NUCLEAR FORCES: A REVIEW

The motivation and goals for this book have been discussed in detail in the preface. Part I of the book is on *Basic Nuclear Structure*, where Refs [N1, N2, N3, N4, N5, N6, N7] provide good backgound texts.[1] This first section is concerned with the essential properties of the nuclear force as described by phenomenological two-nucleon potentials. The discussion summarizes many years of extensive experimental and theoretical effort; it is meant to be a brief *review* and *summary*. It is assumed that the concepts, symbols, and manipulations in this first section are familiar to the reader.

Attractive. That the strong nuclear force is basically attractive is demonstrated in many ways: a bound state of two nucleons, the deuteron, exists in the spin triplet state with $(J^\pi, T) = (1^+, 0)$; interference with the known Coulomb interaction in p-p scattering demonstrates that the force is also attractive in the spin singlet $^1S_0$ state; and, after all, atomic nuclei are self-bound systems.

Short-Range. Nucleon-nucleon scattering is observed to be isotropic, or s-wave with $l = 0$, up to ≈ 10 MeV in the center-of-mass (C-M) system. The reduced mass is $1/\mu_{\rm red} = 1/m + 1/m = 2/m$. This allows one to make a simple estimate of the range of the nuclear force through the relations

$$\begin{aligned} \hbar l_{\rm max} &= rp \\ l_{\rm max} &= r\sqrt{\frac{2\mu_{\rm red}E}{\hbar^2}} \\ l_{\rm max} &\approx r({\rm Fermis})\sqrt{\frac{E}{40}\,{\rm MeV}} \end{aligned} \qquad (1.1)$$

Here we have used the numerical relations (worth remembering)

$$\begin{aligned} 1\,{\rm Fermi} &\equiv 1\,{\rm fm} \\ &\equiv 10^{-13}{\rm cm} \\ \frac{\hbar^2}{2m_p} &\approx 20.7\ {\rm MeV\ fm}^2 \end{aligned} \qquad (1.2)$$

[1]These books, in particular Ref. [N1], provide an extensive set of references to the original literature. It is impossible to include all the developments in nuclear structure in this part of the book. The references quoted in the text are only those directly relevant to the discussion.

A combination of these results indicates that the range of the nuclear force is

$$r \approx \text{few Fermis} \tag{1.3}$$

Spin-Dependent. The neutron-proton cross section σ_{np} is much too large at low energy to come from any reasonable potential fit to the properties of the deuteron alone

$$\begin{aligned} \sigma_{np} &= \frac{3}{4}(^3\sigma) + \frac{1}{4}(^1\sigma) \\ \sigma_{np} &= 20.4 \times 10^{-24}\text{cm}^2 \\ &\equiv 20.4 \text{ barns} \end{aligned} \tag{1.4}$$

At low energies, it is a result of effective range theory that the scattering measures only two parameters

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} r_0 k^2 \tag{1.5}$$

where a is the scattering length and r_0 is the effective range. The best current values for these quantities for np in the spin singlet and triplet states are (Ref. [N1])

$$\begin{aligned} ^1a &= -23.714 \pm 0.013 \text{ fm} & \quad ^3a &= 5.425 \pm 0.0014 \text{ fm} \\ ^1r_0 &= 2.73 \pm 0.03 \text{ fm} & \quad ^3r_0 &= 1.749 \pm 0.008 \text{ fm} \end{aligned} \tag{1.6}$$

The singlet state just fails to have a bound state ($a = -\infty$), while the triplet state has just one, the deuteron, bound by 2.225 MeV.

Noncentral. The fact that the deuteron has a nonvanishing quadrupole moment indicates that there must be some $l = 2$ mixed into the $l = 0$ ground state. Therefore the two-nucleon potential cannot be invariant under spatial rotations alone. The most general velocity-independent potential that is invariant under overall rotations and reflections is

$$\begin{aligned} V &= V_0(r) + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 V_1(r) + S_{12} V_T(r) \\ S_{12} &\equiv \frac{3(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{r})}{r^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \end{aligned} \tag{1.7}$$

The term $S_{12}V_T(r)$ gives rise to the tensor force. Several properties are of interest here:

1. Since

$$\begin{aligned} \mathbf{S} &= \frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \\ 4\mathbf{S}^2 &= 4S(S+1) = 6 + 2\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \end{aligned} \tag{1.8}$$

It follows that

Table 1.1: States of the two-nucleon system.

States	$^1S_0$	$^1P_1$	$^1D_2$	$^3S_1 + ^3D_1$	$^3P_0$	$^3P_1$	$^3P_2 + ^3F_2$	$^3D_2$
Parity	+	−	+	+	−	−	−	+
Particle exchange	−	+	−	+	−	−	−	+
Particles	nn np pp	np	nn np pp	np^a	nn np pp	nn np pp	nn np pp	np

[a] The deuteron.

$$\begin{aligned} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 &= -3\,; & \text{singlet } (S=0) \\ &= +1\,; & \text{triplet } (S=1) \end{aligned} \tag{1.9}$$

2. The total spin S is a good quantum number for the two-nucleon system if the hamiltonian H is symmetric under interchange of particle spins [as in Eq. (1.7)], for then the wave function must be either symmetric ($S = 1$) or antisymmetric ($S = 0$) under this symmetry.[2]

3. Higher powers of the spin operators can be reduced to the form in Eq. (1.7) for spin-1/2 particles.

4. Since the total spin operator annihilates the singlet state, $(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)^1\chi = 0$, so does the tensor operator S_{12}

$$S_{12}[^1\chi] = 0 \tag{1.10}$$

Charge Independent. Charge independence states that the force between any two nucleons is the same $V_{pp} = V_{pn} = V_{nn}$ in the same state. The Pauli principle limits the states that are available to two identical nucleons. For two spin-1/2 nucleons, a complete basis can be characterized by eight quantum numbers, for clearly the states $|\mathbf{p}_1, s_1; \mathbf{p}_2, s_2\rangle$ form such a basis. Alternatively, one can take as the good quantum numbers $|E, J, M_J, S, \pi, \mathbf{P}_{\text{CM}}\rangle$. Table 1.1 lists the first few states available to the two-nucleon system. The Pauli principle states that nn and pp must go into an overall antisymmetric state.[3] Charge independence states that the forces are equal in those states where one can have all three types of particles including np; the nuclear force is independent of the charge in these states. At low energy, the cross sections are given in terms of the singlet

[2] If P_σ is the spin exchange operator then $P_\sigma[^1\chi(1,2)] \equiv {}^1\chi(2,1) = -{}^1\chi(1,2)$ is odd and, similarly, $P_\sigma[^3\chi(1,2)] = +^3\chi(1,2)$ is even. Thus from Eqs. (1.9) $P_\sigma = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2$.

[3] In terms of isospin we assign $T = 0$ to the states that are even under particle interchange and $T = 1$ to those that are odd, so that the overall wave function is antisymmetric.

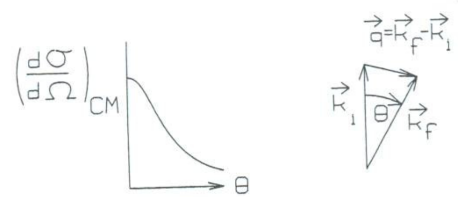

Figure 1.1: Sketch of cross section in Born approximation.

and triplet amplitudes by

$$\left(\frac{d\sigma}{d\Omega}\right)_{np} = \frac{1}{4}|f(^1S_0)|^2 + \frac{3}{4}|f(^3S_1)|^2$$
$$\left(\frac{d\sigma}{d\Omega}\right)_{nn} = \frac{1}{4} \times 4|f(^1S_0)|^2 = |f(^1S_0)|^2 \tag{1.11}$$

Exchange Character. At higher energies more partial waves contribute to the cross section. At high enough energies, one can use the Born approximation

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm CM} = \left|\frac{2\mu_{\rm red}}{4\pi\hbar^2}\int e^{-i\mathbf{q}\cdot\mathbf{r}}V(r)d^3r\right|^2 \tag{1.12}$$

where the momentum transfer $\mathbf{q}$ is defined in Fig. 1.1. For large $\mathbf{q}$ the integrand oscillates rapidly and the integral goes to zero as sketched in Fig. 1.1. The experimental results for np scattering are shown in Fig. 1.2. There is significant backscattering, in fact, the cross section is approximately symmetric about 90°. If $f(\pi-\theta) = f(\theta)$ then only even l partial waves contribute to the cross section; the odd l's will distort $d\sigma/d\Omega$.

To describe this situation one introduces the concept of an exchange force — a force that depends on the symmetry of the wave function. The interaction is written $V(r)P_{\rm M}$ where the Majorana space exchange operator is defined by[4]

$$P_{\rm M}\phi(\mathbf{r_2},\mathbf{r_1}) \equiv \phi(\mathbf{r_1},\mathbf{r_2}) \tag{1.13}$$

Hence since $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$

$$P_{\rm M}\phi(\mathbf{r}) = \phi(-\mathbf{r})$$
$$P_{\rm M}Y_{lm}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right) = (-1)^l Y_{lm}\left(\frac{\mathbf{r}}{|\mathbf{r}|}\right) \tag{1.14}$$

The odd l in the amplitude can evidently be eliminated with a Serber force defined by

$$V \equiv V(r)\frac{1}{2}(1 + P_{\rm M}) \tag{1.15}$$

[4]Since the overall wave function is antisymmetric $P_{\rm M}P_\sigma P_\tau = -1$ (Note $P_\sigma^2 = P_\tau^2 = +1$). Thus $P_{\rm M} = -P_\sigma P_\tau = -(1 + \vec{\sigma}_1\cdot\vec{\sigma}_2)(1 + \vec{\tau}_1\cdot\vec{\tau}_2)/4$ provides an alternate definition.

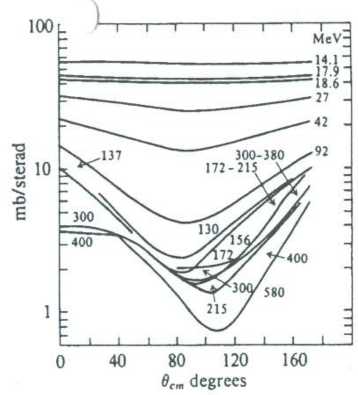

Figure 1.2: The n–p differential cross section in C-M system as a function of laboratory energy. From Ref. [N1].

The differential cross section in Born approximation with this interaction is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm CM} = \left|\frac{2\mu_{\rm red}}{4\pi\hbar^2}\int e^{-i\mathbf{k}_f\cdot\mathbf{r}}V(r)\frac{1}{2}(1+P_{\rm M})e^{i\mathbf{k}_i\cdot\mathbf{r}}d^3r\right|^2$$
$$= \left|\frac{2\mu_{\rm red}}{4\pi\hbar^2}\int e^{-i\mathbf{k}_f\cdot\mathbf{r}}V(r)\frac{1}{2}(e^{i\mathbf{k}_i\cdot\mathbf{r}} + e^{-i\mathbf{k}_i\cdot\mathbf{r}})d^3r\right|^2 \tag{1.16}$$

This result is sketched in Fig. 1.3. The nuclear force has roughly a Serber exchange nature; it is very weak in the odd-l states.

Hard Core. The pp cross section is illustrated in Fig. 1.4. Recall that since the particles are here identical, one necessarily has the relation $[d\sigma(\pi-\theta)/d\Omega]_{\rm CM} = [d\sigma(\theta)/d\Omega]_{\rm CM}$. Although the cross sections shown in Figs. 1.2 and 1.4 are very different, it is possible to make a charge-independent analysis of np and pp scattering as first shown in detail by Breit and coworkers (Ref. [N8]). The overall magnitude of the pp cross section indicates that more than s-wave

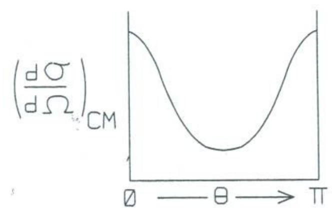

Figure 1.3: Sketch of cross section in Born approximation with a Serber force.

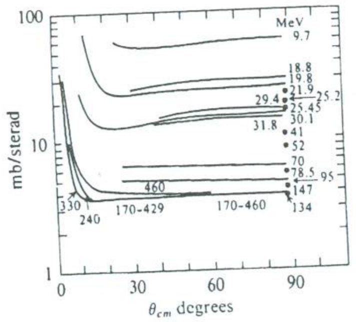

Figure 1.4: Same as Fig. 1.2 for p-p scattering. From Ref. [N1].

nuclear scattering must be included (recall the unitarity bound of π/k^2), and the higher partial waves must interfere so as to give the observed flat angular distribution beyond the Coulomb peak. A hard core will change the sign of the s-wave phase shifts at high energy and allow the ${}^1S - {}^1D$ interference term in pp scattering to yield a uniform angular distribution as first demonstrated by Jastrow (Ref. [N9]); with a Serber force, it is only the states (${}^1S_0$, ${}^1D_2$) in Table 1.1 that contribute to nuclear pp scattering. Recall that for a pure hard core potential the s-wave phase shift is negative $\delta_0 = -ka$ as illustrated in Fig. 1.5. With a finite attractive well outside of the hard core, one again expects to see the negative phase shift arising from the hard core at high enough energy. The experimental situation for the s-wave phase shifts in both pp and np scattering is sketched in Fig. 1.6. From an analysis of the data, one concludes that there is a hard core of radius

$$r_c \approx 0.4 \text{ to } 0.5 \text{ fm} \qquad (1.17)$$

in the relative coordinate in the nucleon-nucleon interaction.

Spin-Orbit Force. It is difficult to explain the large nucleon polarizations observed perpendicular to the plane of scattering with just the central and

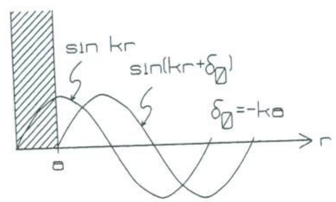

Figure 1.5: The s-wave phase shift for scattering from a hard-core potential.

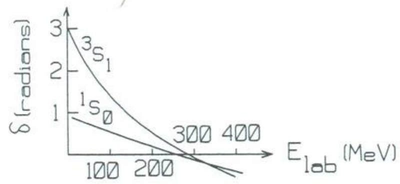

Figure 1.6: Sketch of s-wave nucleon-nucleon phase shifts. After Ref. [N1].

tensor forces discussed above. To explain the data one must also include a spin-orbit potential of the form

$$\begin{aligned} V &= -V_{SO}\mathbf{L}\cdot\mathbf{S} \\ \mathbf{L}\cdot\mathbf{S} &= \frac{1}{2}[J(J+1) - l(l+1) - S(S+1)] \end{aligned} \qquad (1.18)$$

This last expression vanishes if either $S = 0$ ($l = J$) or $l = 0$ ($S = J$). The spin-orbit force vanishes in s-states and is empirically observed to have a short range; thus it is only effective at higher energies.

In *summary*, the present situation with respect to our phenomenological knowledge of the nucleon-nucleon force is the following:

- The experimental scattering data can be fit up to laboratory energies of $\approx$ 300 MeV with a set of potentials depending on spins and parities ${}^1V_C^+$, ${}^3V_C^+$, ${}^1V_C^-$, ${}^3V_C^-$, ${}^3V_T^+$, ${}^3V_T^-$, etc.
- The potentials contain a hard core[5] with $r_c \approx 0.4$ to 0.5 fm.
- The forces in the odd-l states are relatively weak at low energies, and on the average slightly repulsive.
- The tensor force is necessary to understand the quadrupole moment of the deuteron (and its binding).
- A strong, short-range, spin-orbit force is necessary to explain the polarization at high energy.

Commonly used nucleon-nucleon potentials include the "Bonn potential" in Ref. [N6], the "Paris potential" in Ref. [N10], and the "Reid potential" in Ref. [N11]. The first two contain the one-meson (boson) exchange potentials (OBEP) at large distances.

Meson Theory of Nuclear Forces. The exchange of a neutral scalar meson of Compton wavelength $1/m \equiv \hbar/mc$ (Fig. 1.7) in the limit of infinitely heavy sources gives rise to the celebrated Yukawa potential (Ref. [N12])

[5] Or, more generally, a strong, short-range repulsion.

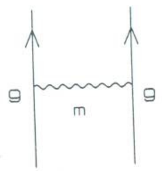

Figure 1.7: Contribution of neutral scalar meson exchange to the N–N interaction.

$$V(r) = -\frac{g^2}{4\pi c^2}\frac{e^{-mr}}{r} \tag{1.19}$$

A derivation of this result, as well as the potentials arising from other types of meson exchange, is given in Appendix A. In charge-independent pseudoscalar meson theory with a nonrelativistic coupling of $\boldsymbol{\tau}(\boldsymbol{\sigma}\cdot\nabla)$ at each vertex, one obtains a tensor force of the correct sign in the N–N interaction. In fact, for this reason, Pauli (Ref. [N13]) claimed there had to be a long-range pseudoscalar meson exchange before the π-meson was discovered. Since the π is the lightest known meson, the 1-π exchange potential is exact at large distances $r \to \infty$; mesons with higher mass $\bar{m}$ give a potential that goes as $e^{-\bar{m}r}/r$ by the uncertainty principle. The existence of this 1-π exchange tail in the N–N interaction has by now been verified experimentally in many ways.

The Paris and Bonn potentials in Refs. [N10, N6] include the exchange of $(\pi, \sigma, \rho, \omega)$ mesons with spin and isospin $(J^\pi, T) = (0^-,1), (0^+,0), (1^-,1), (1^-,0)$, respectively, in the long-range part of the N–N potential. The short-distance behavior of the interaction is then parameterized.

One can get a qualitative understanding of the short-range repulsion and spin-orbit force in the strong N–N interaction by considering meson exchange and using the analogy with quantum electrodynamics (QED). Suppose one couples a neutral vector meson field, the ω, to the conserved baryon current. Then just as with the Coulomb interaction in atomic physics, which is described by the coupling of a neutral vector meson field (the photon) to the conserved electromagnetic current:

1. Like baryonic charges repel;
2. Unlike baryonic charges (e.g., p-$\bar{p}$) attract;
3. There will be a spin-orbit force;
4. While the range of the Coulomb potential $1/r$ is infinite because the mass of the photon vanishes $m_\gamma = 0$, the range of the strong nuclear effects will be $\sim \hbar/m_\omega c$. Since the ω has a large mass, the force will be shortrange.

2

NUCLEAR MATTER

Nuclear Radii and Charge Distributions. The best information we have about nuclear charge distributions comes from electron scattering, where one uses short-wavelength electrons to explore the structure (Ref. [N14]). In the work of Hofstadter and colleagues at Stanford (Ref. [N15]) a phaseshift analysis was made of elastic electron scattering from an arbitrary charge distribution through the Coulomb interaction. The best fit to the data, on the average, was found with the following shape

$$\rho = \frac{\rho_0}{1+e^{(r-R)/a}} \tag{2.1}$$

This is illustrated in Fig. 2.1. Several features of the empirical results are worthy of note:

1. $(A/Z)\rho_0$, the central nuclear density, is observed to be *constant* from nucleus to nucleus.

2. The radius to 1/2 the maximum ρ is observed to vary with nucleon number A according to[6]

$$R = r_0 A^{1/3} \tag{2.2}$$

where the half-density radius parameter r_0 is given by

$$r_0 \approx 1.07 \text{ fm} \tag{2.3}$$

We assume that the neutron density tracks the proton density and that the neutrons are confined to the same nuclear volume.[7] This means that the nuclear density A/V is given by

$$\begin{aligned}\frac{A}{V} &= \frac{3}{4\pi r_0^3} \\ &\approx 1.95\times 10^{38}\ \text{particles/cm}^3\end{aligned} \tag{2.4}$$

One thus concludes *nuclear matter has a constant density* from nucleus to nucleus.

[6] The nucleon number A is identical to the baryon number B, which, to the best of our current experimental knowledge, is an exactly conserved quantity; the notation will be used interchangeably throughout this book.

[7] This assumption is verified quite well in nucleon-nucleus scattering.