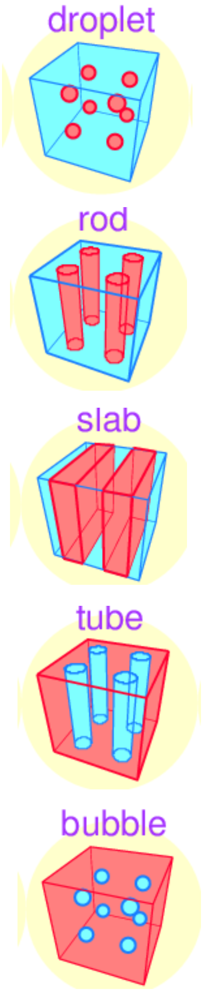
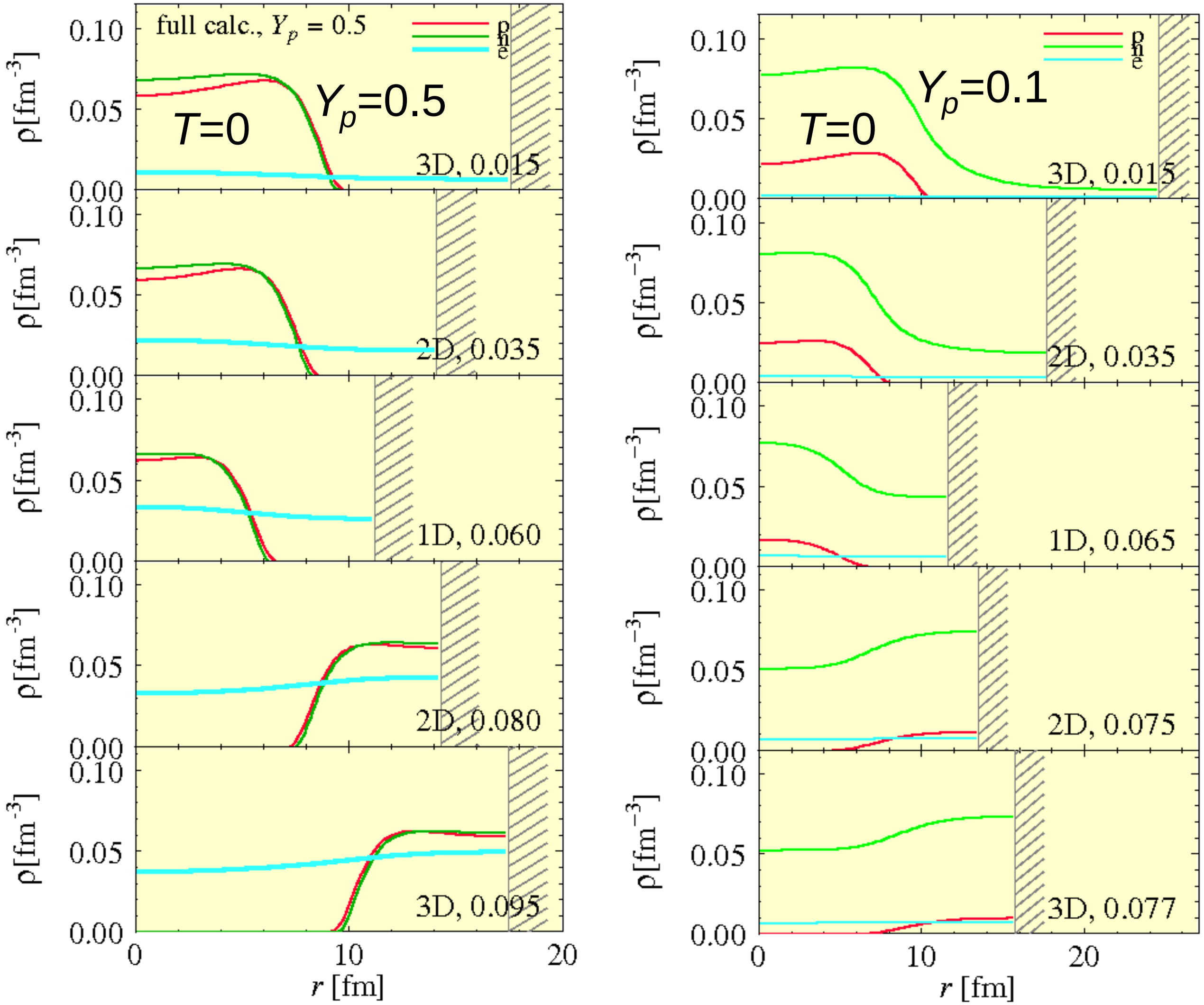


Figure from K. Oyamatsu, NP**A561**, 431 (1993)

Pasta structures in matter (case of fixed Y_p)

Density profiles in WS cells



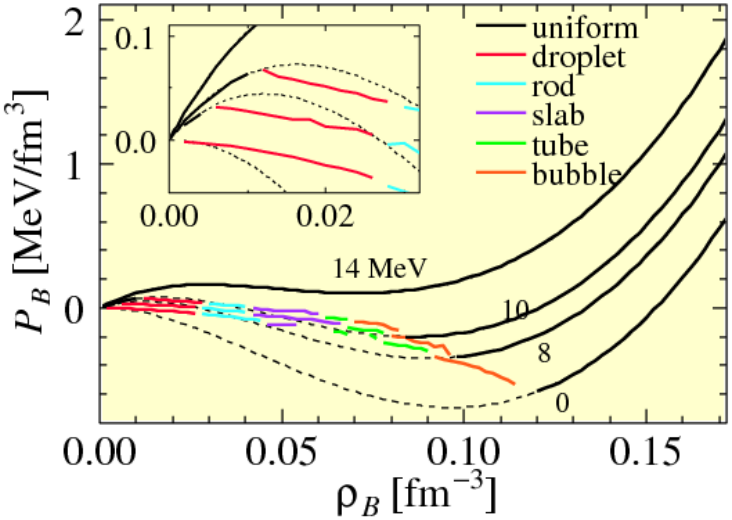


EOS with pasta structures in nuclear matter at $T \geq 0$

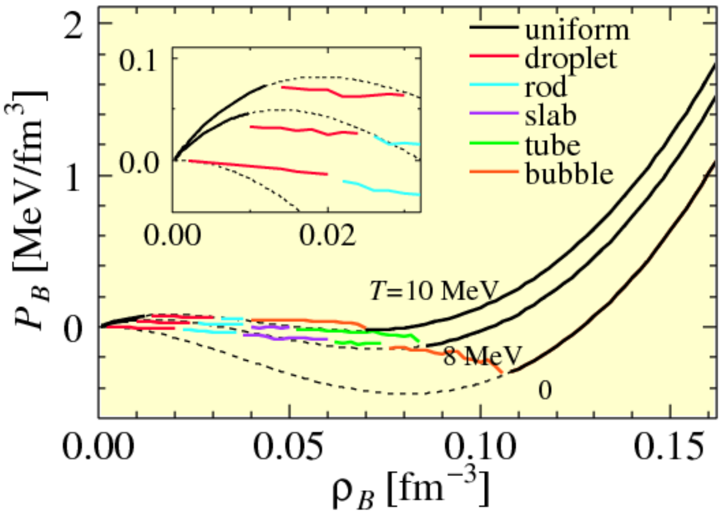
Pasta structures appear at $T \leq 10$ MeV

coexistence region (Maxwell for Y_p=0.5 and bulk Gibbs for Y_p<0.5) is meta-stable.
Uniform matter is allowed in some coexistence region due to finite-size effects.

Symmetric matter Y_p=0.5



Asymmetric matter Y_p=0.3



More realistic case for supernova core: Neutrino-trapped matter

- We discuss roles of neutrinos trapped in nuclear matter at **supernova cores** or **proto neutron stars**.

- The relevant density is around or below the normal nuclear density (saturation density $\rho_0 \cong 0.16\ \mathrm{fm}^{-3}$).

- Mean free path of neutrino

$$\lambda_\nu = \frac{1}{\rho\sigma}, \quad \rho \approx 0.1\ \mathrm{fm}^{-3}, \quad \sigma = 10^{-38}\mathrm{cm}^2$$

→ $\lambda \approx 1\mathrm{cm} \ll$ size of compact stars

→ **neutrinos are trapped**.

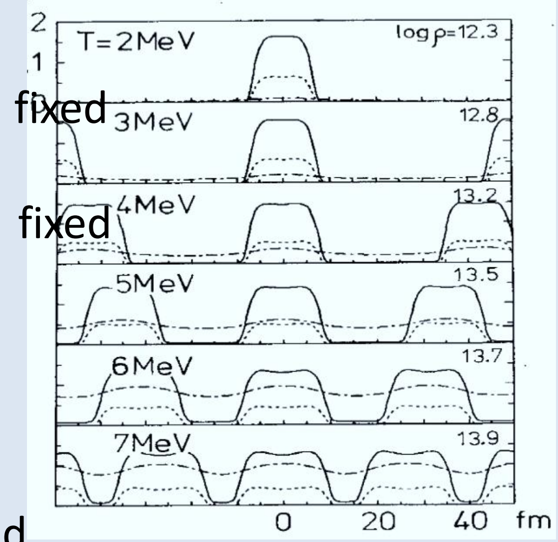
- The ingredients of matter are p, n, e^-, ν (trapped neutrino).

Neutrino degenerate and inhomogeneous matter.

Preceding studies:

- Ogasawara & Sato PTP68(1982)222
  Effective interaction + Thomas-Fermi, $T = 0$, Y_l = fixed
- Ogasawara & Sato PTP70(1983)1569
  Effective interaction + Thomas-Fermi, $T > 0$, Y_l = fixed

  → Enhancement of inhomogeneous structures (droplet & bubble)



- Watanabe, Iida, Sato NPA687(2001)512
  Effective interaction + flat density , $T = 0$, Y_l = fixed

  → Enhancement of pasta phases.

Our present study:

Relativistic mean field + Thomas-Fermi , $T > 0$, Y_l = fixed
Fully consistent density distribution.

Equations to be solved

For Fermions, we employ Thomas-Fermi approx. at finite T

$$f_{i=n,p}(\mathbf{r};\mathbf{p},\mu_i)=\left(1+\exp\left[\left(\sqrt{p^2+m_N^*(\mathbf{r})^2}-\sqrt{p_{Fi}(\mathbf{r})^2+m_N^*(\mathbf{r})^2}\right)\Big/T\right]\right)^{-1},$$

$$f_e(\mathbf{r};\mathbf{p},\mu_e)=\left(1+\exp\left[\left(p-(\mu_e-V_C(\mathbf{r}))\right)/T\right]\right)^{-1},$$

$$f_\nu(\mathbf{p},\mu_\nu)=\left(1+\exp\left[\left(p-\mu_\nu\right)/T\right]\right)^{-1}$$

$$\rho_{i=p,n,e,\nu}(\mathbf{r})=2\int_0^\infty\frac{d^3p}{(2\pi)^3}f_i(\mathbf{r};\mathbf{p},\mu_i),$$

$$\mu_p=\sqrt{p_{Fp}(\mathbf{r})^2+m_N^*(\mathbf{r})^2}+g_{\omega N}\omega_0(\mathbf{r})+g_{\rho N}R_0(\mathbf{r})-V_C(\mathbf{r}),$$

$$\mu_n=\sqrt{p_{Fn}(\mathbf{r})^2+m_N^*(\mathbf{r})^2}+g_{\omega N}\omega_0(\mathbf{r})-g_{\rho N}R_0(\mathbf{r}),\qquad \mu_n=\mu_p+\mu_e-\boldsymbol{\mu_\nu}$$

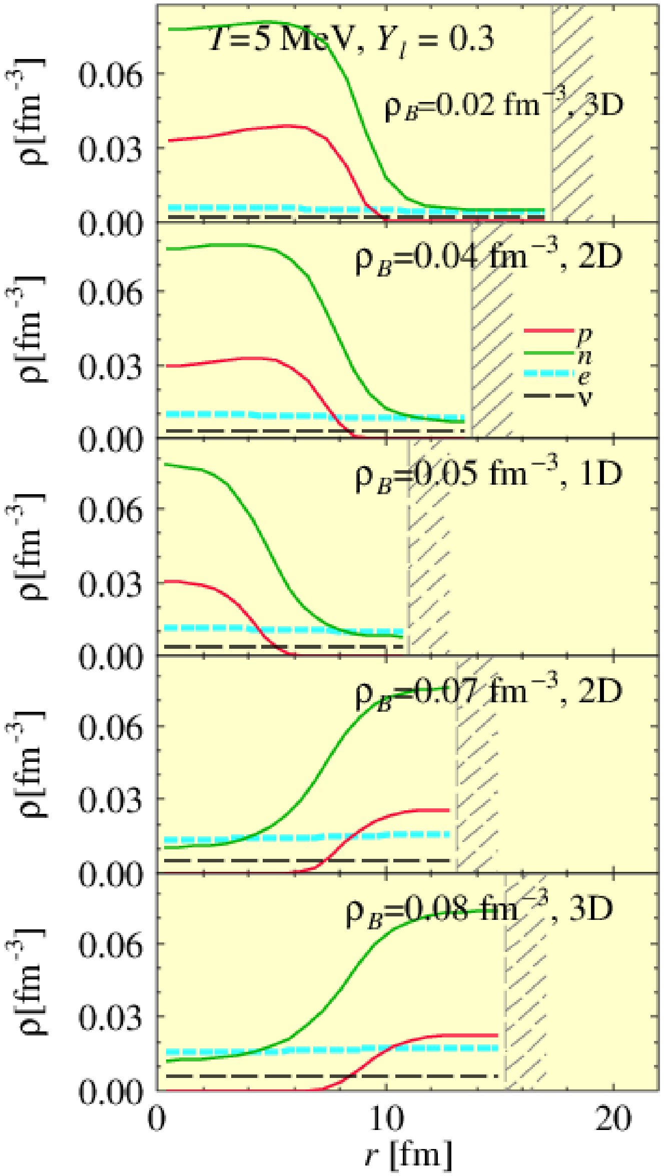
$$\int_V d^3r\left[\rho_p(\mathbf{r})+\rho_n(\mathbf{r})\right]=\text{const},\quad \text{(baryon number)}\qquad \int_V d^3r\,\rho_p(\mathbf{r})=\int_V d^3r\,\rho_e(\mathbf{r}),\quad \text{(total charge)}$$

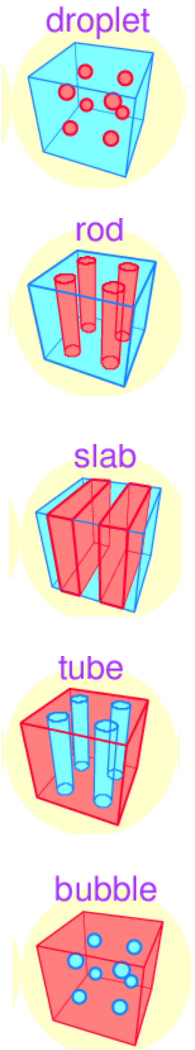
$$\int_V d^3r\left[\rho_e(\mathbf{r})+\rho_\nu(\mathbf{r})\right]=\text{const}\quad \text{(lepton number)}$$

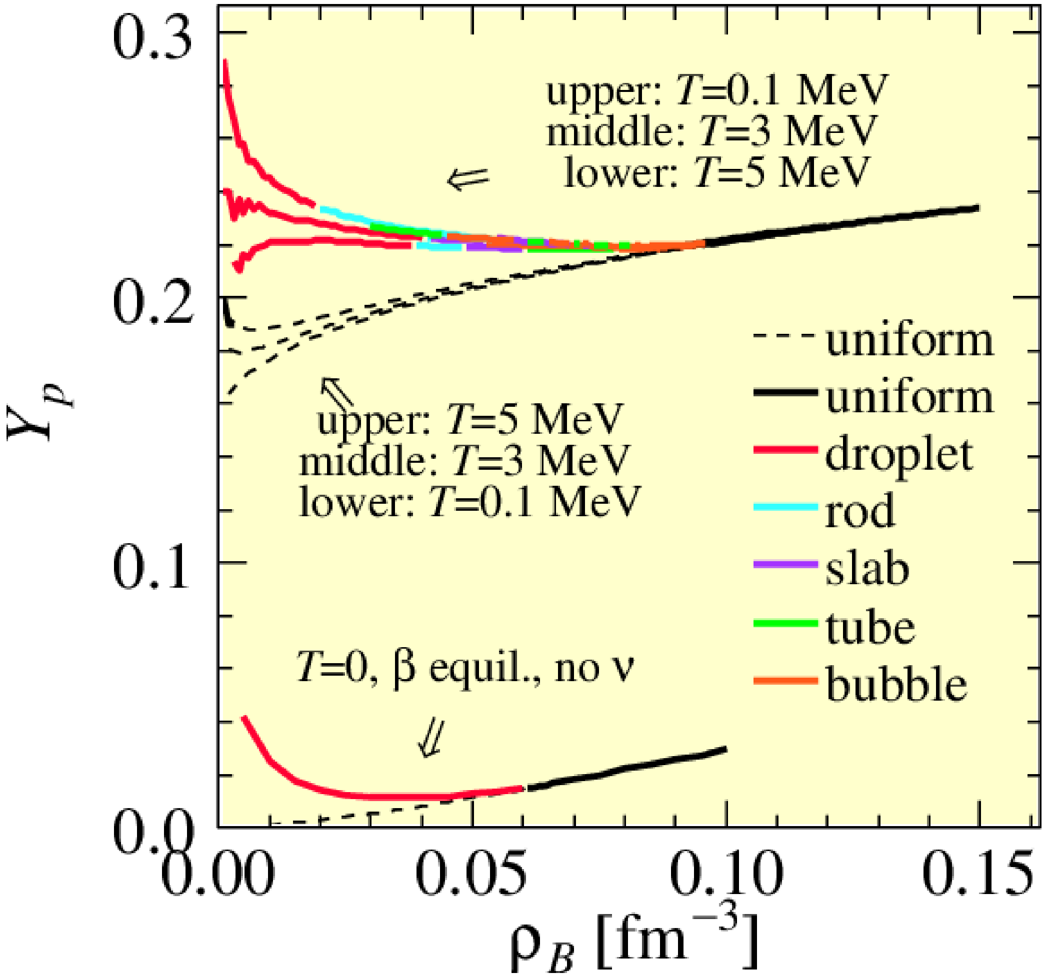
Density distribution of particles in the WS cells.

Looking for free-energy minimum states (solving the coupled equations for fields and chemical potentials), we obtain density distribution in a cell. The optimum geometry (dimensionality of the cell) changes from “Droplet” to “Rod”, “Slab”, “Tube”, “Bubble”, and to “Uniform” with increasing the density.

The electron fraction is larger than the neutrino fraction $Y_e > Y_\nu$ due to the attraction between e^- and p.







Proton fraction of matter depends on the density and the appearance of non-uniform structures.

Constant lepton fraction Y_l gives proton fraction Y_p about 3/4 of Y_l, which is much larger than the case of beta-equilibrium without neutrinos.

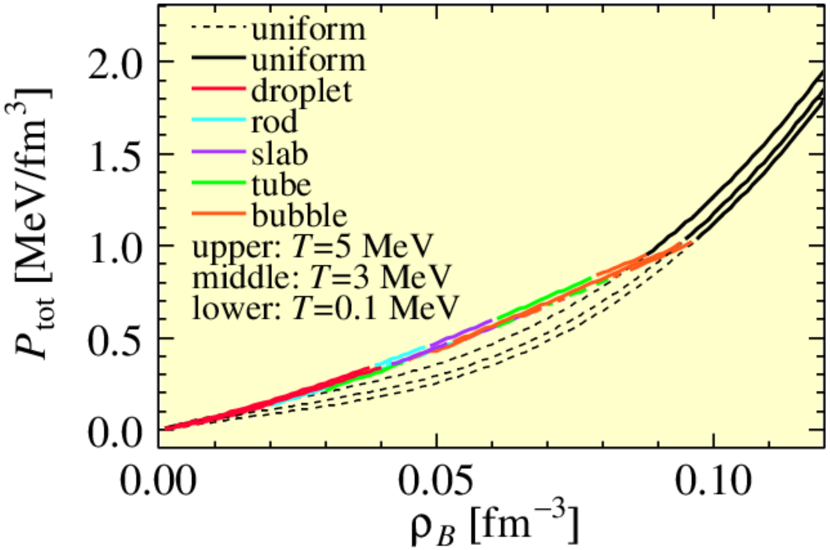
Higher temperature suppresses the appearance of “pasta” (rod, slab, etc) and enlarges the region of droplet.

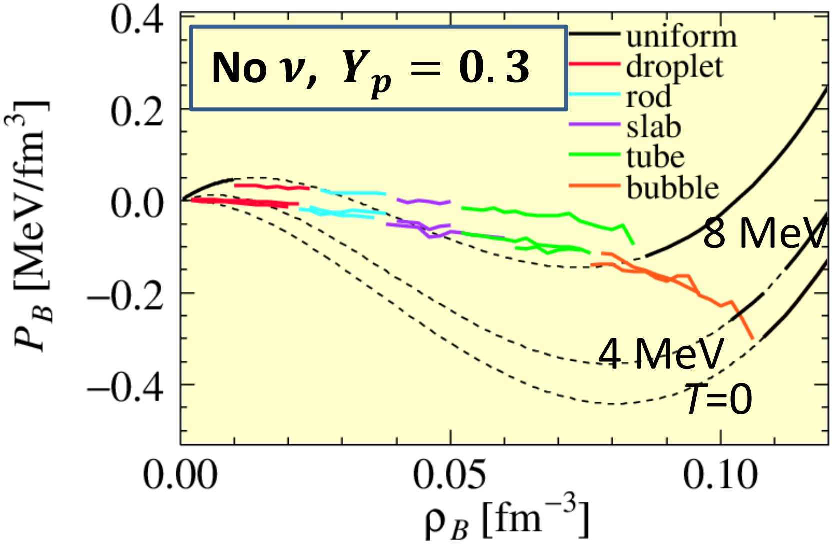
Trapped **neutrino enhances** the appearance of **inhomogeneous** structure.

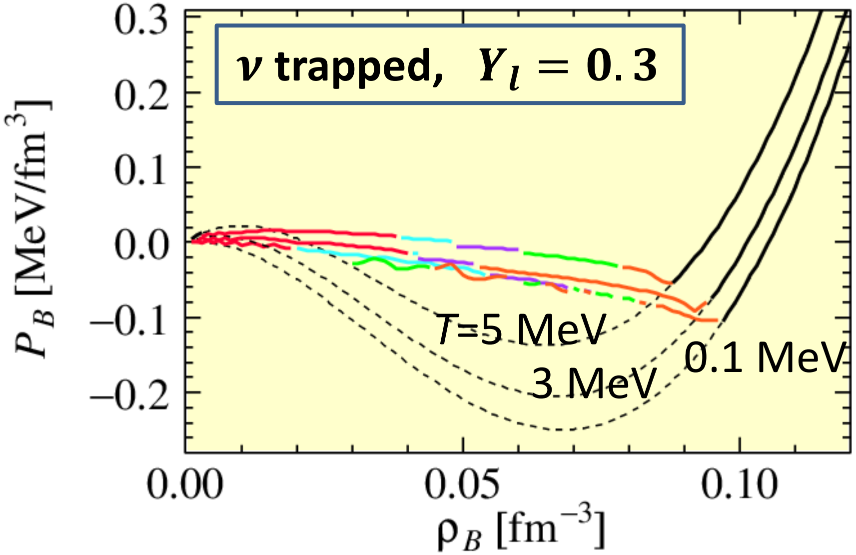
EOS (pressure) of nuclear matter at finite temperature with a fixed proton fraction Y_p=0.3 (below) and a fixed lepton fraction Y_l=0.3 (right panels).

The appearance of inhomogeneous structure softens the EOS for both cases.

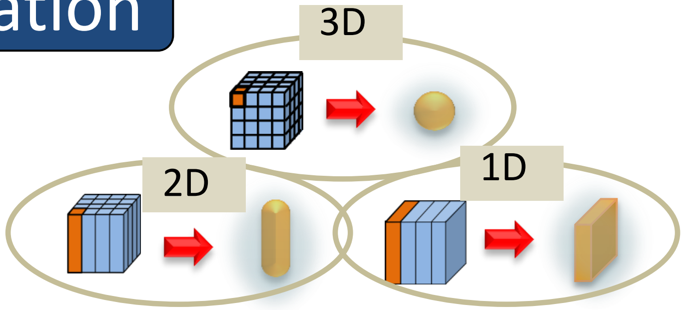
They show similar dependence on the density.





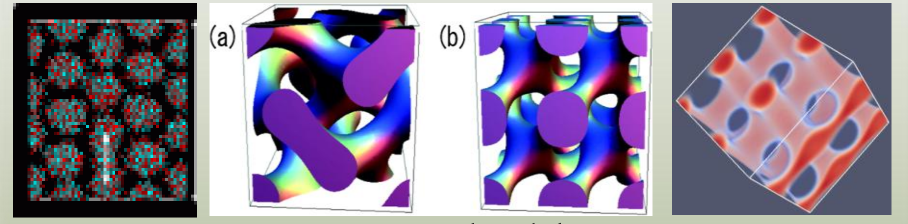


Fully 3 dimensional (3D) calculation



Use of the Wigner-Seitz cell

→ Limitation of emerging structures.

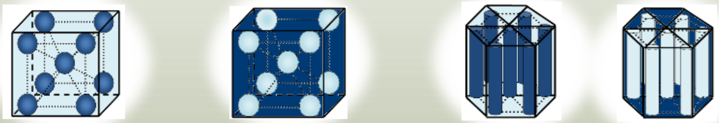


dumbbell Gyroid double-diamond network

Complex structures were missing

Crystalline structures could not be discussed.

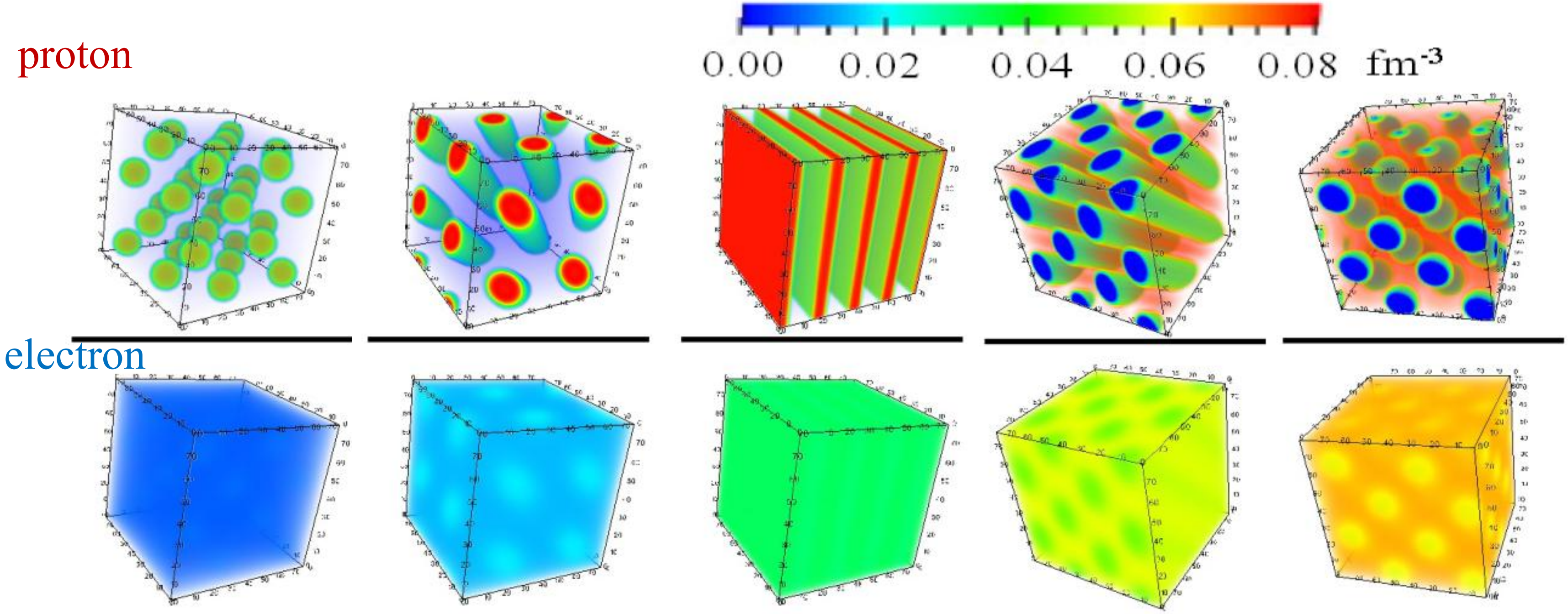
“droplet” “bubble” “rod” “tube”



bcc lattice **Honeycomb**

Results of our 3D RMF calculations

$Y_p = Z/A = 0.5$

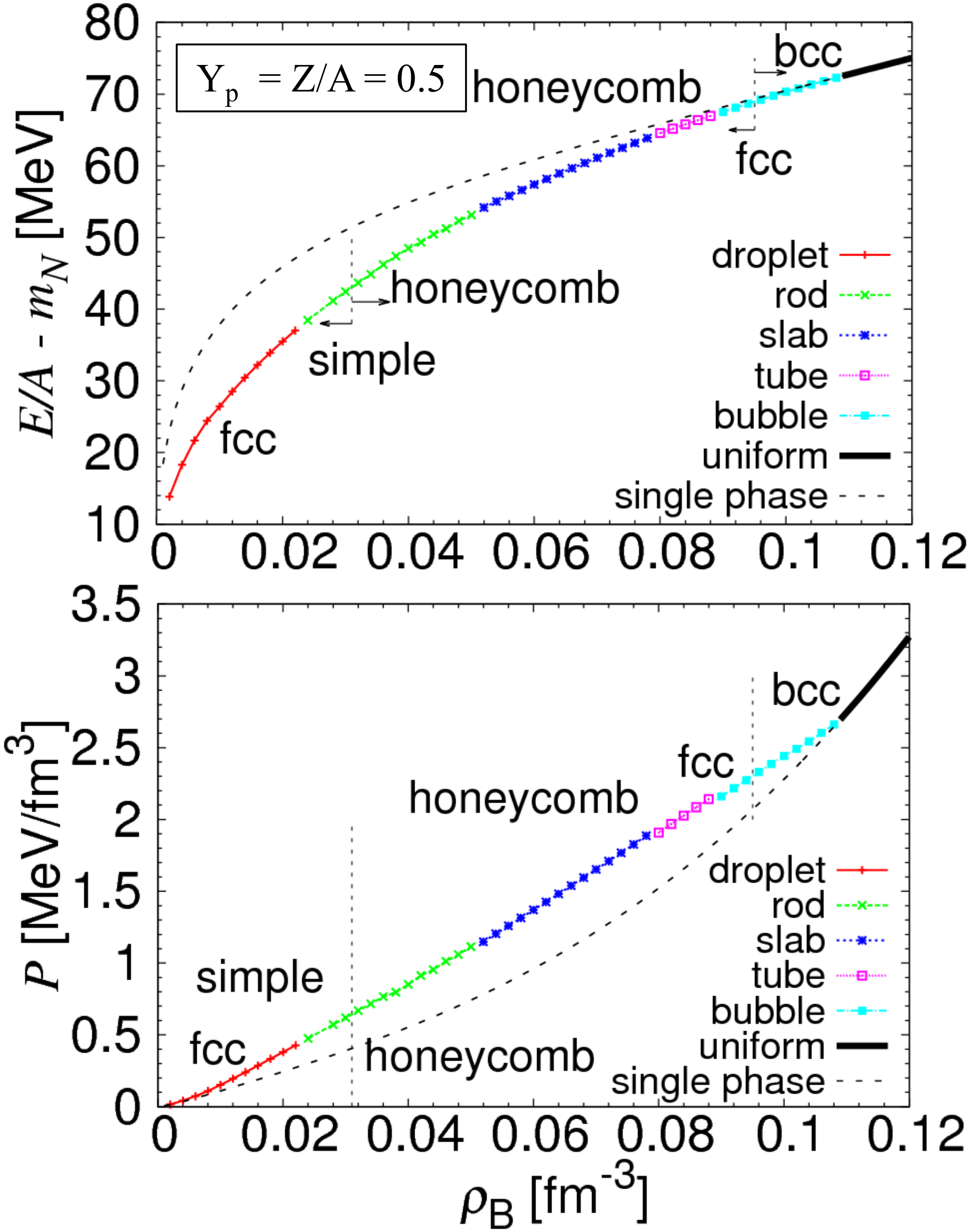


“Droplet”	“Rod”	“Slab”	“Tube”	“Bubble”
[fcc]	[honeycomb]		[honeycomb]	[fcc]
ρ_B = 0.012 fm^{-3}	0.024 fm^{-3}	0.05 fm^{-3}	0.08 fm^{-3}	0.094 fm^{-3}

EOS has a similar behavior to that of the conventional studies.

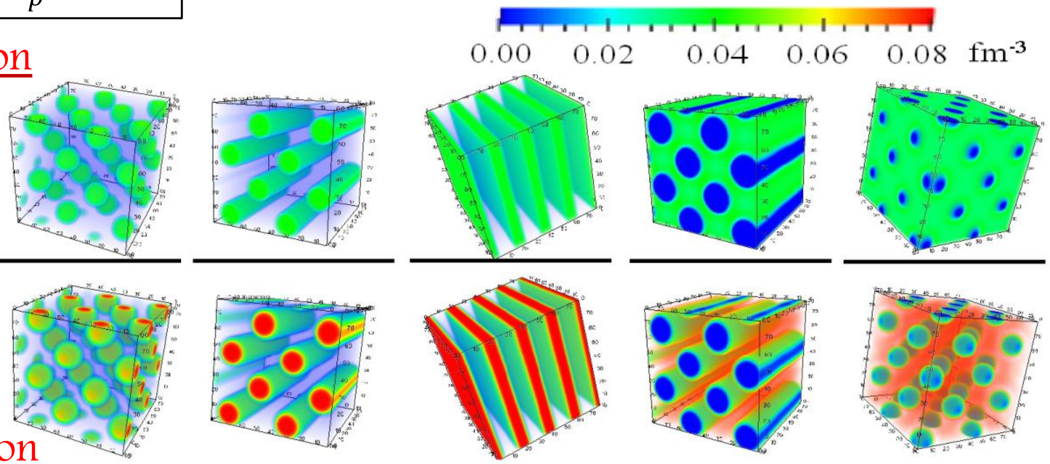
Novelty:

fcc lattice of droplets can be the ground state at some density.
← Not the Coulomb interaction among “point particles” but the change of the droplet size is relevant.



$Y_p = 0.3$

proton



neutron

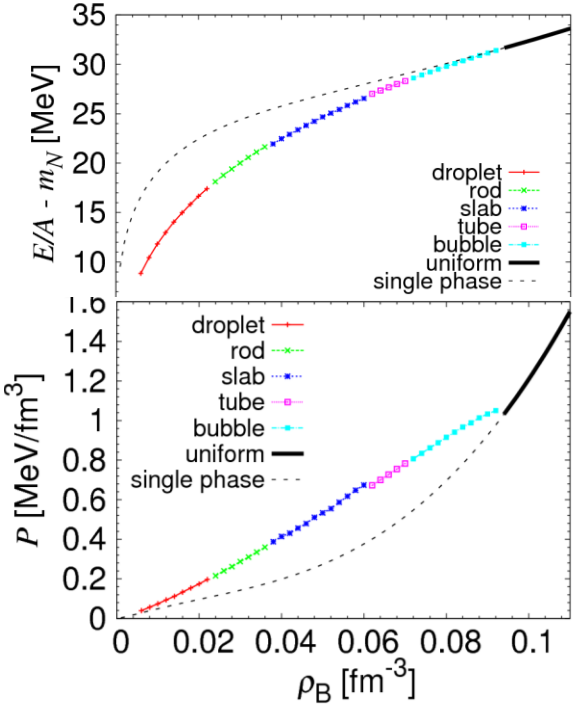
"droplet" [fcc] ρ_B = 0.016 fm^{-3}

"rod" [simple] 0.030 fm^{-3}

"slab" 0.05 fm^{-3}

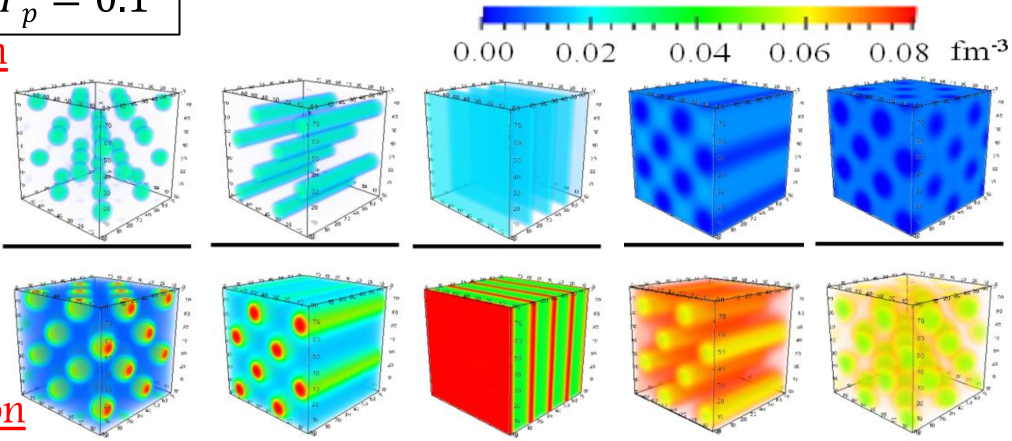
"tube" [simple] 0.066 fm^{-3}

"bubble" [fcc] 0.080 fm^{-3}



$Y_p = 0.1$

proton



neutron

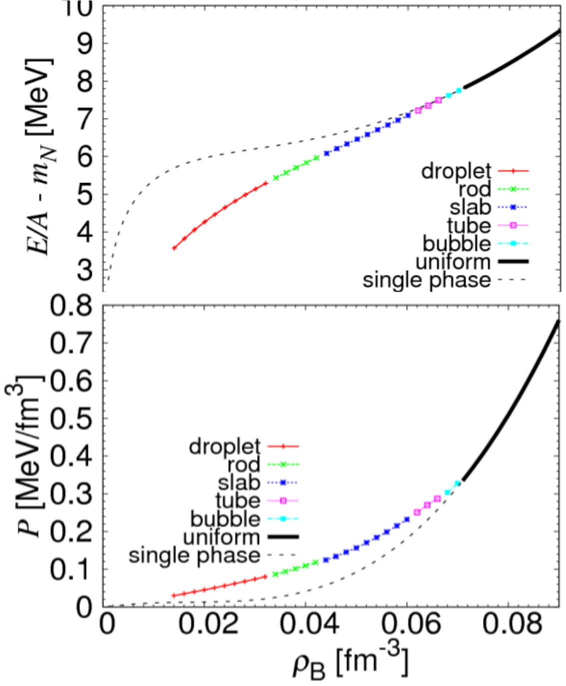
"droplet" [fcc] ρ_B=0.020 fm^{-3}

"rod" [simple] 0.040fm^{-3}

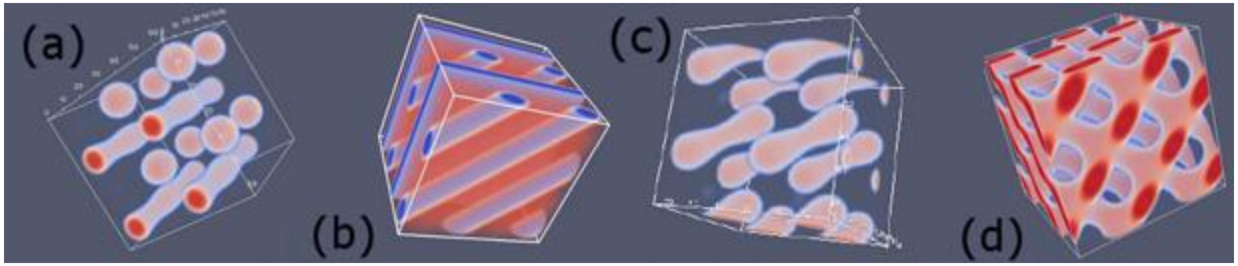
"slab" 0.05 fm^{-3}

"tube" [simple] 0.066 fm^{-3}

"bubble" [fcc] 0.070 fm^{-3}



Some complex metastable states in our 3D RMF calc.



Mixture of
Droplet and
Rod

Mixture of
Slab and
Tube

Dumbbell

Network

⟨ High–density matter ⟩

Structure of compact stars

TOV equation

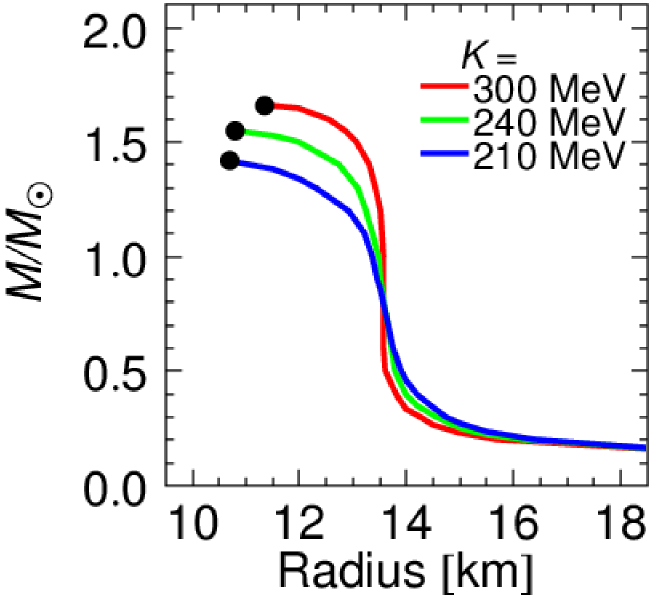
$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}\left(1+\frac{4\pi r^3 P}{m}\right)\left(1+\frac{P}{\rho}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

$P = P(\rho)$ Pressure (EOS --- the input)

$\rho = \rho(r)$ Density at position r

$m = m(r) = \int_0^r 4\pi s^2 \rho(s) ds$ mass inside the position r

$M = m(R), \quad R = R(\rho \approx 0)$ total mass and radius.



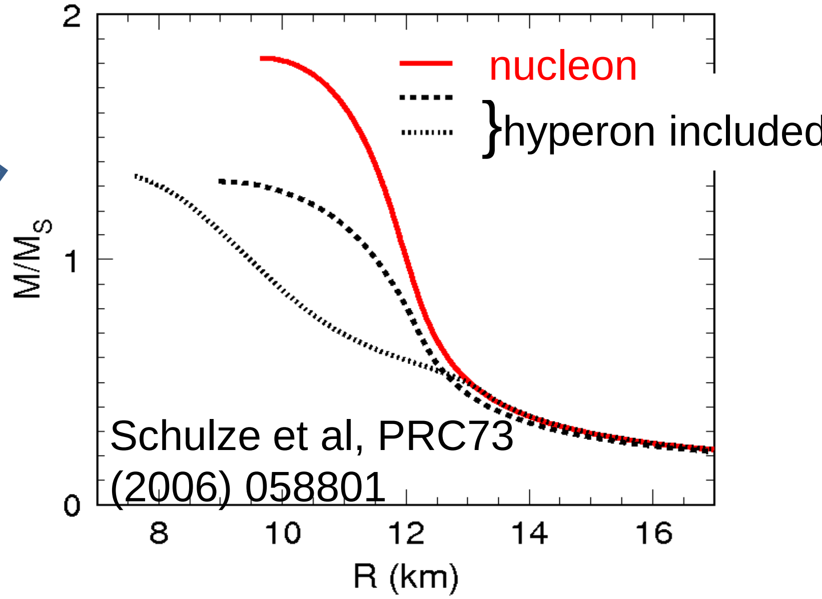
Mass–radius relation
and the **maximum mass**
is determined by the EOS
of matter.

Softening of EOS by hyperon

Hyperon mixture at $2-3\rho_0$

→ EOS gets soft

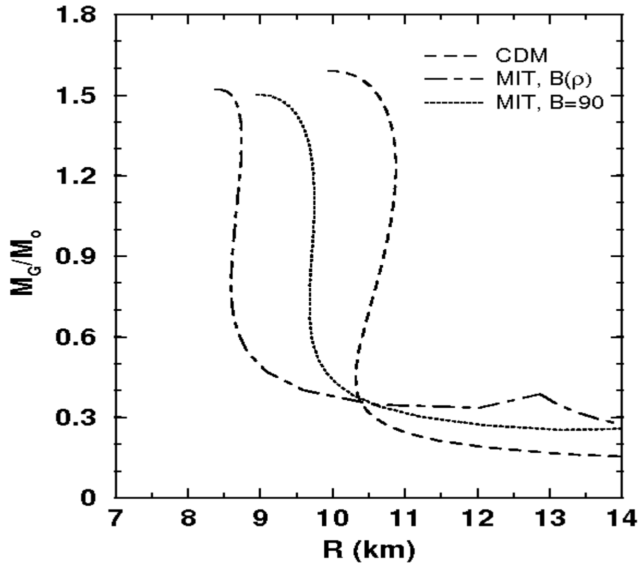
→ too small maximum mass



Schulze et al, PRC73 (2006) 058801

Improve the maximum mass by introducing transition into quark matter (?)

[Maieron et al, PRD70 (2004) 043010].



However, mixed phase may soften the EOS.

A bulk calculation suggests wide region of mixed phase. [Glendenning, PRD46,1274].

Hadron-Quark mixed-phase structure and EOS (*T*=0)

- Assume regularity in structure: divide whole space into **equivalent neutral cells** with a geometrical **symmetry** (3D: sphere, 2D : cylinder, 1D: plate). → Wigner-Seitz approx.
- Give a **sharp boundary between H and Q** phases and a **geometry** (Unif/Dropl/...) .
- Solve the **field equations** numerically and get **density profiles**. Optimize the cell size and H-Q boundary position (choose the energy-minimum).
- Choose **energy-min geometry** (Unif H, droplet, rod, slab, tube, bubble, Unif Q).

Hadron phase

Brueckner Hartree Fock model

$$\varepsilon(\rho) = \frac{3}{5}\frac{k_F^2}{2m}\rho + \frac{1}{2}\sum_{k,k'\le k_F}\langle kk'|AG[\rho;e(k)+e(k')]|kk'\rangle$$

$$G[\rho;w] \equiv v + \sum_{k_a,k_b} v\frac{|k_a k_b\rangle Q\langle k_a k_b|}{w-e(k_a)-e(k_b)}G[\rho;w]$$

$$e(k) \equiv \frac{k^2}{2m} + \mathrm{Re}\sum_{k'\le k_F}\langle kk'|AG[\rho;e(k)+e(k')]|kk'\rangle$$

v: AV18 + UIX + NSC89

Quark phase

MIT bag model

$$\varepsilon(\rho) = \sum_f\left[\varepsilon_f^{\text{kin}} + \varepsilon_f^{\text{Fock}}\right] + B \quad (B\text{: bag constant})$$

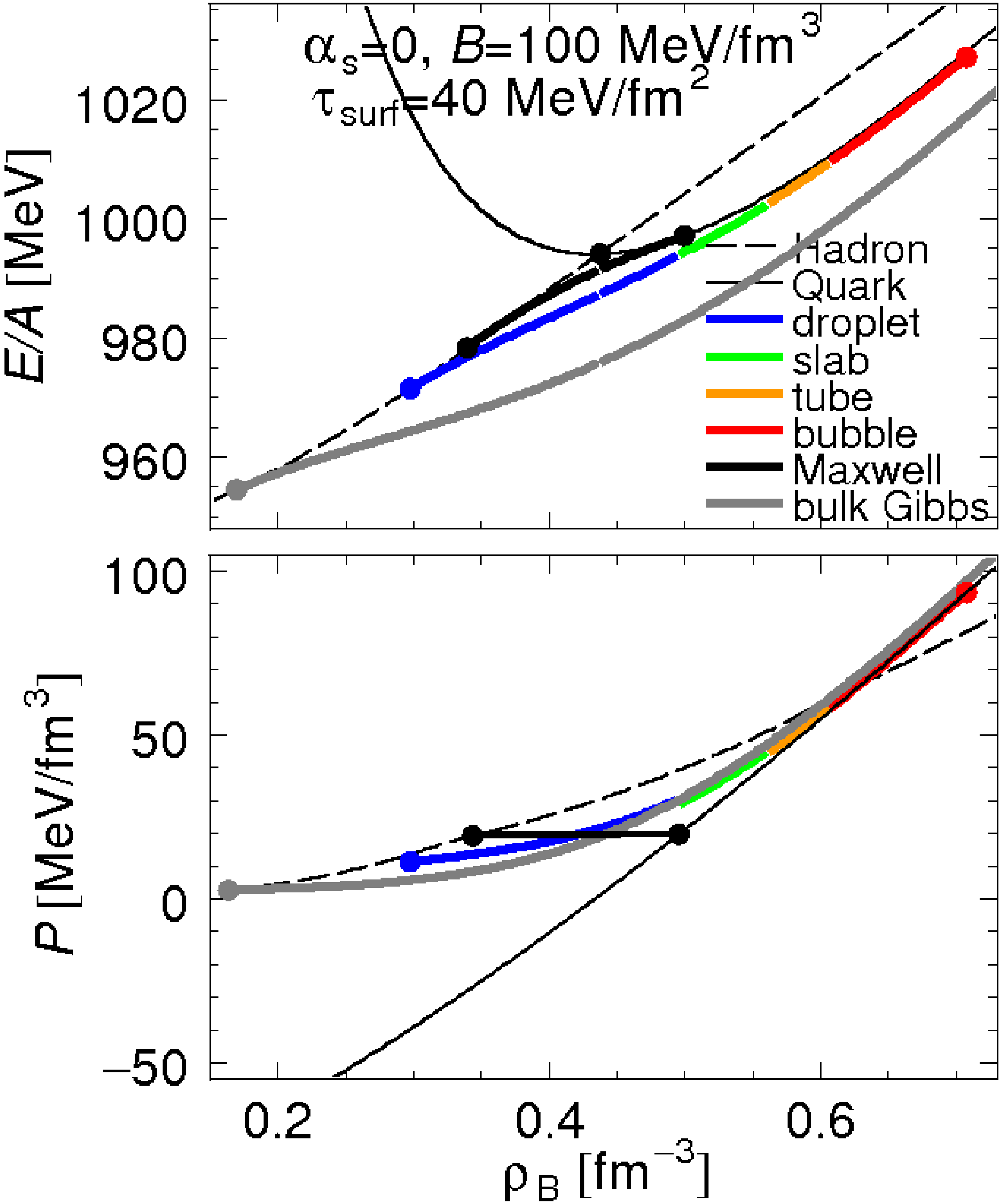
$$\varepsilon_f^{\text{kin}} = \frac{3}{8\pi^2}m_f^4\left[x_f\eta_f\left(2x_f^2+1\right) - \log\left(x_f+\eta_f\right)\right]$$

$$\varepsilon_f^{\text{Fock}} = \frac{\alpha_S}{8\pi^3}m_f^4\left[x_f^4 - \frac{3}{2}\left[x_f\eta_f - \log\left(x_f+\eta_f\right)\right]^2\right]$$

$$x_f \equiv p_F^{(f)}(\rho_f)/m_f, \quad \eta_f \equiv \sqrt{1+x_f^2}$$

EOS of matter

Full calculation is close to the Maxwell construction (local charge neutral).
Far from the bulk Gibbs calculation (neglects the surface and Coulomb).

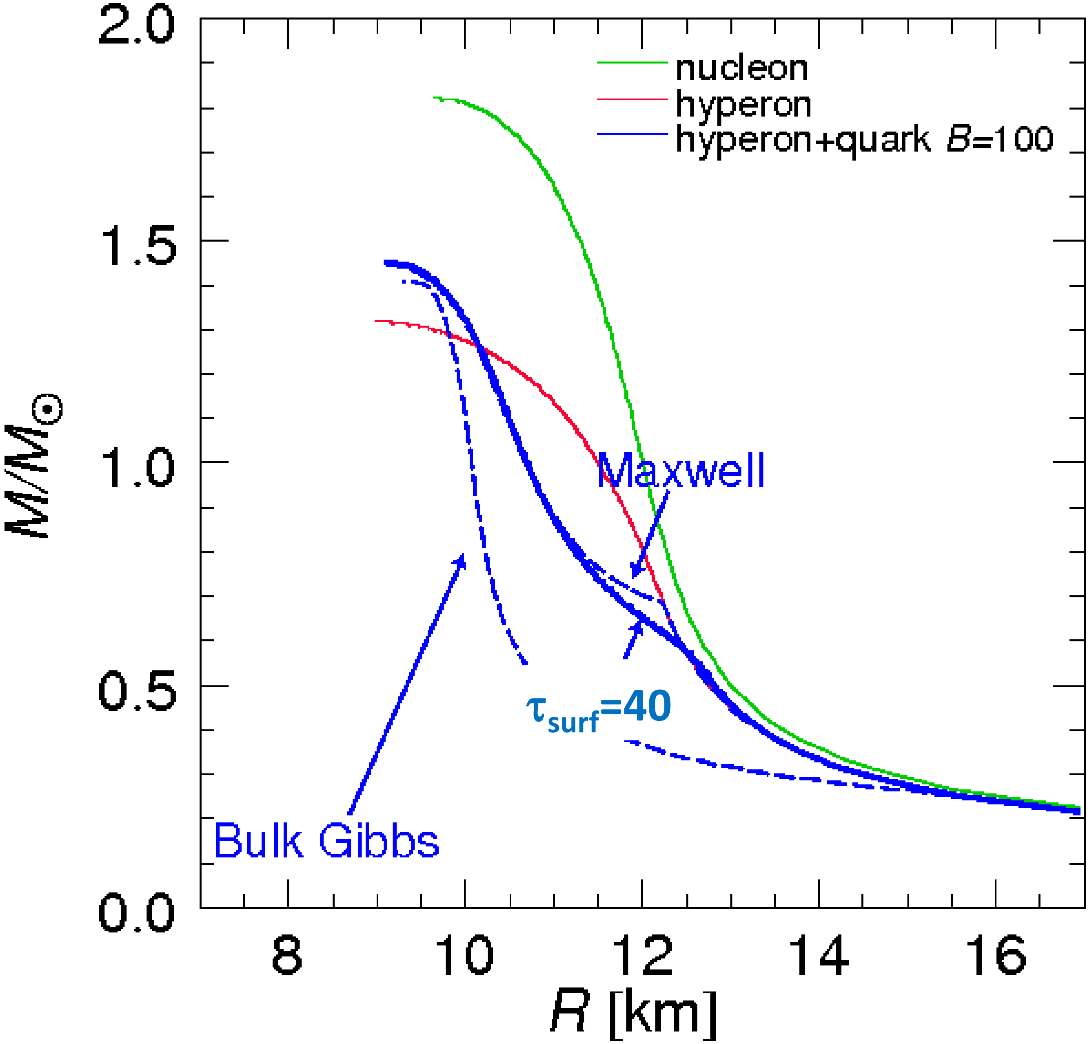


Mass-radius relation of a cold neutron star

Full calculation with pasta structures yields similar result to the Maxwell construction.

Maximum masses are almost the same for 3 cases.

We need to improve largely the quark EOS or hadron EOS to get $\sim 2M_\odot$



Summary

- RMF + Thomas Fermi calculation for low-density nuclear matter
- "Pasta" structure appears and affects on the EOS.
- For neutrino-trapped matter, pasta structures are enhanced by neutrinos
- The EOS depends largely on the structure and the existence of neutrinos.
- Fully 3D calculation（Okamoto *et al*）is developing.

- Hadron-quark mixed phase is important for structure and mass of compact stars.
- We have developed a method to calculate EOS of mixed phase. But each EOS should be improved to sustain neutron stars

Thank you for your attention!

END

Finite-size effects

Strong surface tension and weak Coulomb

→ large *R*

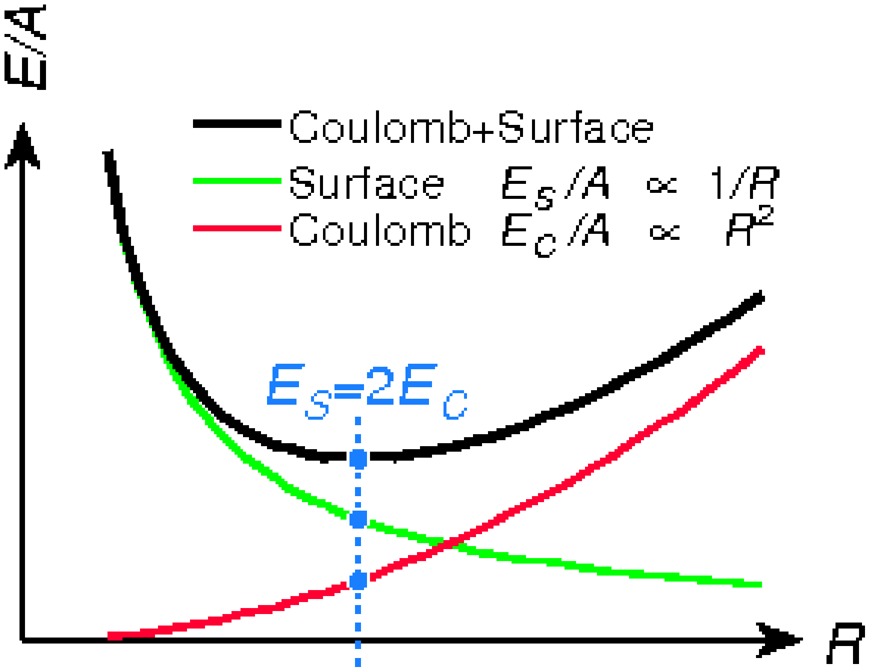
extreme case

→ no minimum.

(pasta unstable)

[Voskresensky et al, PLB541(2002)93; NPA723(2003)291; "amorphous" PRD(2012)]

Dependence of *E/A* on *R*.



Structure of compact stars

TOV equation

$$\frac{dP}{dr} = -\rho\frac{Gm}{r^2}\left(1+\frac{4\pi r^3 P}{m}\right)\left(1+\frac{P}{\rho}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

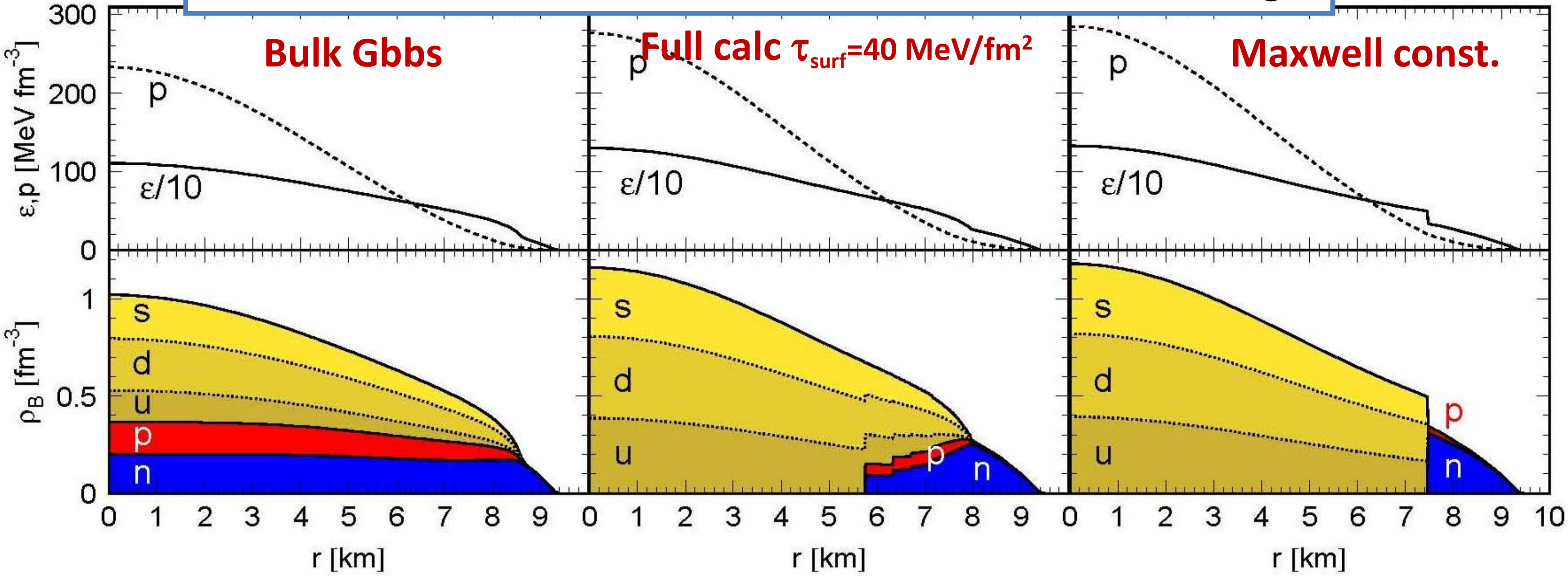
$P = P(\rho)$ Pressure (input of TOV eq.)

$\rho = \rho(r)$ Density at position r

$m = m(r) = \int_0^r 4\pi s^2 \rho(s) ds$ mass inside the position r

$M = m(R), \quad R = R(\rho \approx 0)$ total mass and radius.

Density profile of a compact star ($M = 1.4\, M_\odot$)



Bulk Gbbs

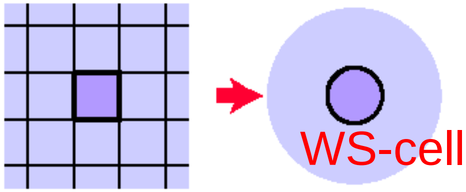
Full calc τ_{surf}=40 MeV/fm²

Maxwell const.

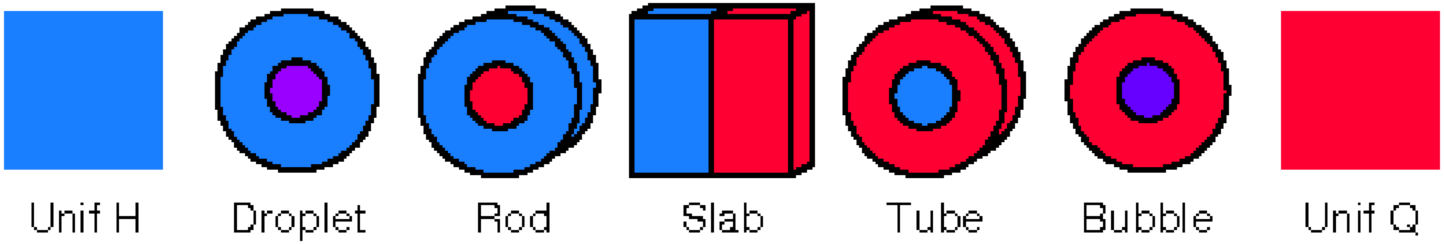
Hadron-Quark mixed-phase structure and EOS (*T*=0)

- Assume regularity in structure: divide whole space into equivalent and neutral cells with a geometrical symmetry (3D: sphere, 2D : cylinder, 1D: plate).

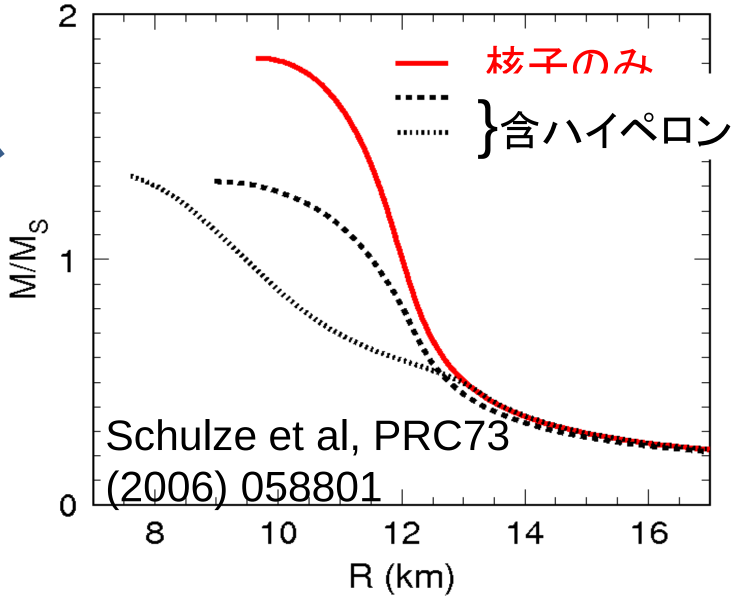
  → Wigner-Seitz cell approx.



- Divide a cell into hadron phase and quark phases.
- Give a geometry (Unif/Dropl/Rod/...) and a baryon density ρ_B.
- Solve the field equations numerically. Optimize the cell size and H-Q boundary position (choose the energy-minimum).
- Choose an energy-minimum geometry among 7 cases (Unif H, droplet, rod, slab, tube, bubble, Unif Q).



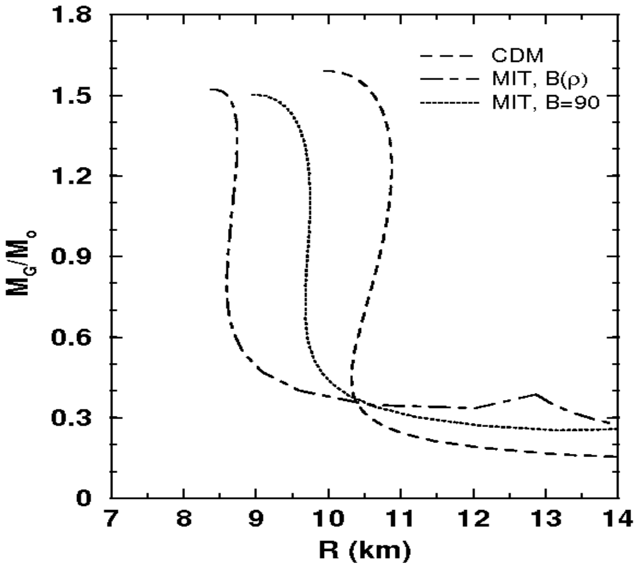
$2\text{-}3\ \rho_0$ でハイペロン混合。→ EOS軟化
→ 中性子星最大質量 $< 1.4 M_{sol}$
→ 観測値 $> 1.5\ M_{sol}$ と矛盾。



Schulze et al, PRC73 (2006) 058801

クォーク物質への相転移で最大質量の問題が解決(?) [Maieron et al, PRD70 (2004) 043010 etc].

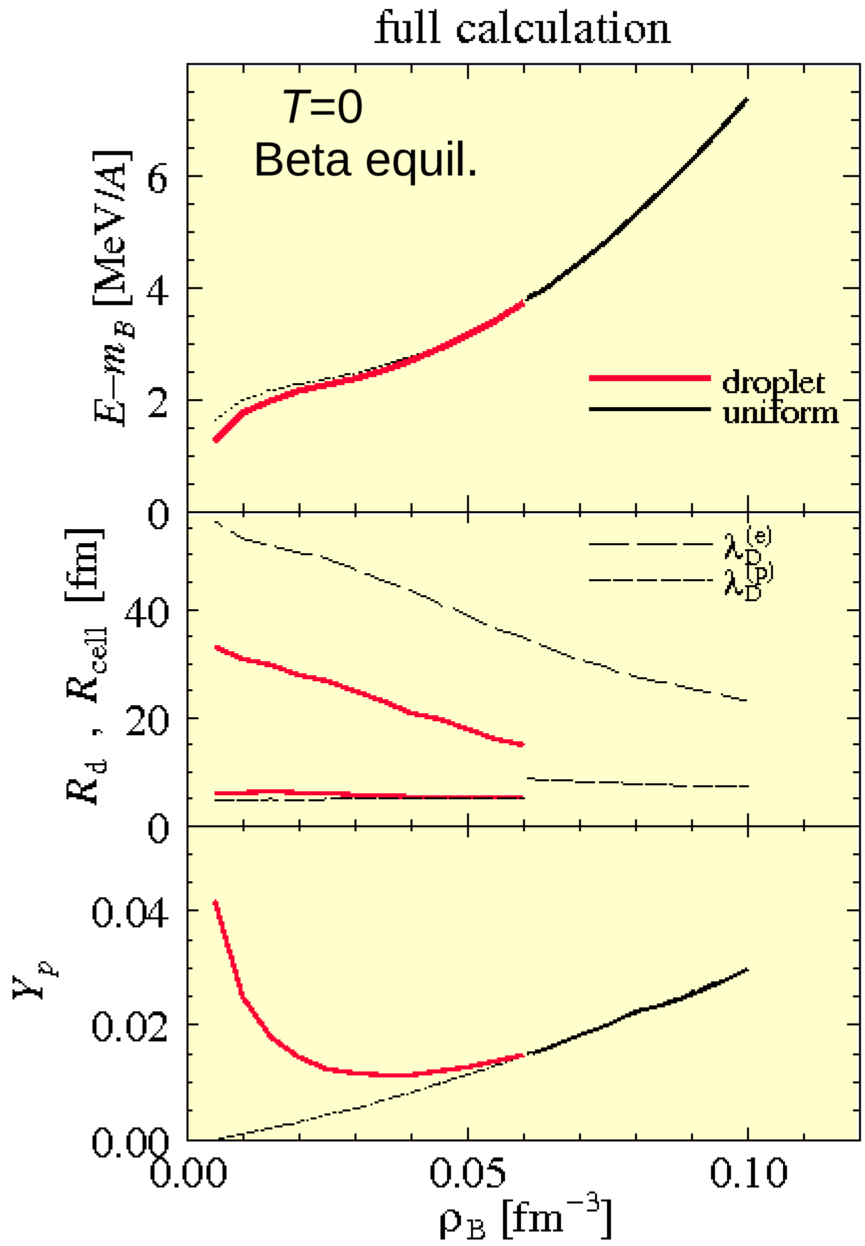
しかし 混合相の存在で再び軟化する可能性。
バルク（構造を無視）なGibbs計算では広い範囲で混合相。
[Glendenning, PRD46,1274].



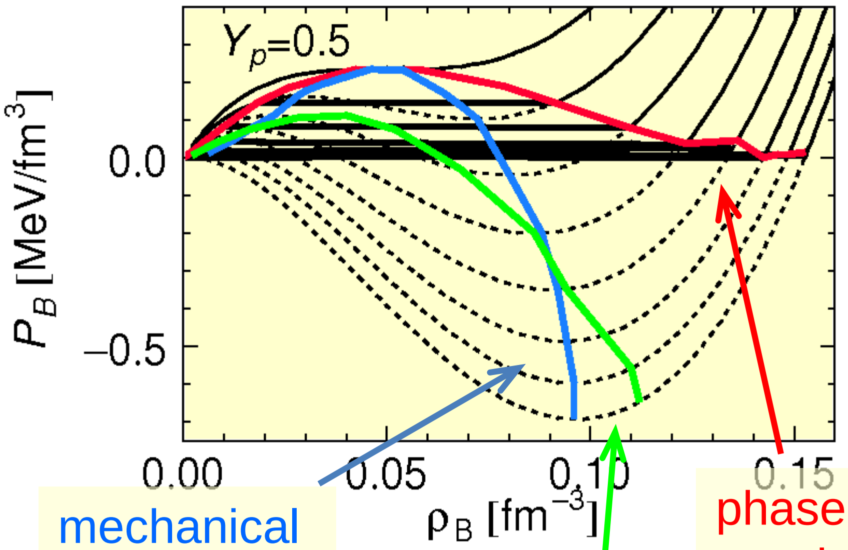
Beta-equilibrium case

Only “droplet” structure appears.

The change of EOS due to the non-uniform structure is small.



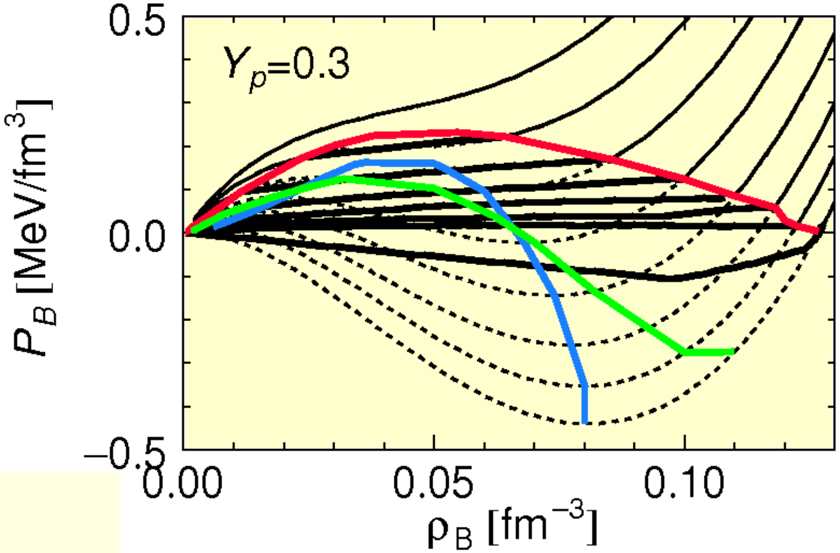
Instability of uniform matter at finite *T*

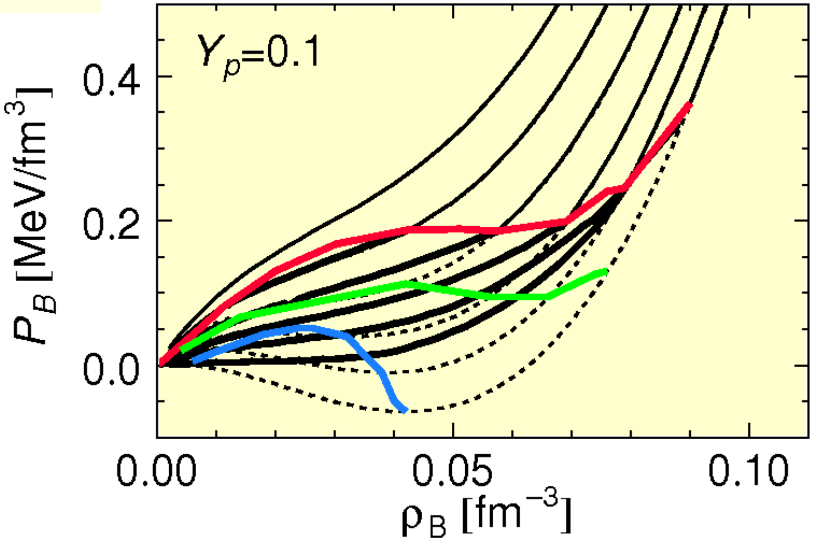


mechanical instability

formation of "pasta"

phase coexistence





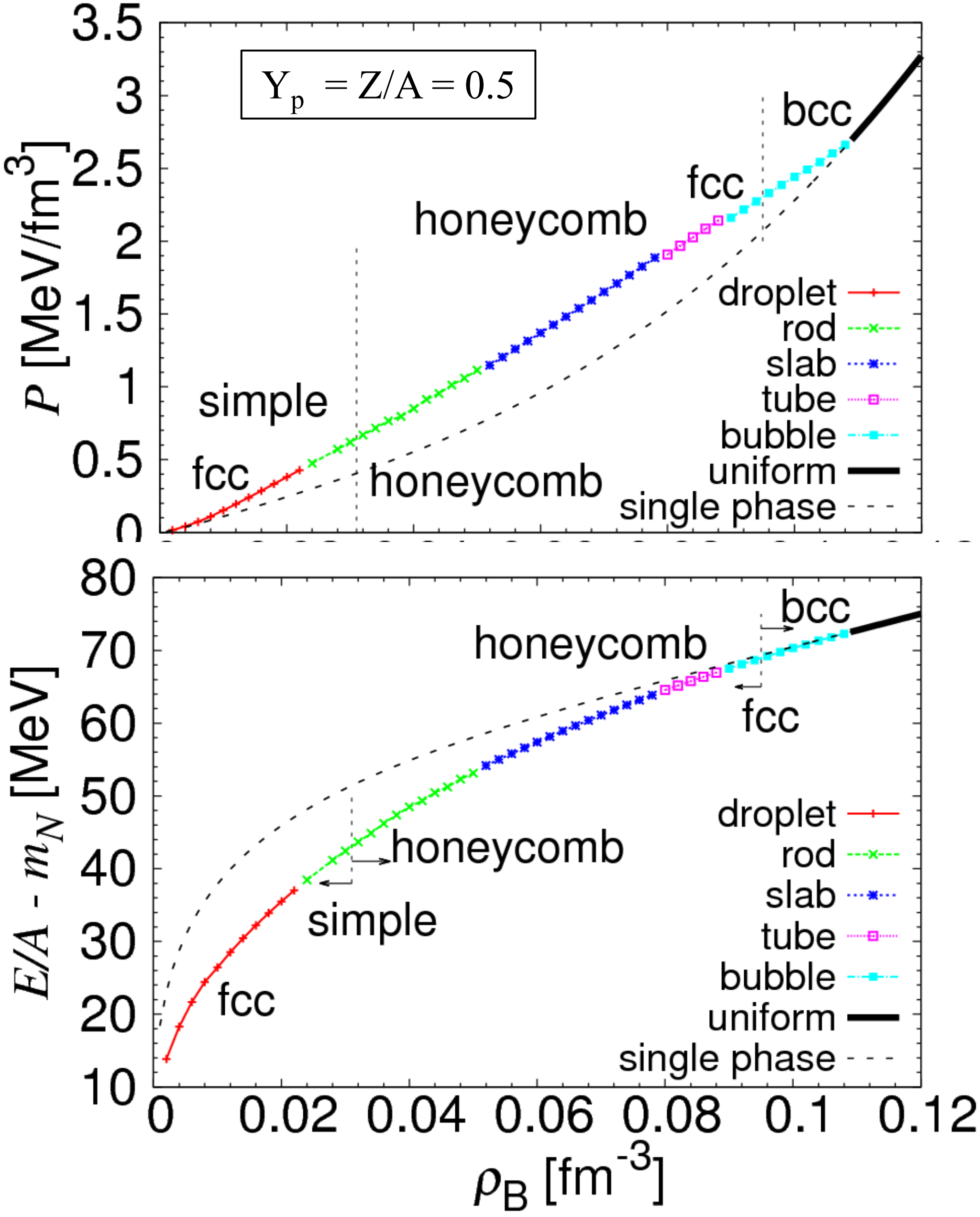
Due to the surface tension and the Coulomb interaction, the region of inhomogeneous matter is limited ("pasta" < coexistence).
But Mechanical instability is not crucial for pasta formation!

EOS has a similar behavior to that of the conventional studies.

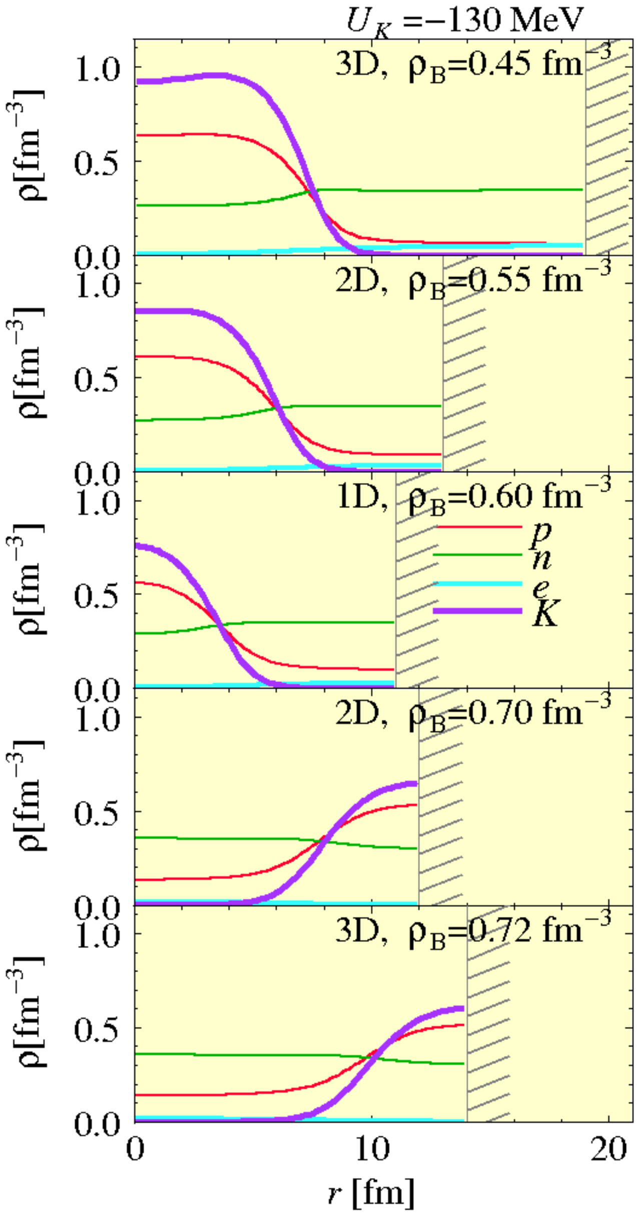
Novelty:

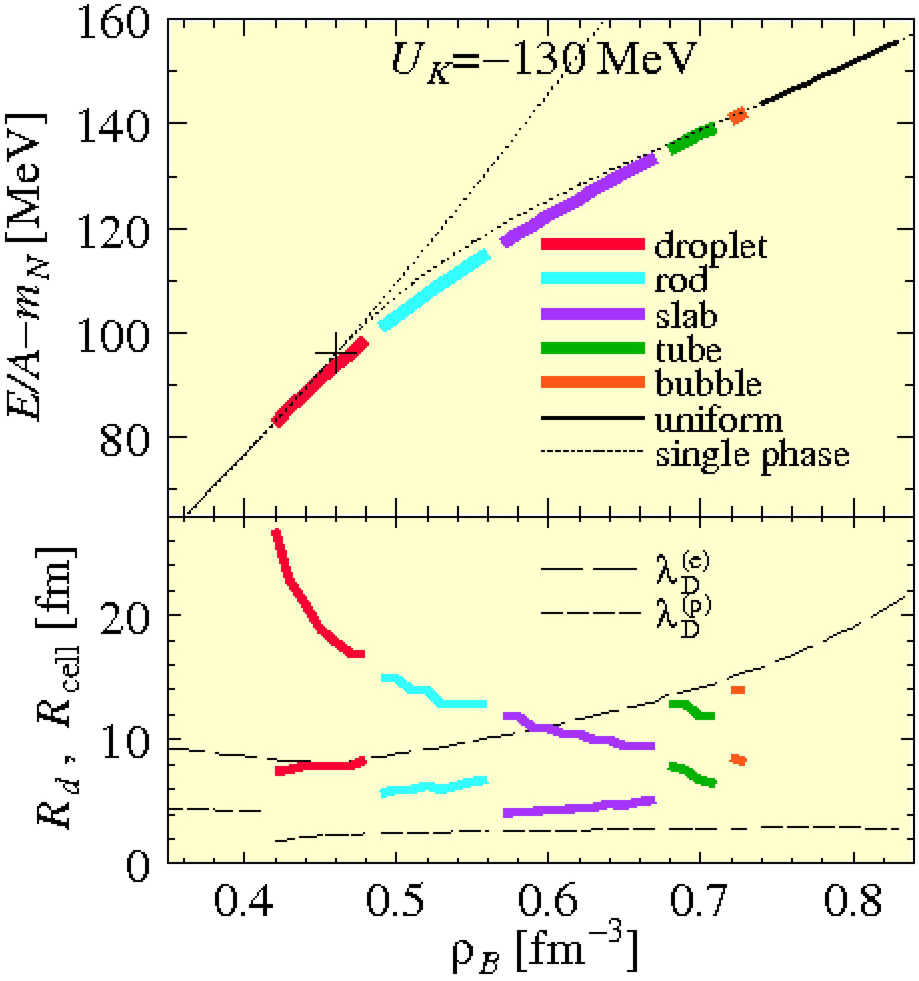
fcc lattice of droplets can be the ground state at some density.

← Not the Coulomb interaction among “point particles” but the change of the droplet size is relevant.



Kaonic pasta structure





ハイペロン抑制機構

ハドロン相、クォーク相それぞれでの荷電中性の条件が無いためにハイペロンが抑制される。

(バルクGibbs計算よりは局所中性に*近い*が、**厳密には中性でない。**)

中性物質 → $\rho_{th}=0.34\ \mathrm{fm}^{-3}$

電子や中性子を減らすためにはハイペロンが出た方が得。

荷電物質 → $\rho_{th}=1.15\ \mathrm{fm}^{-3}$

重いハイペロンは出なくても良い。

混合相は**負荷電のクォーク相**と**正荷電のハドロン相**とからなるためハイペロンが抑制される。