

Introduction to Nuclear Reactions

2.1 INTRODUCTION

When we wish to observe an object, we usually illuminate it with a beam of light. The light is then reflected, refracted, diffracted, absorbed, in various ways. By interpreting our measurements on the scattered light, we learn about the shape and size of the object and its other properties. For example, we may learn whether the object is transparent or opaque; if the former, we may measure its refractive index, if the latter, we can find its absorptivity, and so on. If we study further by using monochromatic light, we find these various properties vary with the wavelength (in particular, we may find the object is coloured; i.e. it preferentially scatters light of a particular wavelength). In some cases we may find more than just scattering or absorption of the light. The object may continue to re-emit light after the incident beam is removed (phosphorescence), it may emit radiation of a different wavelength from that used to illuminate it (fluorescence) or it may emit radiation of a different kind (e.g. electrons in the photo effect).

So it is with nuclear reactions. The light we use to illuminate a nucleus may also be electromagnetic radiation. More usually, it consists of the matter waves associated with a beam of energetic particles such as electrons, mesons, hyperons, neutrons, protons or other nuclei. These waves may also be reflected, refracted, diffracted and absorbed. The various processes which can occur are investigated by making scattering experiments.

If we wish to see the details of an object, it is necessary to illuminate it with radiation of wavelength λ which is shorter than the size of the object. Thus an optical microscope operating with visible light ($\lambda \sim 10^{-7}$ m) is useless for studying objects much smaller than about $1 \mu\text{m}$ (10^{-6} m). The de Broglie wavelength of the matter waves associated with a beam of particles with momentum p is $\lambda = h/p$. Electrons with a kinetic energy of $E = 1$ keV have $\lambda = h/(2m_e E)^{1/2} \sim 0.3 \times 10^{-10}$ m, so an electron microscope operating with 1-keV electrons can

see much finer details than an optical microscope and can even resolve features of atomic or molecular dimensions. Nuclei have radii of about 5 fm and particles like neutrons and protons are about 10 times smaller; thus it is necessary to use radiation with a very short wavelength and hence a high energy to study details of nuclear or particle structure. A photon with a reduced wavelength $\bar{\lambda} = \lambda/2\pi = 1$ fm has an energy of 197 MeV. The reduced de Broglie wavelength for a massive particle at non-relativistic energies is $\bar{\lambda} \approx 4.5/[m(\text{amu})E(\text{MeV})]^{1/2}$ fm, where m is its mass and E its kinetic energy, so that a proton with $\bar{\lambda} = 1$ fm has an energy of about 20 MeV. Table 2.1 lists values of $\bar{\lambda}$ for several particles commonly used in scattering experiments.* A heavy particle has a smaller de Broglie wavelength than a light particle with the same kinetic energy E provided it is non-relativistic ($E \ll mc^2$, the rest energy). At very relativistic energies ($E \gg mc^2$) the particle momentum $p \approx E/c$ and the corresponding wavelength is more or less independent of the particle's rest mass, $\bar{\lambda} \approx 197/E(\text{MeV})$ fm.

TABLE 2.1 Reduced de Broglie wavelengths $\bar{\lambda}$, in fm, for various particles and energies

Energy	Photon	Electron	Pion	Proton	α -Particles	^{16}O	^{40}Ar	^{208}Pb
1 MeV	197	140	12	4.5	2.3	1.14	0.72	0.32
10 MeV	19.7	18.7	3.7	1.4	0.72	0.36	0.23	0.10
100 MeV	2.0	2.0	1.0	0.45	0.23	0.11	0.072	0.032
1 GeV	0.20	0.20	0.17	0.12	0.068	0.035	0.023	0.010

If the wavelength of the incident radiation is very short (and hence the energy or mass or both is very high), the correspondence principle of wave mechanics tells us that we may describe the collision in the particle language of classical mechanics. This classical description is often useful because of the physical insight it provides. However, it is well to remember that over the energy domain of most interest for nuclear physics, 0–1000 MeV say, quantal or wave effects are generally very important and quantum mechanics is required for a quantitative description of the phenomena.

2.2 THE CENTRE-OF-MASS COORDINATE SYSTEM

In the usual nuclear-reaction experiment we have target nuclei A at rest being bombarded by projectiles a . Usually, but not necessarily, the target A is heavier than the projectile a . Anyway, the same reactions could be induced by firing nuclei A at a target containing particles a . Hence it is convenient to use a reference frame which reflects this inherent symmetry of the scattering process. One

*Note that neutrons with the very low energy of, say, 1/40 eV, which is typical of the energies of the thermal neutrons obtainable from many nuclear reactors, have a wavelength $\bar{\lambda} \approx 0.3 \times 10^{-10}$ m = 0.3 Å. Hence diffraction will occur on an atomic scale, such as Bragg reflection by crystals. Indeed, such slow neutrons are an important tool for the study of the structure of materials.

quantity we know to be conserved in a collision between two systems is their *total* momentum; our symmetry requirement is then satisfied by choosing a reference system in which this total momentum is zero. Referred to a coordinate system fixed in the laboratory (the LAB system) this means a system moving with the same velocity V_{CM} as the centre of mass of the colliding pair (hence called the CMS or centre-of-mass system). The kinetic energy, $\frac{1}{2}(M_A + M_a)V_{CM}^2$, associated with this centre-of-mass motion in the laboratory system is conserved and thus is not available for producing nuclear excitations, etc. In the CMS, this energy has been transformed away, the centre of mass is at rest and the two nuclei approach one another with equal and opposite momenta, $p_A^{CM} = -p_a^{CM}$ (see Figure 2.1). Similarly, if there are only two products, b and B, of a reaction, these separate in the CMS with equal and opposite momenta, $p_b^{CM} = -p_B^{CM}$. The kinematic consequences of this transformation from the LAB to the CM system are described in Appendix B. If the target A is much heavier than the projectile a, there is little difference between the two systems but for light targets the differences may be considerable. For example if a and A have equal masses, V_{CM} is one-half the velocity of the projectile in the LAB and one-half the bombarding energy is taken up in motion of the centre of mass. Further, the way the products are distributed in angle will be different in the two systems. In the elastic scatter-

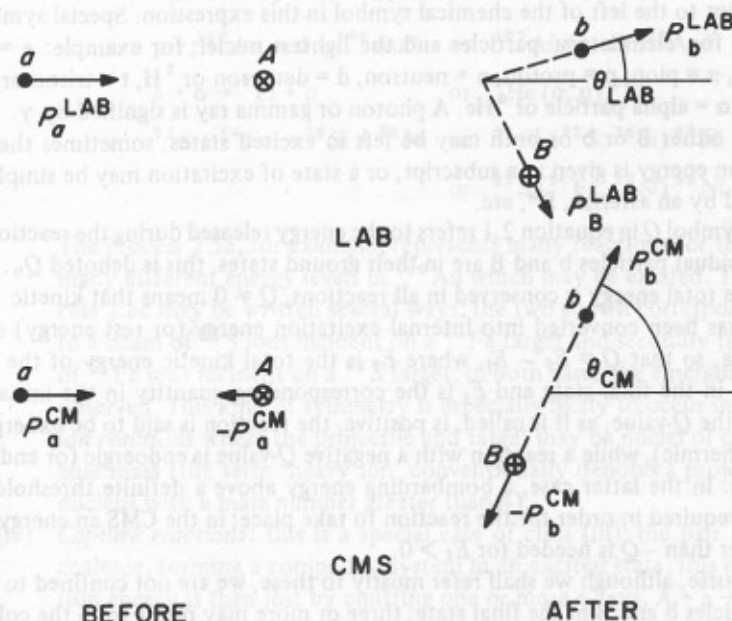


Figure 2.1 LAB and CMS coordinates for the reaction $A(a, b)B$. In the LAB system, the target A is at rest and the projectile a is incident with momentum p_a^{LAB} . The products b and B are shown moving forward. In the CMS, a and A approach one another with equal but opposite momenta and the products b and B separate with equal but opposite momenta

ing of two equal masses, conservation of momentum and energy ensures that the scattering angle θ_{LAB} (Figure 2.1) can never exceed 90° , whereas in the CMS all angles θ_{CM} up to 180° are possible. Indeed in the special case where the scattering intensity was proportional to $\cos \theta_{\text{LAB}}$ in the LAB system we should find it to be isotropic, that is of equal intensity at all angles θ_{CM} , in the CMS (see Appendix B).

The CMS can also be used for scattering experiments at relativistic energies although it is often more convenient to use relativistic invariants, such as the square of the four-momentum transfer, which have the same value in all co-ordinate systems.

2.3 TYPES OF REACTION

Many different processes may take place when two particles collide. A typical nuclear reaction may be written



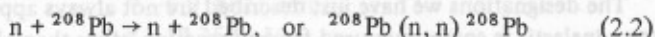
If A is the symbol for the target nucleus, a that for the projectile, while B is the residual nucleus and particle b is observed, this reaction would be written $A(a, b)B$. When specific isotopes are intended, the mass number is written as a superscript to the left of the chemical symbol in this expression. Special symbols are used for 'elementary' particles and the lightest nuclei; for example: e = electron, π = pion, p = proton, n = neutron, d = deuteron or ${}^2\text{H}$, t = triton or ${}^3\text{H}$, and α = alpha particle or ${}^4\text{He}$. A photon or gamma ray is signified by γ . Further, either B or b or both may be left in excited states; sometimes the excitation energy is given as a subscript, or a state of excitation may be simply indicated by an asterisk, B^* , etc.

The symbol Q in equation 2.1 refers to the energy released during the reaction; if the residual particles b and B are in their ground states, this is denoted Q_0 . Since the total energy is conserved in all reactions, $Q \neq 0$ means that kinetic energy has been converted into internal excitation energy (or rest energy) or vice-versa, so that $Q = E_f - E_i$, where E_f is the total kinetic energy of the particles in the final state and E_i is the corresponding quantity in the initial state. If the Q -value, as it is called, is positive, the reaction is said to be exoergic (or exothermic), while a reaction with a negative Q -value is endoergic (or endothermic). In the latter case, a bombarding energy above a definite threshold value is required in order for the reaction to take place; in the CMS an energy E_i greater than $-Q$ is needed for $E_f > 0$.

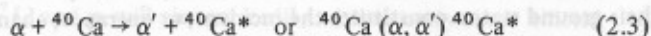
Of course, although we shall refer mostly to these, we are not confined to just two particles b and B in the final state; three or more may result from the collision. If sufficient energy is available, it is possible for the two colliding system to be broken up entirely into their constituents, although such an event is rather unlikely. When there is an appreciable number of reaction products, the collision is often called a spallation reaction.

There are several major classes of reactions.

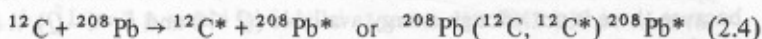
- (i) *Elastic scattering*: here $b = a$ and $B = A$. The internal states are unchanged so that $Q = 0$ and the kinetic energy in the CMS is the same before and after the scattering. We have $a + A \rightarrow a + A$; for example



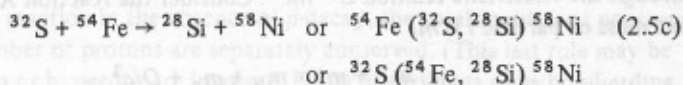
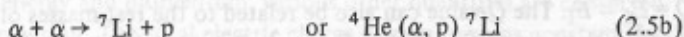
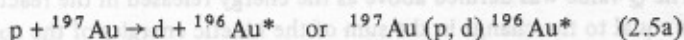
- (ii) *Inelastic scattering*: this term most often means collisions in which $b = a$ but A has been raised to an excited state, $B = A^*$ say, consequently $Q = -E_x$, where E_x is the excitation energy of this state. Since a is then emitted with reduced energy, it is commonly written a' , $A + a \rightarrow A^* + a' - E_x$; for example



If a is itself a complex nucleus, it may be left in an excited state instead of the target, or both may be excited through a *mutual excitation* process. An example of the latter is

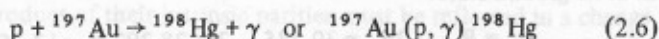


- (iii) *Rearrangement collision or reaction*: here $b \neq a$ and $B \neq A$ so that there has been some rearrangement of the constituent nucleons between the colliding pair (a transmutation). There are many possibilities; $A + a \rightarrow B_1 + b_1 + Q_1$ or $\rightarrow B_2 + b_2 + Q_2$, etc. Some examples are:



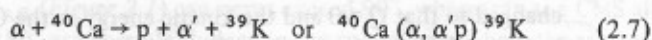
The process 2.5a, for example, represents many reactions for there are many different energy levels in ${}^{196}\text{Au}$ which may be excited. The process 2.5c may be written several ways; the two shown correspond firstly to a beam of ${}^{32}\text{S}$ ions incident on a ${}^{54}\text{Fe}$ target and secondly to a beam of ${}^{54}\text{Fe}$ ions incident on a ${}^{32}\text{S}$ target; in both cases the emerging ${}^{28}\text{Si}$ is observed. This kind of symmetry is especially likely to occur in *heavy-ion reactions* where the projectile and target may be nuclei of comparable mass (the term 'heavy ion' conventionally denotes a projectile nucleus with a mass number greater than 4).

- (iv) *Capture reactions*: this is a special case of class (iii); the pair $A + a$ coalesce, forming a compound system in an excited state; this excitation energy is then lost by emitting one or more γ -rays, $A + a \rightarrow C + \gamma + Q$. For example



- (v) *Other reactions*: we mean here reactions not included in equation 2.1 because there are more than two particles in the final state, such as

$A + a \rightarrow B + b + c + Q$. For example



The designations we have just described are not always applied rigidly. The term inelastic is sometimes used for any reaction other than elastic scattering; these processes may also be referred to as non-elastic. The name nuclear reaction may also be applied to any scattering process involving nuclear particles.

Each possible combination of particles may be called a *partition*. Each partition is further distinguished by the state of excitation of each nucleus and each such pair of states may be called a *channel*. The initial partition $A + a$, both in their ground states, constitutes the incident, or entrance, channel; the various possible sets of products in their various possible energy states become the exit channels. Thus there is an inelastic channel like equation 2.3 for each excited state of the target A and a reaction channel like equation 2.5 for each pair of excited states of the residual nuclei B and b . If a channel cannot be reached because there is not enough energy available ($Q < 0$ and $E_i < -Q$), it is said to be *closed*. *Open channels* are those which are energetically available.

2.4 ENERGY AND MASS BALANCE

The Q -value was defined above as the energy released in the reaction. Hence it is equal to the change in the sum of the kinetic energies of the colliding particles, $Q = E_f - E_i$. The Q -value can also be related to the rest masses of the particles through the relativistic relation $E = mc^2$. Consider the reaction $A(a, b)B$. If the rest mass of particle i is m_i

$$m_A + m_a = m_B + m_b + Q/c^2 \quad (2.8)$$

Alternatively, Q is equal to the change in the binding energies B_i of the particles

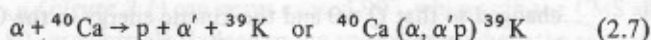
$$B_A + B_a = B_B + B_b - Q \quad (2.9)$$

(If i is an elementary particle, such as a nucleon, we regard $B_i = 0$ in this equation. Also Q appears here with a sign opposite of that in equation 2.8 because of the convention that binding energies are positive—see Chapter 1.) Hence an exoergic (positive Q) reaction results in systems more tightly bound than in the entrance channel, and an endoergic (negative Q) reaction results in less tightly bound systems.

The Q -value can be deduced from tables of masses or binding energies (e.g. Mattauch *et al.*, 1965); often, a measurement of the Q -value of a reaction is used to give the mass or binding energy of one of the particles if the others involved are known. As an example, consider the reaction 2.5b, $\alpha + \alpha \rightarrow {}^7\text{Li} + \text{p}$, leaving ${}^7\text{Li}$ in its ground state:

$$Q_0 = B_{\text{Li}} - 2B_\alpha = 39.245 - 2 \times 28.297 = -17.35 \text{ MeV}$$

$A + a \rightarrow B + b + c + Q$. For example



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$$Q_0 = B_{\text{Li}} - 2B_\alpha = 39.245 - 2 \times 28.297 = -17.35 \text{ MeV}$$

Alternatively,

$$\begin{aligned} Q_0 &= 2m_\alpha - m_{\text{Li}} - m_p = 2 \times 4.001506 - 7.014357 - 1.007276 \\ &= -0.018621 \text{ amu} \\ &= -17.35 \text{ MeV} \end{aligned}$$

This is an endoergic reaction and will not be initiated unless the sum of the kinetic energies of the two α -particles is greater than 17.35 MeV in the CMS. (It is not sufficient to have a ${}^4\text{He}$ target at rest bombarded with 17.35-MeV α -particles in the LAB system. Since the total momentum must be conserved, the two residual nuclei, ${}^7\text{Li}$ and p , will always recoil and thus carry some kinetic energy. In the CMS the total momentum is zero by definition and all the kinetic energy is available for excitation. In this simple case of two identical particles the CMS energy is exactly one-half the LAB bombarding energy; thus a LAB energy of at least 34.7 MeV is required for this reaction. At the threshold energy of 34.7 MeV, the ${}^7\text{Li}$ and p would be formed at rest in the CMS.) If we wish to leave ${}^7\text{Li}$ in an excited state, the Q -value will be even more negative and additional energy will have to be supplied.

2.5 OTHER CONSERVED QUANTITIES

Besides the total energy and momentum, various other quantities are conserved during a nuclear reaction. The total electric charge always remains constant. In the usual nuclear reaction in the absence of β -decay, the total number of neutrons and the total number of protons are separately conserved. (This last rule may be violated if mesons or hyperons are involved, either as products or as bombarding particles, for then nucleons may be transmuted into other particles or into each other. For example, the capture of μ -mesons by nuclei, which is analogous to the inverse of β -decay, may transmute a neutron into a proton or vice-versa. The capture of a π^- -meson by a proton may produce a K -meson and a hyperon or a π^0 and a neutron. If the bombarding energy in a reaction is sufficiently high, mesons or hyperons may be produced. For example, the threshold for charged pion production is about 140 MeV in the CMS. An energy of about 2 GeV enables one to create a nucleon-antinucleon pair. However, in all these processes another quantum number, the total *baryon* number, remains constant.)

Two other important conserved quantities are *parity* and the total *angular* momentum. Any change in the total (vector sum) of the internal angular momenta (spins) of the nuclei must be compensated for by a corresponding change in the orbital angular momentum of their relative motion. Similarly, any change in the product of their intrinsic parities must be reflected in a change in the parity of their relative motion. We return to these matters in Chapter 3.

2.6 CROSS-SECTIONS

We need a quantitative measure of the probability that a given nuclear reaction will take place. For this we introduce the concept of a *cross-section* which we define in the following way. Consider a typical reaction $A(a, b)B$. If there is a flux of I_0 particles of type a per unit area incident on a target containing N nuclei of type A , then the number of particles b emitted per unit time is clearly proportional to both I_0 and N . The constant of proportionality is the cross-section, σ , and has the dimensions of area. Then the cross-section for this particular reaction will be

$$\sigma = \frac{\text{number of particles } b \text{ emitted}}{(\text{number of particles } a \text{ incident/unit area}) (\text{number of target nuclei within the beam})}$$

A convenient unit of area for nuclear physics is the *barn* (symbol b : $1 \text{ barn} = 10^{-28} \text{ m}^2 = 100 \text{ fm}^2$) and cross-sections are usually given in barns or the sub-units millibarn, $1 \text{ mb} = 10^{-3} b$, and microbarn, $1 \mu b = 10^{-6} b$, etc.

If we ask for the number of particles b emitted per unit time within an element of solid angle $d\Omega$ in the direction with polar angles (θ, ϕ) with respect to the incident beam, clearly this is proportional to $d\Omega$ as well as I_0 and N (see Figure 2.2). The constant of proportionality in this case is the *differential* cross-section, $d\sigma/d\Omega$. Since solid angles are dimensionless, this also has the dimensions of area.

In general, the probability of emission of b , and hence the differential cross-section, will depend upon the angles θ and ϕ . Only in special cases will the angular distribution be isotropic. To emphasise this, the differential cross-section $d\sigma/d\Omega$ is sometimes written as $d\sigma(\theta, \phi)/d\Omega$. However (unless the spins of one or more of the particles are polarised—see section 2.13), the scattering process is quite symmetrical about the direction of the incident beam which means that the differential cross-section cannot depend on the azimuthal angle ϕ . In that case, we may write it simply as $d\sigma(\theta)/d\Omega$.

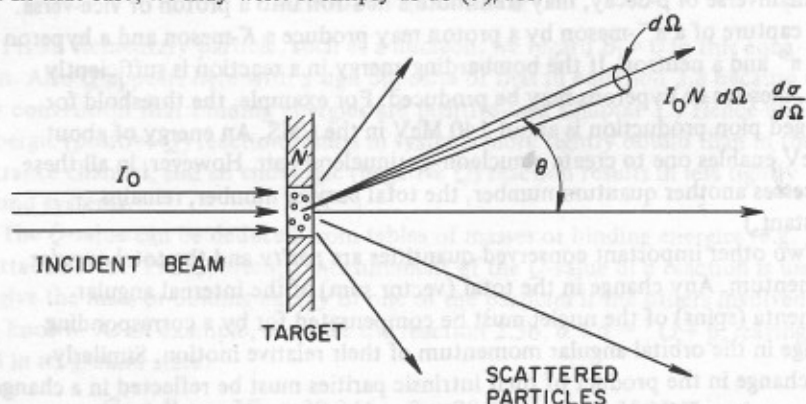


Figure 2.2 Diagram for the definition of differential cross-section. Usually the size of the irradiated area of the target is small, very much smaller than the distance to the detector, so that the scattering angle θ is well defined

Clearly the two kinds of cross-sections are related by

$$\sigma = \int_0^{4\pi} (d\sigma/d\Omega) d\Omega \quad (2.10)$$

or, since the solid angle $d\Omega = \sin \theta d\theta d\phi$

$$\sigma = \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi (d\sigma/d\Omega)$$

If there is no spin polarisation so that $d\sigma/d\Omega$ is independent of ϕ , this becomes

$$\sigma = 2\pi \int_0^\pi (d\sigma/d\Omega) \sin \theta d\theta$$

Cross-sections are measures of probabilities. At a given bombarding energy, we may define a cross-section for each available set of states of each possible set of residual nuclei; that is, for each open channel. Since different channels correspond to nuclei in different energy states, there is no quantum interference between the corresponding probability amplitudes. We may simply add the cross-sections for different reaction channels. The sum of all these cross-sections for non-elastic processes is then called the *reaction* or *absorption* cross-section for the pair at that energy. When the elastic cross-section is added also, we have the *total* cross-section; it is a measure of the probability that *something* will happen during the collision.

To make the concept of cross-section more physical, consider the collision of two classical spheres (Figure 2.3). Let sphere 1 be at rest and let sphere 2 be impinging upon it. The two spheres will not collide unless their impact distance b is less than the sum of their radii, $b \leq R_1 + R_2$. The effect is the same as for the collision of a point particle with a disc of radius $R_1 + R_2$. The area of this disc, $\pi(R_1 + R_2)^2$, is the cross-section for the collision.

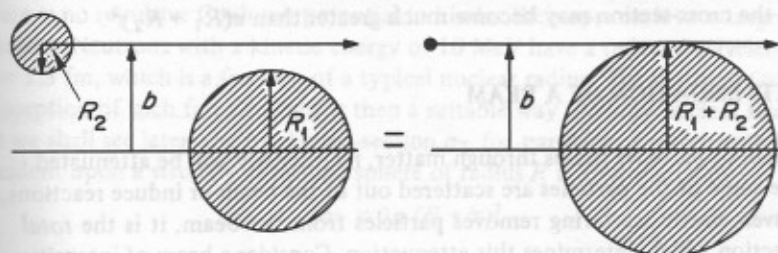


Figure 2.3 The collision of two spheres (left) has the same cross-section as a point particle incident upon a sphere whose radius is the sum $R_1 + R_2$ (right)

We learn one important feature from this picture. The cross-section is not a property of the target alone, but reflects properties of the projectile also. In our classical example, it is the *sum* of the radii which enters. A different projectile will give a different cross-section for the same target if its radius R'_2 is different, and this must be taken into account if we wish to extract the radius of the target

from the results of a scattering measurement. In addition, physical systems like nuclei do not have sharp edges. Their surfaces are diffuse and, as they approach one another, there is a transition region in which they are only partly in contact. The situation is further complicated by the fact that the forces acting between the systems have non-zero ranges. The interaction assumed in our classical picture is a contact interaction but actual nuclear forces act over finite distances. This has the same kind of blurring effect as the diffuse surface of the density distributions. The two effects make it impossible to completely characterise the interaction between two systems simply in terms of a radius. At least one other parameter is required, such as a range or a surface diffuseness. (The electrostatic or Coulomb force, which is inversely proportional to the square of the distance between the two systems, has a very long range and requires special treatment. As we shall see later (Chapter 3), its effects can be taken into account explicitly so that we can separate out the effects due to the short-ranged nuclear forces.)

As we have mentioned before, wave effects such as refraction and diffraction are important for actual nuclear scattering. Even with contact interactions and in the absence of surface diffuseness, these wave effects would modulate the cross-section so that only in an average sense (over a region of bombarding energy, say) would it be $\pi(R_1 + R_2)^2$. Nuclear structure effects may also make the cross-section vary with bombarding energy and even to exhibit sharp resonances—see section 2.7 below. Further, the cross-section for a particular reaction need not be comparable to $\pi(R_1 + R_2)^2$. Although the *total* cross-section for two strongly interacting systems is usually close to this geometric area, the probability of the interaction leading to a particular final state may be very much smaller—even zero—because of nuclear structure or other effects. If the systems are not strongly interacting, the cross-section can also be very small; for example, the cross-section for neutrinos scattering from a nucleus is almost vanishingly small because nuclei are almost transparent to neutrinos. At the opposite extreme, in a strongly resonant situation, such as sometimes occurs for slow neutrons on nuclei, the cross-section may become much greater than $\pi(R_1 + R_2)^2$.

2.7 ATTENUATION OF A BEAM

As a beam of particles passes through matter, its intensity will be attenuated because some of the particles are scattered out of the beam or induce reactions. Since even elastic scattering removes particles from the beam, it is the *total* cross-section which determines this attenuation. Consider a beam of intensity I_0 particles per unit time per unit area (Figure 2.4) incident upon a slab of material. At a depth x , the intensity is I . Let there be N target particles per unit volume in the material. The number of collisions per unit time per unit area in the slice of thickness dx is then proportional to the number of incident particles I and the number of target particles $N dx$. By definition the constant of proportionality is the total cross-section, σ_T . Then

$$dI = -\sigma_T I N dx$$

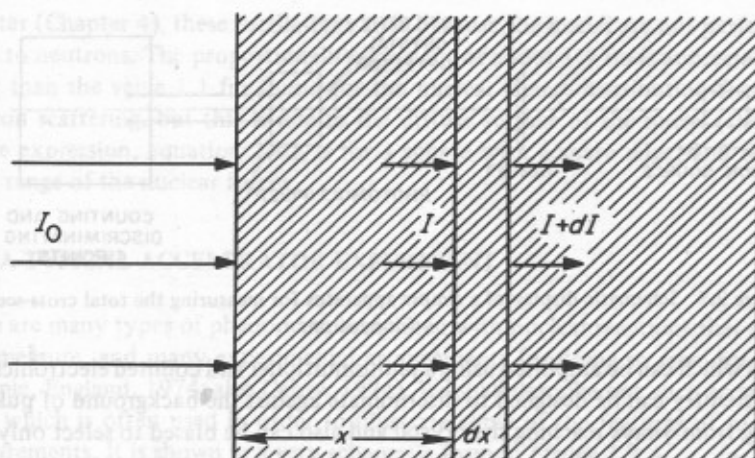


Figure 2.4 Diagram for the attenuation of a beam passing through a slab

Integrating and applying the initial condition $I = I_0$ at $x = 0$

$$I = I_0 \exp(-N\sigma_T x) = I_0 \exp(-x/\Lambda) \quad (2.11)$$

where $\Lambda = 1/N\sigma_T$ is the 'mean free path' between collisions. Clearly Λ is the distance over which the beam intensity falls by e^{-1} . The attenuation 2.11 may be measured in order to give a value for Λ and hence σ_T if N is known. This is known as a transmission experiment.

2.8 NUCLEAR SIZES FROM NEUTRON SCATTERING AND A SIMPLE TRANSMISSION EXPERIMENT

As probes of nuclei, neutrons have the advantage of not being charged so that there is no repulsive Coulomb potential to hinder their approach to a target nucleus. Neutrons with a kinetic energy of 10 MeV have a reduced wavelength $\lambda \approx 1.5$ fm, which is a fraction of a typical nuclear radius. The scattering or absorption of such fast neutrons is then a suitable way to study nuclear sizes. As we shall see later, the total cross-section σ_T for particles of wavelength λ incident upon a strongly absorbing sphere of radius R is approximately

$$\sigma_T = 2\pi (R + \lambda)^2 \quad (2.12)$$

Measurement of σ_T in a transmission experiment will then give a measure of the nuclear radius R . An experiment of this kind (suitable for an undergraduate laboratory) has been described by Fowler *et al.* (1969). The basic apparatus is shown in Figure 2.5. The Pu- α -Be source contains Pu and Be; the α -particles from the decay of the Pu induce an (α, n) reaction with the Be. This provides neutrons (typically $\sim 10^6$ s $^{-1}$) with a spread of energies with a maximum at about 10.7 MeV. Neutrons which pass through the target collide with protons in the organic material of the detector. The recoiling protons produce light

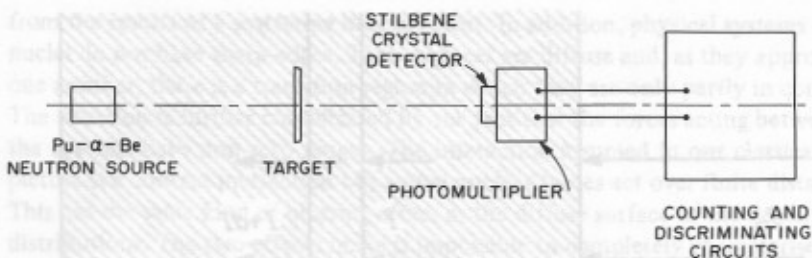


Figure 2.5 Schematic display of a simple apparatus for measuring the total cross-section for neutrons

pulses which can be amplified using a photomultiplier and counted electronically. The circuitry can be designed to discriminate against the background of pulses arising from γ -rays reaching the crystal and also can be biased to select only pulses from a known segment at the upper end of the neutron energy spectrum. Counting neutrons reaching the detector with and without the target in place tells us the attenuation produced by the target. If we know the target thickness and the number of nuclei per unit volume, equation 2.11 then tells us σ_T .

Some results from more sophisticated experiments (see, for example, England, 1974) with 14-MeV neutrons ($\bar{\lambda} \approx 1.2$ fm) are shown in Figure 2.6. The quantity $(\sigma_T/2\pi)^{1/2}$ is plotted against $A^{1/3}$, where A is the mass number of the target nucleus. According to equation 2.12, this is $(R + \bar{\lambda})$ so we see evidence that nuclear radii are proportional to $A^{1/3}$. The straight line is drawn for $R = 1.4 \times A^{1/3}$ fm; the measured values show some oscillations about this line. As we shall

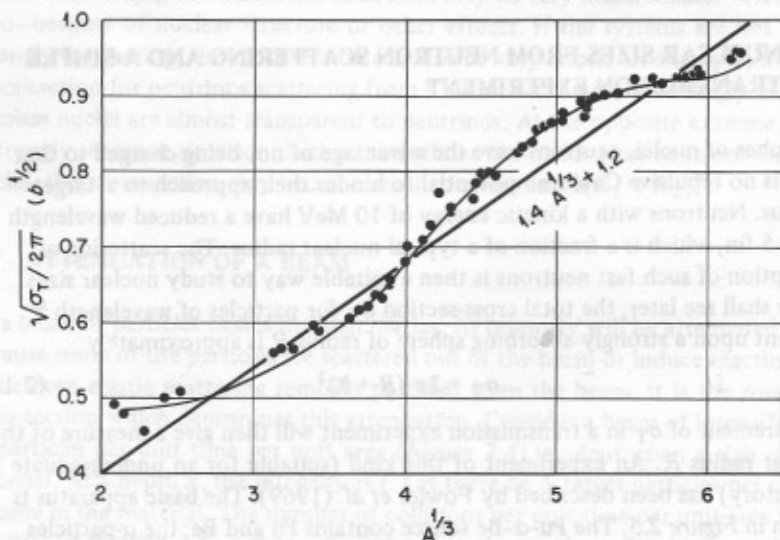


Figure 2.6 Total cross-sections σ_T for 14-MeV neutrons plotted against $A^{1/3}$ where A is the mass number of the target nucleus. The curve which follows the data was obtained from an optical model (see Chapter 4) which allows for nuclei to be partially transparent to neutrons

see later (Chapter 4), these oscillations arise because the nuclei are not perfectly black to neutrons. The proportionality constant of about 1.4 fm is appreciably larger than the value 1.1 fm found for the nuclear charge distribution from electron scattering, but this is due to the diffuse surface of the nuclei (the simple expression, equation 2.12, is for a sphere with a sharp edge) and the finite range of the nuclear forces.

2.9 A TYPICAL ACCELERATOR EXPERIMENT

There are many types of phenomena associated with nuclear reactions that one may measure, and many experimental arrangements for doing so (*see, for example, England, 1974; also Cerny, 1974*). We shall describe one such arrangement which is often used in order to illustrate some of the features of such measurements. It is shown in a very schematic form in *Figure 2.7*.

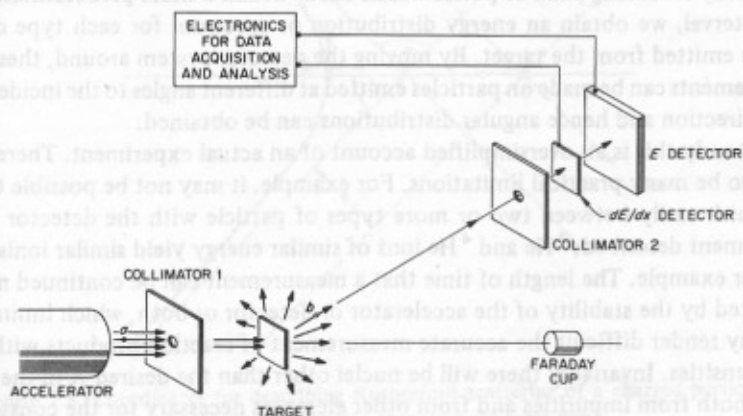


Figure 2.7 Schematic layout of the apparatus for a typical experiment using an accelerator. Compare with an actual set-up as shown in the frontispiece!

First, the accelerator provides a beam of charged particles of type a . (It is possible these have been passed through a bending magnet first in order to select ions of the required energy, etc.) Baffles and shielding (represented by the screen 1) help collimate the beam into a small spot on the target, and to remove stray particles. The target may be a thin film of the required material containing the nuclei A , or this may be deposited on a thin backing of some other material. The target must be thin if we are concerned with precise energy measurements. The beam ions can be scattered by the electrons of the atoms in the target and passage through a thick target will cause an undue spread in the energies of the beam particles. On the other hand, the thicker the target, the more target nuclei there are and the larger is the scattered intensity. Most experiments involve a compromise between these two requirements.

We must know the incident beam intensity as well as the scattered intensity.

For this purpose, and to monitor the uniformity of the beam, we need at least one device. One simple device shown is a Faraday cup in which the beam is collected for a known time, the charge is measured, and hence the current can be estimated.

The points discussed so far are common to most experiments. The mode of detection of the reaction products varies widely. We may wish to distinguish definite types of emitted particles b and to measure the distribution of their energies. One way to do this is shown. The scattered particles are collimated by screens and baffles (represented by screen 2), partly so as to define the scattering angle θ and partly to shield the detectors from background radiation. The scattered particle b first passes through a thin 'dE/dx' counter. The degree of ionisation produced in this distinguishes the type of particle b . The second detector stops the particle b ; the ionisation and hence the size of pulse produced tells us its kinetic energy. By registering these two types of pulses *in coincidence*, i.e. by only recording pairs of pulses which occur within a short predetermined time interval, we obtain an energy distribution or spectrum for each type of particle emitted from the target. By moving the detector system around, these measurements can be made on particles emitted at different angles to the incident beam direction and hence angular distributions can be obtained.

Obviously this is an oversimplified account of an actual experiment. There may also be many practical limitations. For example, it may not be possible to distinguish easily between two or more types of particle with the detector arrangement described; ^3He and ^4He ions of similar energy yield similar ionisation, for example. The length of time that a measurement can be continued may be limited by the stability of the accelerator or detector or both, which limitation may render difficult the accurate measurement of reaction products with low intensities. Invariably there will be nuclei other than the desired A in the target, both from impurities and from other elements necessary for the construction of the target. Some of these, notoriously carbon and oxygen, may give rise to very strong groups of emitted particles which may obscure the radiations one is attempting to measure.

2.10 COULOMB SCATTERING AND RUTHERFORD'S FORMULA

One of the most important cross-sections in physics is the differential cross-section for the scattering of two charged particles. The Coulomb force will cause scattering even in the absence of other, specifically nuclear, forces. Since all nuclei and the majority of elementary particles are charged, Coulomb scattering is a common experience. In a classic paper, Rutherford (1911) gave the formula for the differential cross-section for Coulomb scattering in the non-relativistic case. He used it to interpret his early experiments on the scattering of α -particles which provided evidence for the nuclear atom. Rutherford used classical mechanics, but it was found subsequently that the formula remains true in non-relativistic quantum mechanics (see, for example, Mott and Massey, 1965).

2.10.1 Classical derivation

The Coulomb force between a projectile with a mass m and a charge $Z_1 e$ ($Z_1 = 2$ for the α -particle) and a target nucleus with a charge $Z_2 e$ is $Z_1 Z_2 e^2 / r^2$, where r is the distance between them. The force is repulsive if the two particles have charges of the same sign. The corresponding potential is $Z_1 Z_2 e^2 / r$. If the target is much heavier than the projectile, we may consider it to remain at rest while the projectile describes an orbit which is one branch of a hyperbola (Figure 2.8).

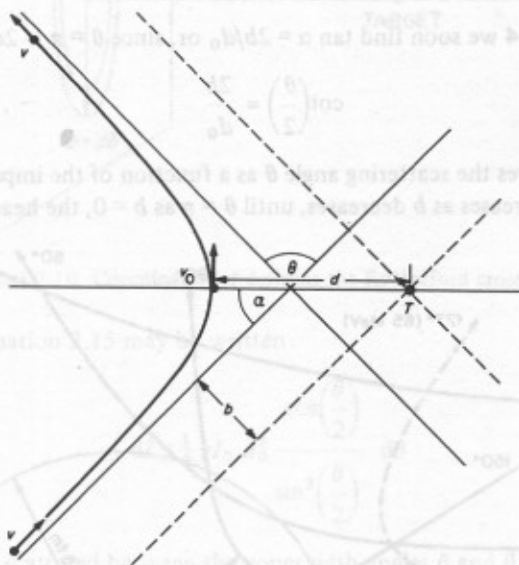


Figure 2.8 Coordinates for describing Rutherford scattering of a charged particle by a target T with a charge of the same sign

Let b be the distance of the target T from the asymptote of the hyperbola; b is called the impact parameter. Let d be the distance of closest approach of the orbit to the target. Further, let the projectile velocity be v at a very great distance from the nucleus where the potential is negligible and let its velocity be v_0 at the point of closest approach, $r = d$. Then conservation of energy gives

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 + \frac{Z_1 Z_2 e^2}{d}$$

or

$$\left(\frac{v_0}{v}\right)^2 = 1 - \frac{d_0}{d}, \quad d_0 = \frac{2Z_1 Z_2 e^2}{m v^2} = \frac{Z_1 Z_2 e^2}{E} \quad (2.13)$$

where $E = \frac{1}{2} m v^2$ is the bombarding energy. (Note that $e^2 = 1.44 \text{ MeV fm}$.) We see that d_0 is the distance of closest approach for which $v_0 = 0$, i.e. a head-on collision ($b = 0$). Conservation of angular momentum implies

$$mvb = mv_0 d$$

Hence with equation 2.13 we have a relation between b and d

$$b^2 = d(d - d_0) \quad (2.14)$$

It is a property of the hyperbola that

$$d = b \cot\left(\frac{\alpha}{2}\right);$$

with equation 2.14 we soon find $\tan \alpha = 2b/d_0$ or, since $\theta = \pi - 2\alpha$

$$\cot\left(\frac{\theta}{2}\right) = \frac{2b}{d_0} \quad (2.15)$$

Equation 2.15 gives the scattering angle θ as a function of the impact parameter b ; note that θ increases as b decreases, until $\theta = \pi$ as $b = 0$, the head-on collision (Figure 2.9).

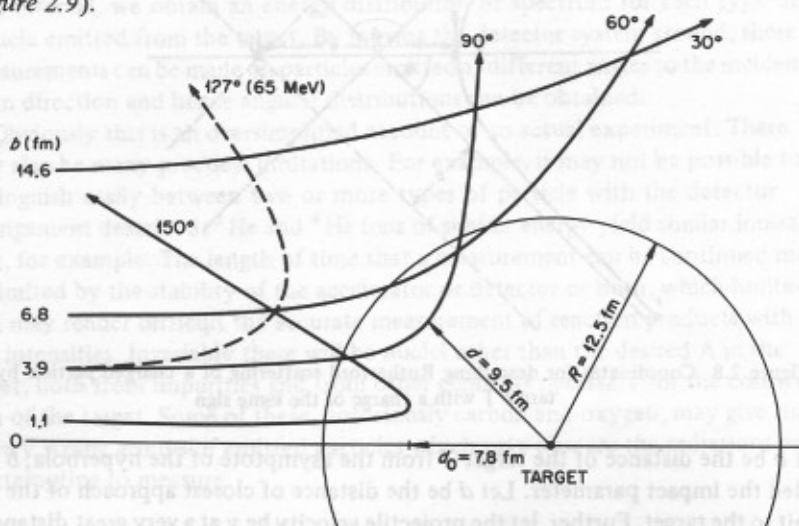


Figure 2.9 Some typical Rutherford (classical) orbits for a charged particle scattered by a charged target for different impact parameters b . The curves are drawn for ^{16}O ions with energy 130 MeV incident on ^{208}Pb , except that the dashed curve for $b = 3.9$ fm is for an energy of 65 MeV. The circle has a radius $R = 12.5$ fm which is equal to the distance of closest approach at which the nuclear forces begin to act for this system. We see that the orbit with $b = 6.8$ fm barely touches this region but orbits with $b < 6.8$ fm approach more closely. In practice the nuclear forces would distort these latter orbits and the scattering would deviate from Rutherford for scattering angles $\theta \gtrsim 60^\circ$

Consider a flux of I_0 particles per unit area and unit time crossing a plane perpendicular to the beam (Figure 2.10). The flux passing through the annulus with radii $b, b + db$ is

$$dI = 2\pi I_0 b db$$

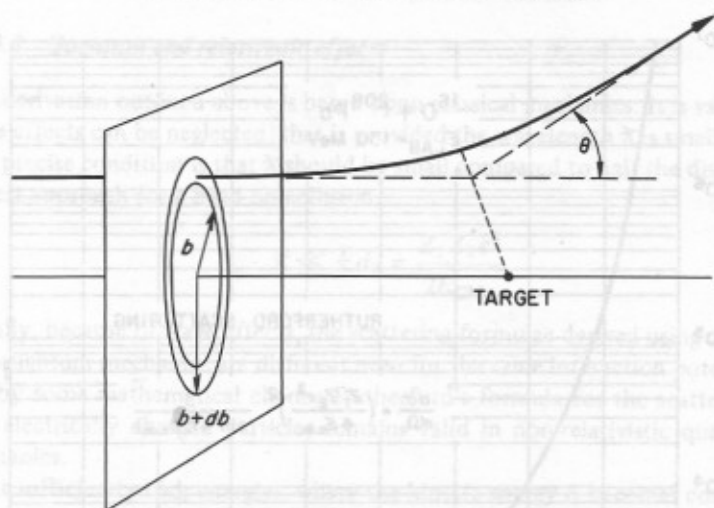


Figure 2.10 Coordinates for defining the Rutherford cross-section

which from equation 2.15 may be written

$$dI = \frac{1}{4} \pi I_0 d_0^2 \frac{\cos\left(\frac{\theta}{2}\right)}{\sin^3\left(\frac{\theta}{2}\right)} d\theta$$

This is the flux scattered between the cones with angles θ and $\theta + d\theta$, which enclose a solid angle $d\Omega = 2\pi \sin \theta d\theta$. Thus the differential cross-section (section 2.6) is

$$\frac{d\sigma}{d\Omega} \equiv \frac{1}{I_0} \frac{dI}{d\Omega} = \left(\frac{d_0}{4}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} = \left(\frac{Z_1 Z_2 e^2}{4E}\right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} \quad (2.16)$$

This is Rutherford's formula for Coulomb scattering. We see that the *angular distribution* has a universal form* (Figure 2.11), while the *magnitude* of the cross-section depends only upon the product $Z_1 Z_2$ of the charges of the two particles and the bombarding energy $E = \frac{1}{2} mv^2$.

The condition that the target mass be much greater than that of the projectile may be relaxed. It turns out that equation 2.16 is still true if we take account of the recoil of the target; we only have to reinterpret E , the energy, and θ , the scattering angle, as being measured in the centre-of-mass system, E_{CM} and θ_{CM} respectively (section 2.2 and Appendix B), then equation 2.16 gives the cross-section in the CMS.

*An apparatus and procedure for demonstrating this result which is suitable for an undergraduate teaching laboratory has been described by Ramage *et al.* (1975).

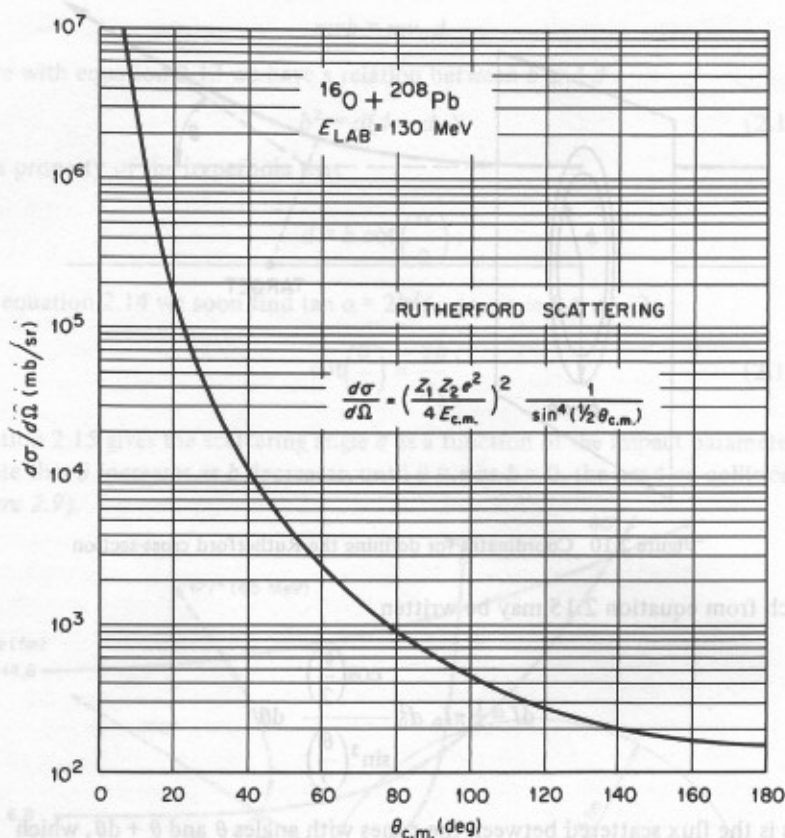


Figure 2.11 The differential cross-section for Rutherford scattering, equation 2.16, drawn for ^{16}O ions of 130 MeV scattered by ^{208}Pb . The cross-sections for 7.68-MeV α -particles on ^{197}Au (as in the original Geiger and Marsden, 1913, experiment) would have the same angular distribution but be 3.86 times larger. Note that the cross-section scale is logarithmic

The formula 2.16 predicts that the cross-section will become infinite as the scattering angle goes to zero (Figure 2.11). Physically this means that the Coulomb potential has a very long range so that even particles with very large impact parameters are deflected slightly by it. If we integrate equation 2.16 to obtain the total cross-section, this is also infinite. In practice, the differential cross-section cannot be measured for *very* small angles; the beam of incident particles has a finite width so that there is an upper bound on the impact parameter b , and those particles scattered in the most forward direction cannot be distinguished from the beam itself. Further, the charge of the target nucleus will usually be screened by atomic electrons so that the potential felt at distances of the order of atomic dimensions is no longer Coulombic.

2.10.2 Quantum and relativistic effects

The derivation outlined above is based upon classical mechanics. It is valid if wave effects can be neglected; that is provided the wavelength λ is small enough. The precise condition is that λ should be small compared to half the distance of closest approach for a head-on collision

$$\lambda \ll \frac{1}{2} d_0 = \frac{Z_1 Z_2 e^2}{2E_{\text{CM}}} \quad (2.17)$$

Usually, because of wave effects, the scattering formulae derived using classical and quantum mechanics are different even for the same interaction potential, but by some mathematical chance Rutherford's formula for the scattering of two electrically charged particles remains valid in non-relativistic quantum mechanics.

At sufficiently high energies, where the kinetic energy E becomes comparable to the energy of the rest mass, mc^2 , relativistic effects must be taken into account. This is very often the case for electrons whose rest energy, $m_e c^2$ is small, ≈ 0.5 MeV. The cross-section for scattering high-energy electrons has to be obtained from Dirac's relativistic wave equation (see, for example, Ueberall, 1971). There is a simple expression for the scattering from a point charge Ze , provided Z is small compared to the fine structure constant, $Z \ll (\hbar c/e^2) = 137$. Then

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E\beta^2} \right)^2 \left(1 - \beta^2 \sin^2 \frac{\theta}{2} \right) \frac{1}{\sin^4 \left(\frac{\theta}{2} \right)}$$

where $\beta = v/c$, the ratio of the electron's speed to the speed of light. Note that here E represents the relativistic expression for the kinetic plus the rest energy of the electron

$$E = \frac{m_e c^2}{(1 - \beta^2)^{1/2}}$$

In the limit of low energy, $\beta \rightarrow 0$ and $E\beta^2 \rightarrow m_e v^2$, so we regain the Rutherford formula 2.16. At extreme relativistic energies $\beta = 1$ and we have the *Mott formula*

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \left(\frac{Ze^2}{2E} \right)^2 \frac{\cos^2 \left(\frac{\theta}{2} \right)}{\sin^4 \left(\frac{\theta}{2} \right)} \quad (2.18)$$

The additional angle dependence, $\cos^2(\theta/2)$, compared to the Rutherford expression which arises from the relativistic treatment, is not negligible.

2.10.3 Extended particles

Strictly, Rutherford's formula is only true for the scattering of *point* charges. However, it also describes the scattering of particles with extended charge distributions provided they do not approach one another too closely. Consider the scattering of a nucleus with radius R_1 and charge Z_1e by another with radius R_2 and charge Z_2e . The interaction potential when their centres are separated by r is

$$\frac{Z_1 Z_2 e^2}{r} \quad \text{if } r \geq R_1 + R_2 = R$$

and is often represented by (see Exercise 1.5)

$$\frac{Z_1 Z_2 e^2}{2R} \left(3 - \frac{r^2}{R^2} \right) \quad \text{if } r \leq R_1 + R_2 = R \quad (2.19)$$

If the distance of closest approach for a head-on collision, equation 2.13, is greater than the sum of the two radii $d_0 > R$, then only the Coulomb force will be experienced and the Rutherford formula will hold. This means the bombarding energy should be less than $Z_1 Z_2 e^2 / R$; this critical energy is often referred to as the *Coulomb barrier*. At energies greater than this, the scattering will deviate from equation 2.16. The scattering angle θ and the distance of closest approach d are related by equations 2.13–2.15

$$\cot\left(\frac{\theta}{2}\right) = 2 \left[\frac{d}{d_0} \left(\frac{d}{d_0} - 1 \right) \right]^{1/2} \quad (2.20)$$

Deviations will occur for those θ for which $d < R$. Since the smaller d result in the larger θ (Figure 2.9), the deviations first appear at $\theta = 180^\circ$ and then at progressively smaller angles as the energy is increased.

In Geiger and Marsden's original experiment of 1913, α -particles with $E = 7.68$ MeV from an RaC (or ^{214}Po) source were scattered from gold. Their results agreed with the Rutherford formula for all scattering angles. Since $d_0 = 3 \times 10^{-14}$ m = 30 fm in this case ($Z_1 = 2$, $Z_2 = 79$), they concluded that the gold nucleus had a radius of less than 30 fm. (It is now known that the radius of the gold nucleus is about 7 fm; the α -particles would need an energy of over 30 MeV to penetrate to $d_0 = 7$ fm.)

If there are also nuclear forces acting between the two particles, the potential 2.19 will be further modified. The nuclear forces are known to have a short range δ , of the order of 1 fm, so they are not felt for separations r which are much greater than $R + \delta$. At larger values of r , the potential 2.19 is still valid; at smaller values it is modified by the addition of the (attractive) nuclear potential. Again the scattering will be described by Rutherford's formula if $d_0 > R + \delta$, but deviations will occur if d_0 is less than this amount. (This discussion assumes that the particles can be precisely localised; it is valid if the condition 2.17 is satisfied. Otherwise wave effects will modify the results.) Figure 2.12 shows the differential cross-sections for O and C ions scattering from various target nuclei,

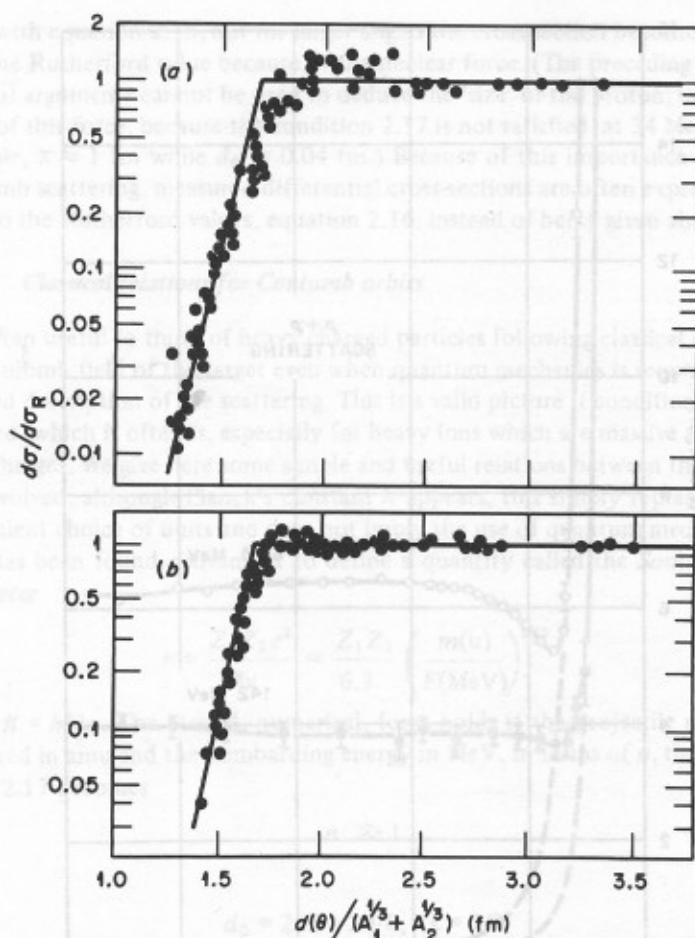


Figure 2.12 The differential cross-sections for O and C ions, in ratio to the Rutherford ones, scattering from various targets plotted against the distance of closest approach d , instead of the scattering angle θ , by using equation 2.20. The distance d has been divided by $(A_1^{1/3} + A_2^{1/3})$, where A_1 is the mass number of nucleus i . The measured cross-sections then fall on a universal curve, showing that nuclear radii are approximately proportional to $A^{1/3}$. (a) $^{16}\text{O} + ^{40,46}\text{Ca}$ at 49 MeV, $^{16}\text{O} + ^{40,46}\text{Ca}$, ^{50}Ti , ^{52}Cr , ^{54}Fe , ^{62}Ni at 60 MeV and $^{18}\text{O} + ^{60}\text{Ni}$ at 60 MeV; (b) $^{12}\text{C} + ^{96}\text{Zr}$ at 38 MeV, $^{16}\text{O} + ^{96}\text{Zr}$ at 47, 49 MeV, $^{16}\text{O} + ^{88}\text{Sr}$, ^{93}Zr at 60 MeV and $^{18}\text{O} + ^{90}\text{Zr}$ at 60, 66 MeV. (After Christensen *et al.*, 1973)

plotted versus the distance of closest approach d instead of the scattering angle, using the relation 2.20. The cross-section is given by the Rutherford value until the two nuclei approach to within a distance of about $1.7(A_1^{1/3} + A_2^{1/3})$ fm, when the cross-section begins to fall rapidly to zero. This decrease occurs because the two nuclei have surmounted their mutual Coulomb barrier and come under the influence of the nuclear forces. These induce non-elastic reactions and the

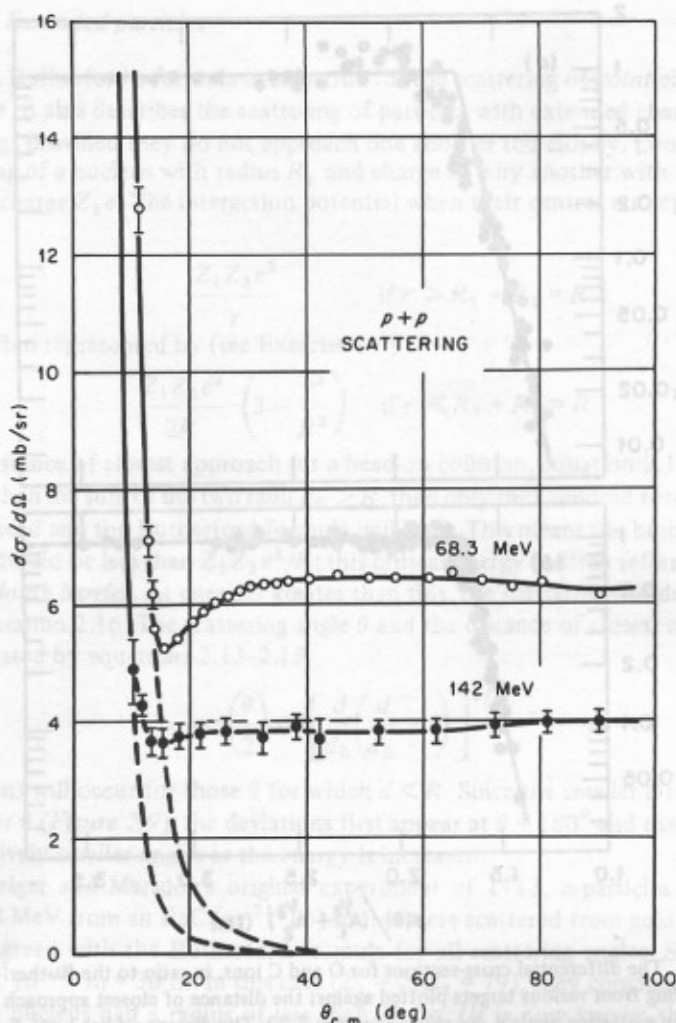


Figure 2.13 Differential cross-sections for protons with LAB energies of 68 and 142 MeV scattering from protons. The dashed curves show the Rutherford cross-sections. The measured values follow the Rutherford curves at small angles but show large deviations at larger angles where the nuclear force is important. The bars attached to the data points indicate the experimental uncertainties. (After Breit *et al.*, 1960)

nuclei are removed from the elastic channel; they are said to be *absorbed*, in analogy to the absorption of light by a black object.

The differential cross-section for the elastic scattering of charged particles is always dominated by Rutherford scattering at small angles. Figure 2.13 shows the cross-sections for the scattering of protons by protons with CMS energies of 34 and 71 MeV. The cross-sections predicted by the Rutherford formula are shown as dashed lines. For small angles the measured values become large and

agree with equation 2.16, but for larger angles the cross-section becomes bigger than the Rutherford value because of the nuclear force. (The preceding semi-classical arguments cannot be used to deduce the 'size' of the proton, i.e. the range of this force, because the condition 2.17 is not satisfied; at 34 MeV, for example, $\lambda \approx 1$ fm while $d_0 \approx 0.04$ fm.) Because of this importance of the Coulomb scattering, measured differential cross-sections are often expressed in ratio to the Rutherford values, equation 2.16, instead of being given absolutely.

2.10.4 Classical relations for Coulomb orbits

It is often useful to think of heavy charged particles following classical orbits in the Coulomb field of the target even when quantum mechanics is required for a detailed description of the scattering. This is a valid picture if condition 2.17 is satisfied, which it often is, especially for heavy ions which are massive and carry large charges. We give here some simple and useful relations between the quantities involved; although Planck's constant h appears, this simply represents a convenient choice of units and does not imply the use of quantum mechanics.

It has been found convenient to define a quantity called the *Sommerfeld parameter*

$$n = \frac{Z_1 Z_2 e^2}{\hbar v} \approx \frac{Z_1 Z_2}{6.3} \left(\frac{m(u)}{E(\text{MeV})} \right)^{1/2} \quad (2.21)$$

where $\hbar = h/2\pi$. The second, numerical, form holds if the projectile mass is measured in amu and the bombarding energy in MeV. In terms of n , the condition 2.17 becomes

$$n \gg 1 \quad (2.22)$$

Also

$$d_0 = 2n\lambda = 2n/k, \quad k = 1/\lambda \quad (2.23)$$

where

$$k = 1/\lambda = (2mE/\hbar^2)^{1/2} \approx 0.22 (m(u)E(\text{MeV}))^{1/2} \text{ fm}^{-1} \quad (2.24)$$

The angular momentum of the projectile about the target is

$$L\hbar = mvb, \quad \text{or} \quad L = b/\lambda = kb \quad (2.25)$$

Then equation 2.15 relates the scattering or deflection angle of the orbit to the angular momentum (in units of $\hbar$) of the particle following that orbit

$$L = n \cot\left(\frac{\theta}{2}\right) \quad (2.26)$$

With equation 2.14 we also find a relation between the angular momentum and the distance of closest approach d of the orbit

$$L^2 = \rho(\rho - 2n), \quad \text{or} \quad \rho = n + (L^2 + n^2)^{1/2} \quad (2.27)$$

where $\rho = kd = d/\lambda$. Of course, if $L = 0$, then $\rho = 2n$ or $d = d_0$. Combining equations 2.26 and 2.27, we also have

$$\rho = n \left(1 + \operatorname{cosec} \frac{\theta}{2} \right), \quad \text{or} \quad \sin \frac{\theta}{2} = \frac{n}{\rho - n} \quad (2.28)$$

2.12 COULOMB EXCITATION

As just remarked, the Coulomb force between an electron and a nucleus may excite the nucleus when the electron is scattered from it. Similarly the scattering of any other charged particle can excite the nucleus. However, the expression 'Coulomb excitation' is usually only applied to the scattering of protons or heavier nuclei. If the bombarding energy of such particles is below the Coulomb barrier (*see* section 2.10.3), they do not approach closely enough for the short-ranged nuclear forces to act, the long-ranged and well-known Coulomb force acts by itself. The interpretation of measurements under these conditions is then unambiguous and much valuable information on transition rates to excited states can be obtained. If the bombarding energy is then raised above the Coulomb barrier, both types of force act. This also is a useful situation since there are *interference* effects between the repulsive Coulomb and attractive nuclear forces; since the Coulomb force is known, observation of these effects can lead to information about the nuclear force.

If the wavelength of the projectile is sufficiently short (*see* section 2.10.2), such as is the case with heavy ions, the collision can be visualised semi-classically. The projectile moves along a classical Coulomb (or Rutherford) orbit and at some point near the distance of closest approach emits a virtual photon, losing kinetic energy and thereby slowing down. The virtual photon is absorbed by the target nucleus which is thus excited. The emission and absorption process have to be treated quantum mechanically; the excitation of the target nucleus is like an

inverse γ -decay, the 'photon' being provided by the field of the scattered projectile.

Detailed expositions of Coulomb excitation can be found in Alder and Winther (1975) and Biedenharn and Brussard (1965); see also McGowan and Stelson (1974).

2.18 QUALITATIVE FEATURES OF NUCLEAR REACTIONS

Nuclear reactions are found in astonishing variety. Nevertheless, some general characteristics can be discerned and some broad categories defined. For example, not all reactions to all possible final states proceed with anything like equal probability. On the contrary, there is often a high degree of *selectivity*. Figure 2.26 illustrates just two examples of this. The reactions $^{12}\text{C} (^{12}\text{C}, \alpha) ^{20}\text{Ne}$ and $^{16}\text{O} (^7\text{Li}, t) ^{20}\text{Ne}$ lead to the same final nucleus ^{20}Ne , but Figure 2.26 shows that they do not excite the same excited states of ^{20}Ne with equal probability. Further, the levels of ^{20}Ne in this region of excitation are known to be many times more numerous than the few excited in either of these reactions. Such selectivity will also vary with bombarding energy. It often allows us to deduce much interesting information about nuclear structure and about the mechanisms of the various reactions.

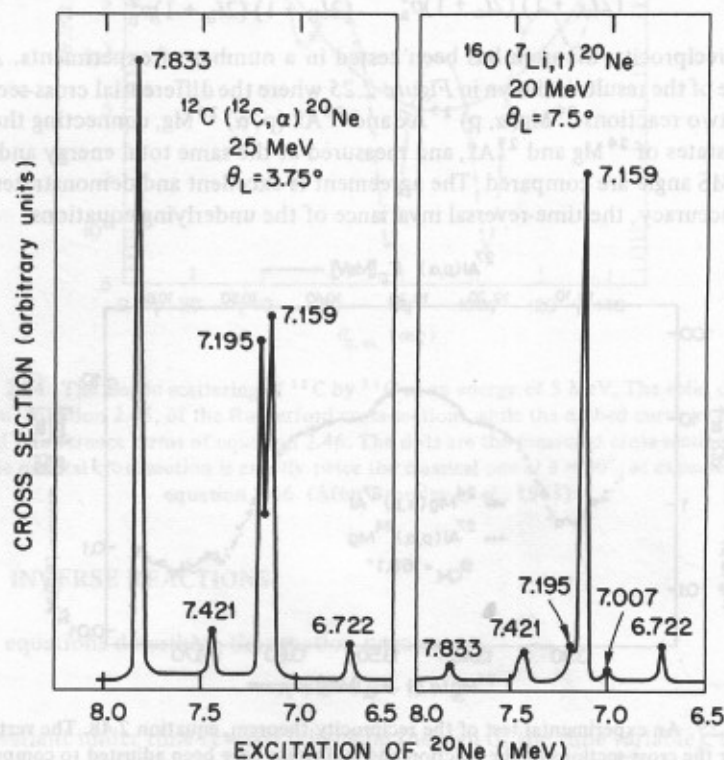


Figure 2.26 A comparison of the energy spectra for the products from the reactions $^{12}\text{C} (^{12}\text{C}, \alpha) ^{20}\text{Ne}$ and $^{16}\text{O} (^7\text{Li}, t) ^{20}\text{Ne}$ which lead to the same final nucleus. This shows that the two reactions do not excite the states of ^{20}Ne in the same way; both reactions are very 'selective'. The peaks are labelled by the excitation energy of the corresponding states in ^{20}Ne . (After Bromley, 1974)

What one learns from a measurement on a nuclear reaction often depends upon the energy resolution available. It might be supposed that it was best to have very sharp energy resolution; in many cases this is true, for example if we wish to resolve two nuclear levels which have almost the same excitation energy. However, in some instances interesting structure may be observed in an experiment with poor resolution which would be obscured by the complexity of the results from a high-resolution study. Of course, the high-resolution results may be deliberately smoothed later by averaging them over a small energy interval. The point is that such smoothing is easy to do once one realises that it is useful to do it. It is the cruder experiment that points the way to this.* *Figure 2.27*

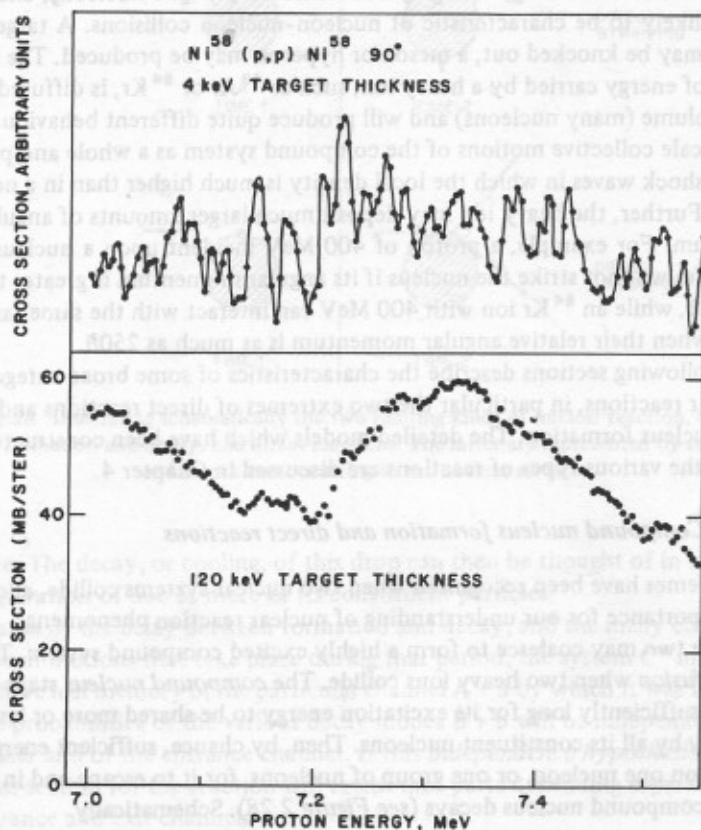


Figure 2.27 Excitation functions (cross-sections as a function of bombarding energy) for protons elastically scattered from ^{58}Ni . The upper portion shows results obtained using a thin target, the lower portion is for a thick target. (After Lee and Schiffer, 1963)

*Fortunately the early stages of a science are usually characterised by poor resolution measurements. Imagine how little progress would have been made systematising the line spectra emitted by atoms if all the fine and hyperfine structure had been observed at the very beginning!

illustrates this for protons scattering elastically from ^{58}Ni to an angle of $\theta_L = 90^\circ$. The upper portion shows the results of high-resolution measurements with a thin target; there are considerable fluctuations as the proton energy is varied. The lower portion shows the result of averaging these results over about 120 keV by using a thick target. The fluctuations are largely removed but some broad 'intermediate' structure remains. (Another example is shown in *Figure 4.38*, Chapter 4.)

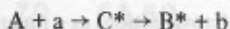
The type of information available from reaction measurements also depends upon the nature of the projectile and the bombarding energy. As a particular example, suppose we bring into a nucleus hundreds of MeV of energy via a high-energy proton. This energy is highly concentrated (on one nucleon), and the result is likely to be characteristic of nucleon-nucleon collisions. A target nucleon may be knocked out, a meson or hyperon may be produced. The same amount of energy carried by a heavy ion, such as ^{40}Ar or ^{84}Kr , is diffused over a large volume (many nucleons) and will produce quite different behaviour such as large-scale collective motions of the compound system as a whole and perhaps exciting shock waves in which the local density is much higher than in a normal nucleus. Further, the heavy ion may deposit much larger amounts of angular momentum. For example, a proton of 400 MeV incident upon a nucleus of radius 6 fm will not strike the nucleus if its angular momentum is greater than about $25\hbar$, while an ^{84}Kr ion with 400 MeV can interact with the same target nucleus when their relative angular momentum is as much as $250\hbar$.

The following sections describe the characteristics of some broad categories of nuclear reactions, in particular the two extremes of direct reactions and compound nucleus formation. The detailed models which have been constructed to describe the various types of reactions are discussed in Chapter 4.

2.18.1 Compound nucleus formation and direct reactions

Two extremes have been recognised when two nuclear systems collide, and both are of importance for our understanding of nuclear reaction phenomena.

(i) The two may coalesce to form a highly excited compound system. This is called *fusion* when two heavy ions collide. The *compound nucleus* stays together sufficiently long for its excitation energy to be shared more or less uniformly by all its constituent nucleons. Then, by chance, sufficient energy is localised on one nucleon, or one group of nucleons, for it to escape and in this way the compound nucleus decays (see *Figure 2.28*). Schematically



If sufficient excitation energy remains in B^* , further particle emissions may occur. Otherwise, it will de-excite by β - or γ -decay.

The picture of a nucleus as a liquid drop is perhaps helpful in visualising these processes. In the compound nucleus reaction, the two colliding droplets combine to form a single compound drop which, because it is excited, is at a high tem-

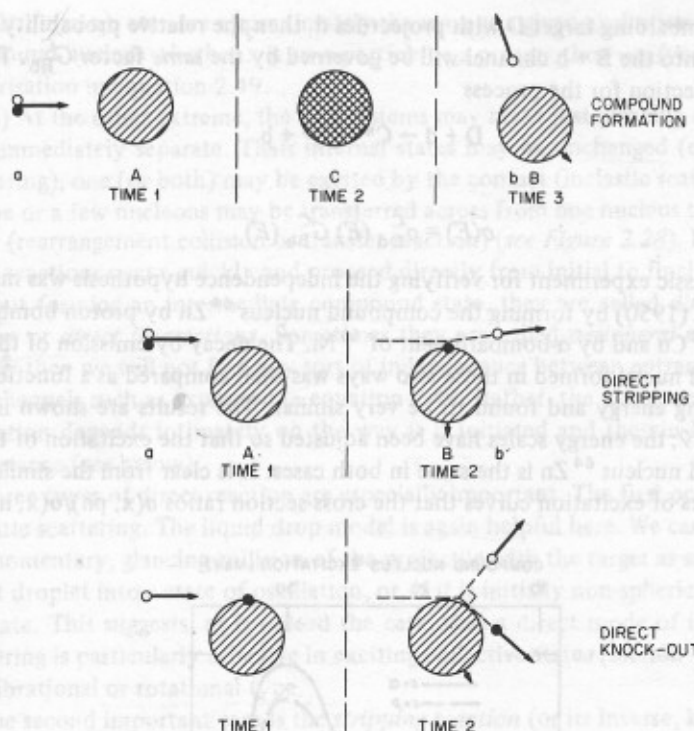


Figure 2.28 Illustrating schematically the two limiting kinds of nuclear reaction, compound nucleus formation and decay, and direct reactions. The latter are represented by stripping and knock-out occurring in the nuclear surface

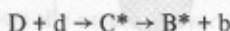
perature. The decay, or cooling, of this drop can then be thought of in terms of the *evaporation* of one or more of its constituent particles.

Because of the delay between formation and decay, and the many complicated nucleon motions that take place during that period, the system C^* may be said to have lost memory of the particular channel $A + a$ by which it was formed, and the probabilities of the various decay modes $B + b$ will be independent of each other and of the entrance channel. If this *independence hypothesis* holds, the cross-section for the reaction will factor into parts depending separately on the entrance and exit channels

$$\sigma = \sigma_{Aa}^C(E) G_{Bb}^C(E) \quad (2.49)$$

where $\sigma_{Aa}^C(E)$ is the cross-section for forming the compound nucleus C^* from $A + a$ with the total energy E , and $G_{Bb}^C(E)$ is the relative probability of C at this energy then decaying into $B + b$. The significance of this is that if the same compound nucleus C^* with the same total energy were formed in some other way,

say by bombarding target D with projectiles d, then the relative probability for its decay into the B + b channel will be governed by the *same* factor G_{Bb}^C . Then the cross-section for the process



would be

$$\sigma(E) = \sigma_{Dd}^C(E) G_{Bb}^C(E)$$

The classic experiment for verifying the independence hypothesis was made by Goshal (1950) by forming the compound nucleus ^{64}Zn by proton bombardment of ^{63}Cu and by α -bombardment of ^{60}Ni . The decay by emission of the compound nuclei formed in these two ways was then compared as a function of bombarding energy and found to be very similar. The results are shown in Figure 2.29; the energy scales have been adjusted so that the excitation of the compound nucleus ^{64}Zn is the same in both cases. It is clear from the similarity of the pairs of excitation curves that the cross-section ratios $\sigma(x, pn)/\sigma(x, n)$ or

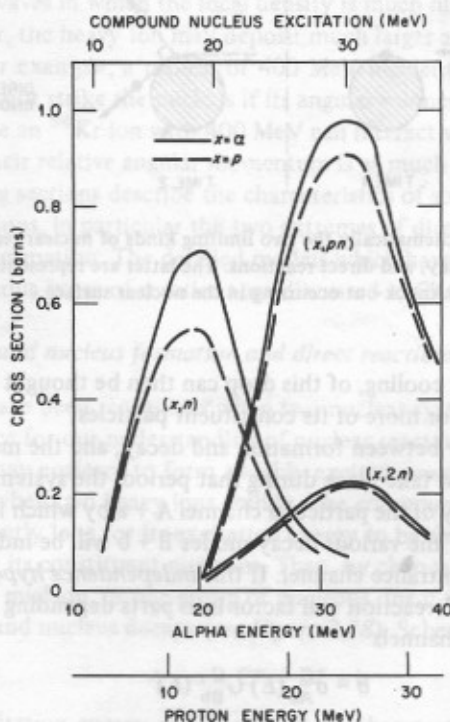


Figure 2.29 Experimental evidence for the independence of formation and decay in a compound nucleus reaction. The same compound nucleus ^{64}Zn , with the same excitation energy, is formed in two different ways, but the excitation curves are very similar. (After Goshal, 1950)

$\sigma(x, 2n)/\sigma(x, n)$, etc., are approximately the same at a given excitation of the compound nucleus whether x is an α -particle or a proton, thus verifying the factorisation in equation 2.49.

(ii) At the other extreme, the two systems may make just glancing contact and immediately separate. Their internal states may be unchanged (elastic scattering), one (or both) may be excited by the contact (inelastic scattering), or one or a few nucleons may be transferred across from one nucleus to the other (rearrangement collision or transfer reaction) (see Figure 2.28). Because these reactions occur quickly and proceed directly from initial to final states without forming an intermediate compound state, they are called *direct reactions* or *direct interactions*. Sometimes they are called *peripheral reactions*. Clearly then we will not find any sort of independence between entrance and exit channels such as expressed in equation 2.49. Rather, the outcome of such a reaction depends intimately on the way it is initiated and therein lies its importance (see below).

Three types of direct reaction are especially important. The first occurs for inelastic scattering. The liquid drop model is again helpful here. We can visualise the momentary, glancing collision of the projectile with the target as setting the target droplet into a state of oscillation, or, if it is initially non-spherical, making it rotate. This suggests, as is indeed the case, that a direct mode of inelastic scattering is particularly effective in exciting collective states (section 1.7.4) of the vibrational or rotational type.

The second important case is the *stripping reaction* (or its inverse, known as a *pick-up reaction*). These are also referred to as *transfer reactions*. Figure 2.28 illustrates this; one (or a few) nucleons is stripped from the projectile as it passes the periphery of the target. The prototype of these reactions is the deuteron stripping reaction $A(d, p)B$ in which a neutron is stripped from the deuteron and transferred to the target.

The third concerns knock-out reactions, in which a nucleon or light nucleus is ejected from the target by the projectile, which itself continues freely (see Figure 2.28). We then have three particles in the final state. These reactions are also referred to as *quasi-free scattering* since the picture is of the projectile and struck particle colliding almost as though the latter was free and the rest of the target nucleus was simply a spectator. Here the prototype is the $(p, 2p)$ reaction. This and other reactions which eject a single nucleon, (p, pn) , $(e, e'p)$, etc., are nuclear analogues of the early experiments of Franck and Hertz and others on the ionisation of atoms under electron bombardment. The ejection of other particles, e.g. a $(p, p'\alpha)$ reaction, may tell us about the probability of finding preformed clusters, such as α -particles, within the nucleus.

A multiple-scattering picture can also give us insight into the relation between direct and compound reactions, especially if the projectile is a nucleon. We can consider this incident nucleon as making successive collisions with the various target nucleons. At each collision, the incident nucleon usually will not lose a large fraction of its energy. Hence after one such collision, the target nucleus

will not be highly excited. If the incident particle then escapes, we will then have a direct-reaction inelastic scattering, leaving the target in one of its low-lying excited states. If the struck nucleon also escapes we have a direct knock-out or, if it emerges bound to the incident nucleon, a pick-up reaction. If instead the incident nucleon suffers another collision, it will tend to lose some more energy. After a number of such collisions, the original energy has been shared with many target nucleons, the incident particle no longer has sufficient energy to escape, and a compound nucleus has been formed.

Direct and compound nuclear reactions are not mutually exclusive; both types of process may contribute to a given reaction leading to a particular final state. For example one may visualise a glancing, peripheral collision resulting in a direct reaction and a more head-on collision leading to fusion into the compound nucleus and its subsequent decay. Their relative importance can also be expected to depend upon the bombarding energy. Figure 2.30 shows excitation functions (cross-section versus bombarding energy) for the inelastic scattering of nucleons exciting the first excited state of a medium-weight nucleus. At the lowest energies the compound nucleus formation dominates but its importance falls off as the energy increases and more open channels become available (see section 2.18.5). Meanwhile the direct component increases steadily until it begins to dominate at energies above 10 MeV or so.

In addition, we must recognise that in a given reaction there will usually be a

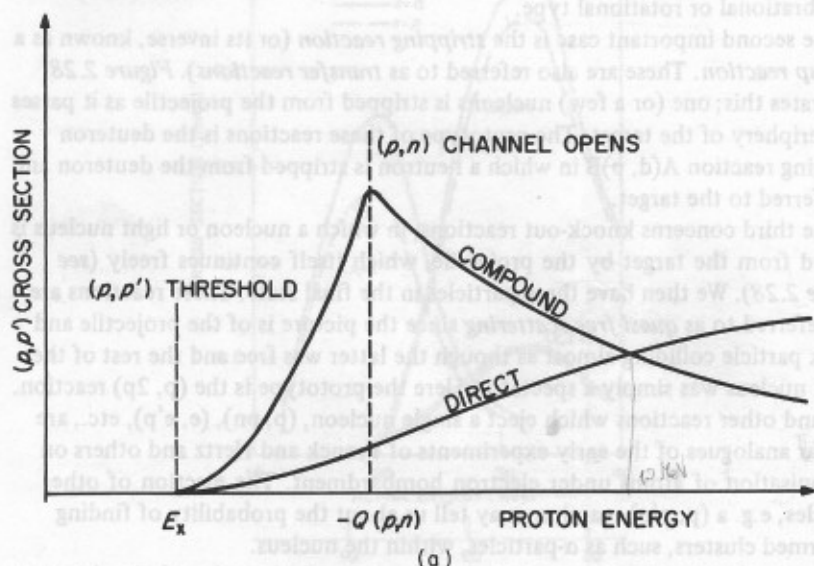


Figure 2.30 (a) Excitation functions for exciting the lowest excited state of a medium-weight nucleus by the inelastic scattering of nucleons: the cross-section for proton scattering is predominantly due to compound nucleus formation at the lowest energies until the threshold for the (p, n) reaction is reached, then neutron decay competes successfully until the direct reaction becomes most important

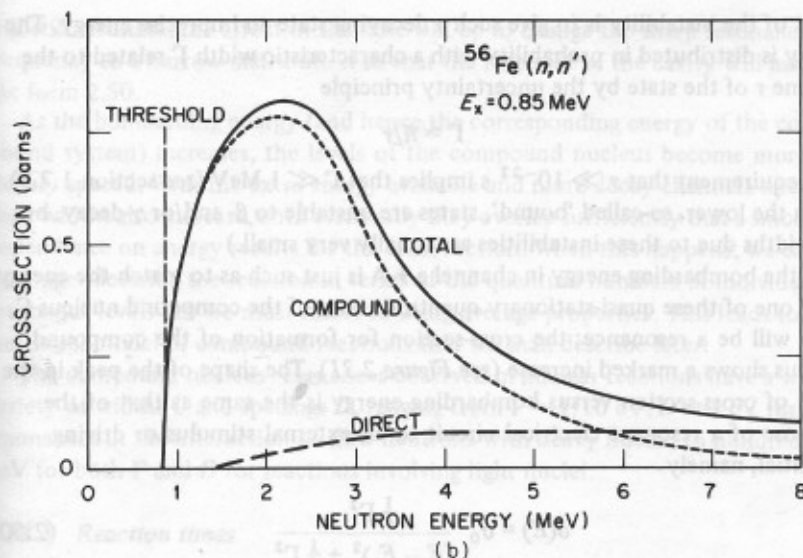


Figure 2.30 (b) Excitation functions for exciting the lowest excited state of a medium-weight nucleus by the inelastic scattering of nucleons: neutron scattering is also dominated by compound nucleus reactions at low energies until the increasing energy makes available many competing (n, n') reactions to higher excited states. (Based upon data and analyses of Kinney and Perey, 1970)

continuous spectrum of reaction processes between the two extreme ones we describe as direct and compound; there will be intermediate processes that do not clearly fall into either category. Until recently there had not been a wide interest in these intermediate processes when they occurred in reactions induced by light-ion projectiles, where they are referred to as *pre-equilibrium reactions* (see, for example, Blann, 1975; Hodgson, 1987, for a theoretical description of them), because they did not seem to exhibit any qualitatively new nuclear phenomena. This was not true for collisions between heavy-ion projectiles and heavy-nucleus targets (see section 2.18.12).

2.18.2 Compound resonances

A compound nucleus which is formed with a relatively low excitation energy may exhibit discrete quantum states even though it is unstable against particle emission. These states are merely an extension to higher energies of the discrete states observed at lower excitation energies below the threshold for particle emission. They have definite spin and parity, for example. Although enough energy is available for the emission of a particle, most of the time the energy is shared by several particles, none of which has by itself enough energy to escape. Hence the compound system may survive for a time long compared to a typical orbital period of a nucleon, 10^{-22} s. However it will decay eventually. The

effect of the instability is to give such a decaying state an imprecise energy. The energy is distributed in probability with a characteristic width Γ related to the lifetime τ of the state by the uncertainty principle

$$\Gamma \approx \hbar/\tau$$

The requirement that $\tau \gg 10^{-22}$ s implies that $\Gamma \ll 1$ MeV (see section 1.7.5). (Even the lower, so-called 'bound', states are unstable to β - and/or γ -decay, but the widths due to these instabilities are usually very small.)

If the bombarding energy in channel $a + A$ is just such as to match the energy E_r of one of these quasi-stationary quantum states of the compound nucleus C , there will be a resonance; the cross-section for formation of the compound nucleus shows a marked increase (see Figure 2.31). The shape of the peak in the curve of cross-section versus bombarding energy is the same as that of the response of a resonant electrical circuit to an external stimulus or driving potential, namely

$$\sigma(E) = \sigma_0 \frac{\frac{1}{4} \Gamma^2}{(E - E_r)^2 + \frac{1}{4} \Gamma^2} \quad (2.50)$$

Here σ_0 is the peak height (at $E = E_r$) and Γ is the full width at half maximum (FWHM). The shape 2.50 is referred to as a *Breit-Wigner resonance*.

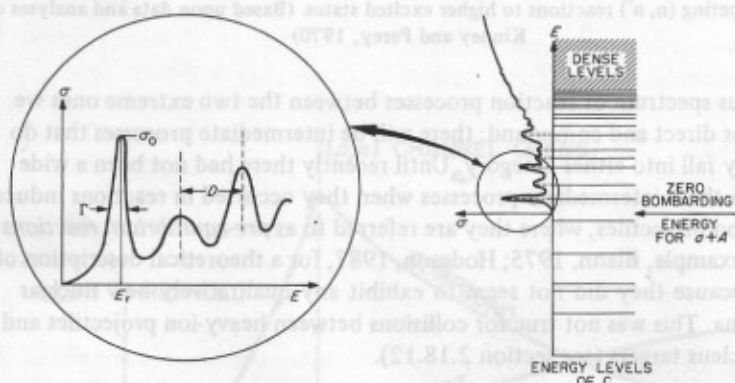


Figure 2.31 Excitation function for a reaction obtained by bombarding nucleus A with ions a . Each peak corresponds to a state of the compound system $C = A + a$, until the density of such levels becomes sufficiently high that individual ones cannot be resolved

Wave-guides provide an analogy to these resonant states of the compound nucleus. A closed cavity with perfectly reflecting walls has definite resonant frequencies for electromagnetic radiation within it which are determined by its shape and dimensions. These are like the bound levels of a nucleus. If we make a hole in the wall of the cavity, the radiation can escape; this is like the unbound nucleus. However if the hole is small so that the rate of loss is low, the cavity

will still resonate; the effect of the hole will be to change the sharp resonance frequency to a narrow distribution so that the response of the cavity will have the form 2.50.

As the bombarding energy (and hence the corresponding energy of the compound system) increases, the levels of the compound nucleus become more closely spaced. With the extra energy available and more decay channels open, their widths also increase, until eventually they overlap sufficiently that a smooth dependence on energy results for the cross-section. When this happens, we can no longer describe the reaction in terms of the quantum numbers of individual resonance levels but we must resort to using average properties. This leads to the statistical model of compound reactions that we shall describe later.

The compound nucleus resonances observed in nuclear reactions have a wide variety of widths Γ and spacings D , ranging from $\Gamma \sim 1/10$ eV, $D \sim 1$ eV for resonances in the interaction of slow neutrons with heavy nuclei, to hundreds of keV for both Γ and D for reactions involving light nuclei.

2.18.3 Reaction times

One essential feature distinguishing the two types of reaction, direct and compound, is the time required for each; the compound reaction is slow, the direct reaction is fast. The time scale by which we judge slow or fast is given by the motion of nucleons within the nucleus. A typical nucleon orbital period is a few times 10^{-22} s (corresponding to a kinetic energy of about 20 MeV) so that it will traverse the nucleus in about 10^{-22} s. If the collision is completed within this time or less, there is no time for appreciable sharing of energy between a struck nucleon and the other nucleons in the target. Formation of a compound system requires interaction over a much longer period, $\gg 10^{-22}$ s.

When we are dealing with distinct resonances, we can estimate the lifetime from the resonance width, using the uncertainty relation (section 1.7.5). Since $\tau \approx (6.6 \times 10^{-16} / \Gamma)$ s if Γ is expressed in eV, we see that lifetimes from 10^{-14} s to 10^{-20} s and less are encountered.

2.18.4 Energy spectra

The two reaction modes, compound and direct, are not mutually exclusive. We can expect both to be present in a given reaction (and perhaps modes intermediate between these two extremes). They will, however, tend to vary in relative importance with the Q -value of the reaction. We can understand this in the following terms. In the compound picture, the available energy is distributed more or less uniformly over the compound nucleus. The energy spectrum of the evaporated particles will reflect a Maxwell-type of distribution of nucleon velocities within the nucleus. The probability of finding one particle carrying most of the energy is small, so the spectrum peaks for the emission of low-energy particles. Consider a particular reaction $A(a, n)B$; the spectrum of energies

for the emitted neutrons might appear like the full curve in *Figure 2.32*, where $n(E)$ is the number emitted with energy between E and $E + dE$. The maximum energy E_m is determined by the bombarding energy E_a and the Q_0 -value; in the CMS, $E_m = E_a + Q_0$. The dashed curve is the spectrum one would expect from simple evaporation. The position of the peak of this distribution is determined by the *temperature*; i.e. the degree of excitation of the compound nucleus. (We return to this in Chapter 4.) Because of the peaking of the evaporation spectrum for neutrons at low energy, if enough energy is available usually we will find two or more low-energy neutrons emitted instead of one high-energy neutron.

In practice, however, one finds an excess (over the evaporation predictions) of particles with the higher energies. These we can identify as arising from direct (or semi-direct) reactions which do not inhibit the emission of energetic particles. Rather, as already remarked, the fast direct collision tends not to excite the nucleus very much, hence the energy loss tends to be small.

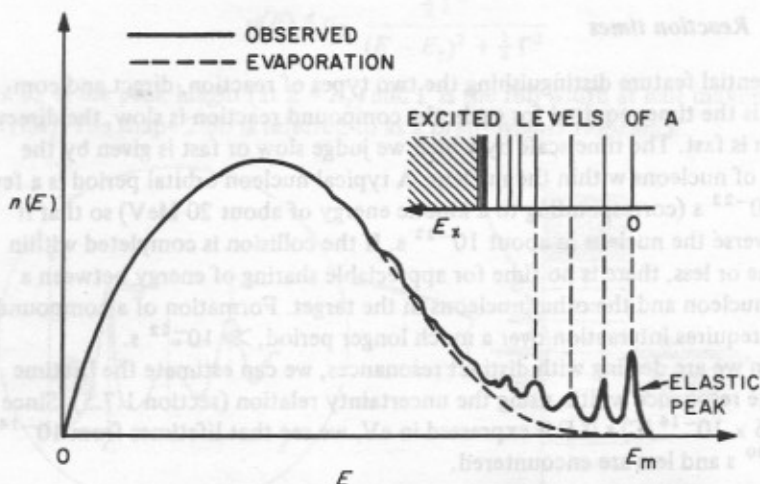


Figure 2.32 Typical spectrum of energies of the inelastic neutrons emitted by evaporation following compound nucleus formation in the reaction $A(n, n')A^*$. Peaks are observed for the neutrons with the highest energies, corresponding to discrete excited states of the target nucleus A

(The peaks shown at the upper end of the spectrum in *Figure 2.32* correspond to discrete, low-lying excited states of the residual nucleus, including the uppermost peak which corresponds to its ground state. If the incident beam had a very precise energy and our detecting system was perfect, each of these would show up as a sharp line. In practice, the beam will have a finite energy spread, the target material will have a finite thickness and the detectors will have a finite energy resolution so the lines are broadened into peaks.)

2.18.5 Branching ratios

By this we mean the relative probabilities for a colliding pair $A + a$ to result in the various exit channels $B + b$, etc. These relative probabilities are determined by *competition* when the reaction proceeds through a compound nucleus C . The hypothesis of the independence of formation and decay was expressed by equation 2.49. In this equation, the G_{Bb}^C represents the branching ratio for a particular $B + b$ channel. Since the total probability for decay is unity, we must have

$$\sum_{bB} G_{Bb}^C = 1$$

where the sum runs over all open channels (including $A + a$ for elastic scattering). Hence the value of G for a given channel depends strongly on how many other competing open channels there may be. For example, the elastic channel is always open, but if the bombarding energy is below the threshold for any other, non-elastic, channel, then we would have the maximum value of unity for G_{Aa}^C .

When the compound nucleus is highly excited and many decay channels are open, there are unlikely to be any special relationships between the compound system and a given final state. We then expect the branching probability G to depend smoothly only on simple properties like the excitation energy of the residual nucleus. This is the basis of the statistical model to be described in Chapter 4.

The factors governing the feeding of a particular channel by direct reaction are quite different. There is no formation and decay of an intermediate system so that competition between different modes of decay of the compound nucleus plays no role. The speed and simplicity of a direct reaction ensures that the target nucleus is only slightly rearranged to form the residual nucleus. In other words, the target and residual nuclei are very similar in structure; their wave functions have good overlap. As a corollary to this, if the structure of a particular residual nuclear state differs considerably from that of the target we will not expect it to be excited by a direct reaction mechanism. Hence there will be selectivity even among the low-lying excited states of the residual nucleus. Some will be fed strongly by the direct mode, some will only be fed weakly by the compound or some intermediate mode.

Although both compound and direct reaction components will generally contribute to a given reaction, some types of reactions will favour one over the other. For example, the spectrum shown in Figure 2.32 is typical of those for the inelastic scattering of neutrons of, say, 10 or 20 MeV. Most of the reactions proceed through the compound mode, but with an important high-energy direct component. Some reactions, such as the (p, d) reaction with protons of a few tens of MeV, appear to proceed predominantly through the direct mode (which, in this case, is the simple pick-up of a neutron from the nuclear surface). The relative importance of direct and compound processes also depends upon the

bombarding energy, as shown in *Figure 2.30*. The behaviour shown in *Figure 2.30* for the compound cross-section can be understood in terms of competition. As other channels become open, they compete for the flux from the compound nucleus decay so that the cross-section for this particular final state decreases. This is particularly marked for proton-induced reactions when the (p, n) threshold is reached; neutrons can escape much more readily than charged particles because they experience no Coulomb barrier (section 2.18.8) and tend to become the most important decay channel.

2.18.6 Importance of direct reactions

This tendency for the low-lying states of nuclei to be populated by direct reactions is one of the main reasons for the importance of these reactions. The nucleus represents a complicated many-body problem and in general we can only expect our theories of nuclear structure to give accurate and detailed explanations of the structure of these isolated low-energy quantum states. The direct reactions have a relatively simple nature, with no complications from an intermediate compound system, and are well suited to giving information about the relationship (overlap) between the ground state of the target nucleus and the ground or a particular excited state of the residual nucleus. For example, with a deuteron stripping (d, p) reaction we may learn to what degree a given residual nuclear state looks like the target ground state with the simple addition of one neutron in a particular shell model orbit. This gives us important information about the validity of the shell model and can provide tests of calculated shell model wave functions.

The dense, highly excited, states of nuclei generally have complicated structures and we can only expect to understand their properties in an average or statistical way. Hence, it is particularly appropriate that the compound reaction mechanism which tends to populate them is also one that can be described in similar terms.

2.18.7 Characteristic angular distributions

We have remarked that the differential cross-section for a reaction $A(a, b)B$ depends upon the angle of emission of the residual particle b relative to the incident beam. The reaction products are not, in general, emitted isotropically, that is to say in all directions with equal probability. This might seem surprising for a compound nucleus reaction, where there is supposed to be independence of formation and decay, so that the reaction products have forgotten the direction of the incident beam. In fact, such reactions exhibit angular distributions which are *not* isotropic although in many cases they do not vary strongly with angle. The explanation is that we can never have *complete* independence of formation and decay because certain constants of the motion must be conserved and their initial values are determined by the conditions in the entrance channel. The particular conserved quantity of importance here is the

angular momentum. The angular momentum L of the incident particle relative to the target nucleus is a vector which is always perpendicular to its direction of motion and hence has zero component in this direction. It is through this property that the incident direction influences the subsequent decay of the compound nucleus. If the intrinsic spins of the projectile and target are zero (or negligible compared to L), then L is also the spin of the compound nucleus, which is then oriented perpendicular to the direction of the incident beam. In the extreme classical limit of very large L , we can visualise the decay particles being flung preferentially from the equator of the rapidly rotating compound system. When we average over the orientation of this spinning system by averaging in azimuth around the incident direction, this 'water wheel' picture results in a preponderance of particles emitted equally in the forward and backward directions. Because the element of solid angle is $d\Omega = \sin \theta d\theta d\phi$, uniform emission from all parts of the equator gives a differential cross-section $d\sigma/d\Omega$ which is proportional to $1/\sin \theta$. However, unless the energy is high or the mass of the projectile is large (a heavy ion), the value of L will be rather small. In addition, the two particles often have intrinsic spins which will be randomly oriented and this will tend to reduce the degree of orientation of the spin of the compound nucleus. As a consequence, these compound reactions often do not show a marked anisotropy in the angular distribution of their products (*see Figure 2.33*). Exceptions occur when large L values are involved, as happens in heavy-ion collisions.

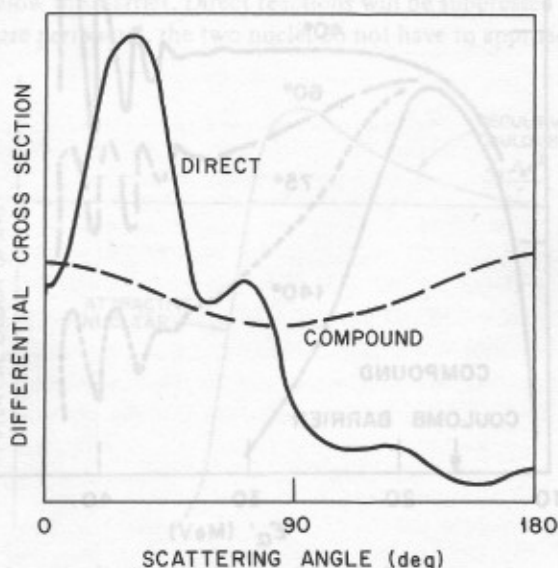


Figure 2.33 Typical angular distributions for direct and compound nucleus reactions induced by light ions with moderate bombarding energies (for example, some (d, p) reaction with 20-MeV deuterons)

Provided we are averaging over a sufficiently large number of compound nucleus states (either because our energy resolution is poor or because the states are dense and overlap strongly), the angular distribution will exhibit backward and forward symmetry. That is, the intensities at θ and $\pi - \theta$ will be equal. This is a characteristic of a compound nuclear reaction.

From our description of a direct reaction, the projectile suffers a glancing or peripheral collision with the target. It may lose some energy, or have one or a few nucleons transferred to or from it, but in any case, it will tend to continue moving in the forward direction. Hence the angular distribution of the emitted particle from a direct reaction will tend to be strongly peaked in the forward direction (Figure 2.33) and not exhibit a backward-forward symmetry. We shall see later that often we can learn from the form of these angular distributions important information about the angular momentum transferred between the particles during the collision, and in turn this can tell us about the spin and parity of the residual nuclear state.

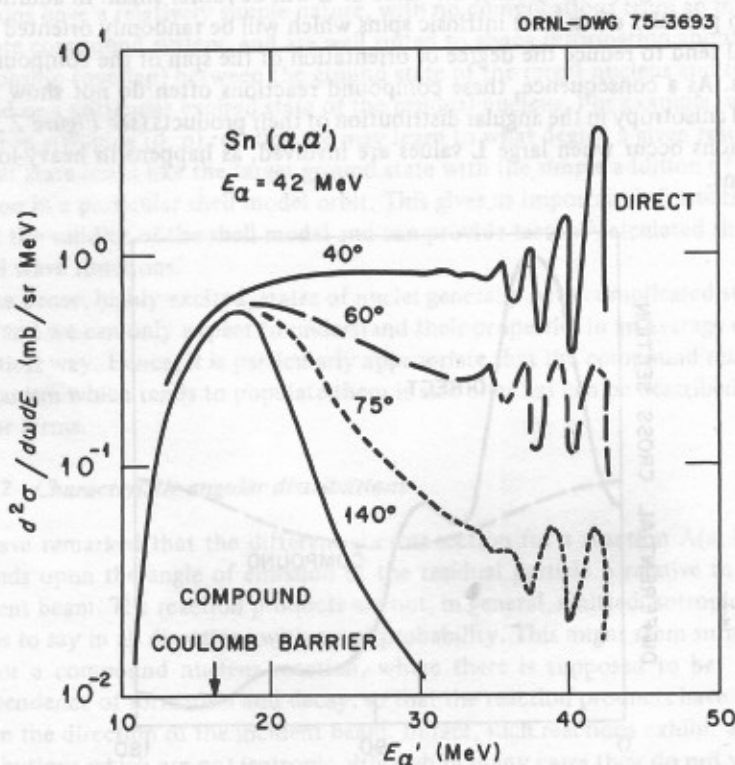


Figure 2.34 Spectra of the energies of α -particles inelastically scattered from Sn nuclei at various angles. α -particles with small energy losses (large $E_{\alpha'}$) show forward-peaked distributions characteristic of direct reactions. The low-energy α -particles are almost isotropic and more characteristic of evaporation from a compound nucleus. (After Chenevert *et al.*, 1971)

An example which illustrates several of the features just discussed is the $\text{Sn}(\alpha, \alpha')$ reaction at 42 MeV. Figure 2.34 shows the inelastic α -particle spectra as measured at several scattering angles. The most backward angle gives an evaporation type of spectrum which is typical of the compound nucleus process and in which the most probable energy is below 20 MeV (representing a large energy loss). As one comes to more forward angles the discrete low-excited states of the nucleus are more strongly excited as direct reactions become more important, illustrating both the feeding of low states (small energy loss) by the direct process and the forward-peaking associated with it.

2.18.8 Coulomb effects

Except for neutrons, all the particles involved in the usual nuclear reaction will be positively charged and hence repel each other. Once two nuclei a and A are close enough for the strong, attractive nuclear forces to act, this Coulomb repulsion is overwhelmed and is usually not very important (see Figure 2.35). At larger separations, however, it is far from being negligible and has an important influence on the probability of two nuclei coming into contact and undergoing a reaction. A proton approaching the lead nucleus, for example, will experience a repulsive Coulomb potential of about 13 MeV before the specifically nuclear forces begin to act. More highly charged projectiles experience correspondingly higher Coulomb barriers. Clearly the cross-section σ^C for forming a compound nucleus will be drastically reduced as the bombarding energy falls below this barrier. Direct reactions will be suppressed also, although, because they are peripheral, the two nuclei do not have to approach as closely

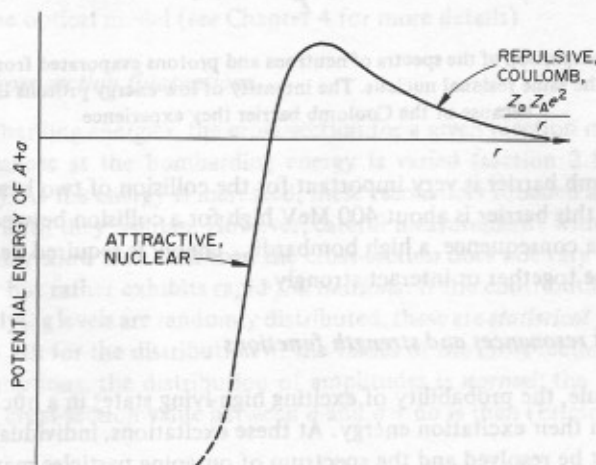


Figure 2.35 Schematic picture of the *Coulomb barrier* for two nuclei $a + A$. As the attractive nuclear potential begins to act, the total potential reaches a maximum then decreases to become negative (attractive). The repulsive part, due to the Coulomb potential, represents a barrier which tends to keep the nuclei apart

to experience a direct reaction as they do for complete coalescence into a compound system.

A charged particle within the nucleus, on the other hand, finds its escape is inhibited by this Coulomb barrier unless the energy available to it is comparable to or higher than the barrier (the classic example is the effect of barrier penetrability on α -decay; see Preston and Bhaduri, 1975). This means that the evaporation of low-energy charged particles from a compound nucleus will tend to be strongly inhibited relative to the emission of neutrons. It follows that the energy spectrum for a charged particle has its maximum shifted to a larger energy than that for neutron emission (see Figure 2.36 for a comparison of the spectra of emitted neutrons and protons).

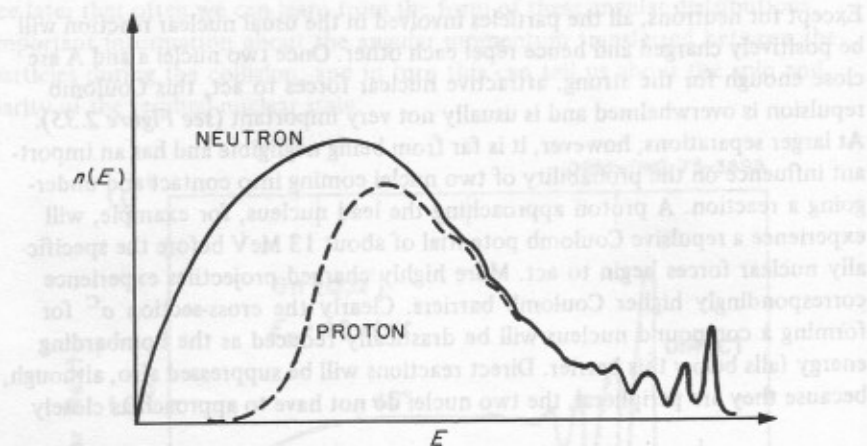


Figure 2.36 Comparison of the spectra of neutrons and protons evaporated from compound nuclei, leaving the same residual nucleus. The intensity of low-energy protons is depressed because of the Coulomb barrier they experience

The Coulomb barrier is very important for the collision of two heavy ions. For example, this barrier is about 400 MeV high for a collision between Xe and Pb nuclei. As a consequence, a high bombarding energy is required before these nuclei will fuse together or interact strongly.

2.18.9 Giant resonances and strength functions

As a general rule, the probability of exciting high-lying states in a nucleus varies smoothly with their excitation energy. At these excitations, individual states usually cannot be resolved and the spectrum of outgoing particles may look like those in Figures 2.32, 2.34, 2.36. Sometimes, however, a gross structure may be seen in the spectrum; this may be a concentration of excitation strength in a given region of excitation energy. An example was shown in Figure 2.27. Such a concentration is often referred to as a *giant resonance*. Although we associate

definite quantum numbers, such as spin and parity, with this peak, we do not interpret it as the excitation of a single level of the nucleus, for such a resonance may have a width of one or more MeV in a region where the underlying fine-structure levels are known to be separated by only a few eV. When seen in inelastic scattering measurements it is natural to identify these resonances with some kind of collective motion (see, for example, Satchler, 1978). For example, a liquid drop will be able to undergo oscillations in shape and a quantum state of such oscillation would be strongly excited by inelastic scattering. However, we realise that this liquid-drop picture is only a rough approximation to the average behaviour of a nucleus; when we take account of the individual nucleon motions, we find that such an oscillatory state is only describing an average property of the very many actual states possible. Each of the actual states has a certain probability of being excited. This probability fluctuates in strength from level to level, but when studied as a function of excitation energy it will show on average a marked increase in the region of the giant resonance. This distribution of average probability can be called a *strength function*. The concept is not restricted to inelastic scattering; indeed it was first introduced to describe the average cross-section for formation of a compound nucleus by the capture of low-energy (*s*-wave) neutrons. This average cross-section shows a giant resonance behaviour as a function of the mass number A of the target nucleus (see Figure 4.43); it is large for nuclei with $A \sim 60$ and $A \sim 150$ –190 but small in between for nuclei with $A \sim 100$. This behaviour is evidence for the partial transparency of nuclei to neutrons of low energy which then experience 'size-resonance' effects according to whether or not an integral number of their half-wavelengths will fit inside the nucleus. This can be described by the optical model (see Chapter 4 for more details).

2.18.10 Cross-section fluctuations

At low bombarding energies, the cross-section for a given reaction may show sharp resonances as the bombarding energy is varied (section 2.18.2 and Figure 2.31). As the energy is increased, these resonances broaden and become more dense until they overlap. However, careful measurements with good energy resolution may show that even then the cross-section does not vary smoothly with energy but rather exhibits rapid *fluctuations*. If the contributions from the many underlying levels are randomly distributed, these are *statistical fluctuations* and we may ask for the distribution of the values of the cross-section. Under simple assumptions, the distribution of amplitudes is *normal*; the probability of finding a cross-section value between σ and $\sigma + d\sigma$ is then (Ericson, 1963)

$$P(\sigma)d\sigma = (d\sigma/\bar{\sigma}) \exp(-\sigma/\bar{\sigma})$$

where $\bar{\sigma}$ is the mean value. With this distribution, the root mean square deviation is equal to the average cross-section itself

$$\left(\overline{\sigma^2} - \bar{\sigma}^2 \right)^{1/2} = \bar{\sigma}$$

Figure 2.37 shows the results of some measurements on the $^{12}\text{C}(^{16}\text{O}, \alpha)^{24}\text{Mg}$ reaction; see also Figure 2.27. A more detailed discussion of fluctuation phenomena is given in Chapter 4.

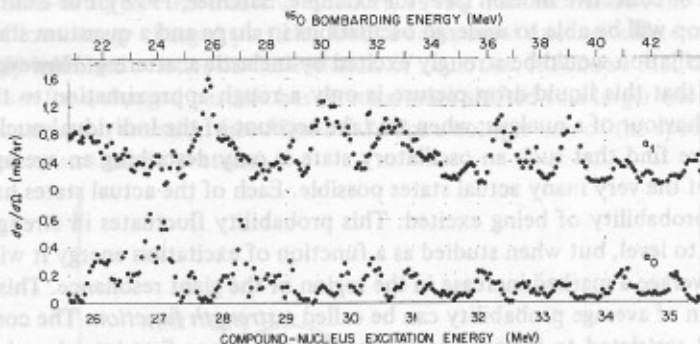


Figure 2.37 Excitation function (variation of the cross-section for a fixed angle as the energy is changed) for the reaction $^{12}\text{C}(^{16}\text{O}, \alpha)^{24}\text{Mg}$ at $\theta_L = 20^\circ$ showing how the cross-section fluctuates rapidly as the bombarding energy is varied. (From Halbert *et al.*, 1967)

2.18.11 Strong and weak absorption: diffraction and the optical model

If a spherical object is black to radiation of wavelength λ falling upon it, the cross-section for absorption is $\pi(R + \lambda)^2$ where R is its radius; when λ is small, this is just the geometric cross-section πR^2 . The angular distribution of the scattered radiation shows a Fraunhofer diffraction pattern. Such patterns are seen in many nuclear reactions, for example in the elastic scattering of α -particles (see Chapter 4 for examples, especially Figures 4.1, 4.5, 4.7, 4.20) and are said to be characteristic of *strong absorption*. Classical diffraction theory predicts that these patterns are a universal function of $R \sin \theta / \lambda = kR \sin \theta$, where θ is the scattering angle, and this is found to be approximately true. However, the scattering of low- and medium-energy neutrons and protons does not show a sharp diffraction pattern; rather it implies that target nuclei are partially transparent to nucleons. For example, the total cross-section for neutrons scattering from nuclei (Figure 2.6) oscillates about the black nucleus value $2\pi(R + \lambda)^2$ when the energy (or λ) or the target mass (or R) is varied. This can only mean that the neutron waves penetrate into the nuclear interior sufficiently for some resonance condition to be important. Thus the mean free path for neutrons within a nucleus must be comparable to the nuclear radius; this we call *weak absorption*. This observation of weak absorption phenomena led to the introduction of the 'cloudy crystal ball' model, now called the optical model (Chapter 4), which can explain these phenomena quantitatively.

2.18.12 *Some characteristics of heavy-ion reactions*

Reactions between two heavy nuclei exhibit an even richer variety of phenomena than those between a light projectile and a heavy target nucleus. (A 'heavy' ion is defined as a nucleus with mass $A > 4$.) However, a few simple characteristics can be remarked upon. Firstly, the large mass means a proportionately greater momentum for a given energy and consequently a greater amount of *angular momentum** about the centre of a struck target nucleus (see equation 2.43). This facilitates the excitation of nuclear states with very high spin; for example, states with spin as large as $60\hbar$ have been excited in (HI, xn) reactions (this means a reaction in which a heavy ion (HI) projectile, e.g. ^{16}O , is absorbed by a target nucleus and some of the excitation energy is lost by the evaporation of a number x of neutrons). This large angular momentum and the associated kinetic energy is distributed over the many nucleons in the heavy projectile which makes the formation of a compound nucleus more probable than if it were concentrated upon one or just a few nucleons as in a high-energy light-ion reaction. We can then study, for example, whether there are limits to the amount of angular momentum that a compound nucleus can sustain so that it becomes unstable and flies apart under the influence of the centrifugal forces. Indeed, it is found that for angular momenta larger than a critical value, the compound nucleus rather quickly fissions into two large fragments.

Secondly, reactions between heavy ions are always characterised by strong absorption; that is to say, once they approach within the range of their strong interactions, some non-elastic event will occur. Further, heavy-ion reactions show much more strongly than reactions with light-ion projectiles classes of phenomena intermediate between the extreme peripheral (or direct) reactions occurring at grazing impact and the complete fusion (or compound nuclear) reactions resulting from more head-on collisions (see, for example, Schroder and Huizenga, 1984). One sees phenomena which appear to result from the two heavy nuclei sticking together for a short time, longer than one would associate with a direct reaction, but not long enough for the two systems to be regarded as having fused into a compound nucleus. There tends to be a large loss of kinetic energy in such collisions (hence they are sometimes called *deep inelastic* or *strongly damped* collisions), the energy reappearing as internal excitation ('heat') energy, but the system still retains 'memory' of how it was formed by showing, for example, a forward-peaked angular distribution. In addition, even when there is complete fusion, it is not always followed by evaporation, as is characteristic of light-ion reactions, but there is often a strong probability that the compound nucleus will fission into two large fragments.

The large mass of a heavy ion means the associated wavelength is shorter than

*For example, a Zn nucleus with a bombarding energy of 400 MeV (only 6.25 MeV per nucleon) carries an angular momentum of nearly $200\hbar$ when it makes a grazing collision with an Sn nucleus.

for a light ion with the same energy. In many cases the wavelength is short enough for it to be meaningful to use classical concepts to describe a collision, such as speaking of the ion following a classical trajectory. This has led to a resurgence of interest in semi-classical theories of scattering. Even if a full quantal treatment is needed to account for all the details of a reaction, often considerable physical insight can be gained from the semi-classical description.

Because the charge on a heavy-ion projectile may be quite large, high energies are required to overcome the Coulomb repulsion between projectile and target nuclei. As an extreme example, the Coulomb barrier to be surmounted before two uranium nuclei make contact is about 700 MeV. This requires a bombarding energy in the laboratory system of about 1400 MeV. However, the *average energy per nucleon* in the projectile is not high; only about 6 MeV per nucleon in this example and 5–50 MeV per nucleon are typical figures for many experiments, although some measurements are now being made at several hundred MeV per nucleon. The study of heavy-ion interactions is a rapidly expanding subject; accelerators are now available that can produce projectile ions with energies ranging from a few MeV per nucleon to as much as 200 GeV per nucleon, and which can accelerate ions as heavy as uranium.

It is useful to break up this enormous range of possibilities in a way which reflects the kind of physics which can be studied. At energies below the threshold for pion production (about 140 MeV in the CMS), we need only consider nuclei as made of individual nucleons. A characteristic length is ~ 1 fm (or the wavelength of the most energetic nucleon in a Fermi gas with the density of nuclear matter, $\lambda_F \approx 0.7$ fm). A characteristic energy is a few tens of MeV (or the maximum kinetic energy of a nucleon in the same Fermi gas, $E_F \approx 40$ MeV).

The velocity of sound in nuclear matter is estimated to be equal to that of a nucleon with a kinetic energy of about 20 MeV (or a speed of about one-fifth the velocity of light). This leads us to subdivide the 'low' energy region into subsonic and supersonic sections. In the subsonic region we expect nuclei to behave as though they were incompressible. However, when we have reactions at supersonic energies we may see compressibility playing a role; for example, we might generate shock waves in nuclei. The properties of nuclear matter at above-normal density have not yet been studied.

Above the pion-production threshold we can no longer expect to be able to ignore the substructure of nucleons. Nucleon isobars may be produced, for example. Eventually we reach energies (~ 1 GeV per nucleon) which are comparable to or greater than the energy of the rest mass of a nucleon; in this domain we cannot escape the consequences of relativity. It has been suggested that we might create a new abnormal or condensed state of nuclear matter in which the nucleons dissolve into a plasma of their constituent quarks and gluons. To induce this state, we need nuclear matter that is hotter and considerably more dense than normal. The possibility of producing this in the collision of ultrarelativistic heavy ions (~ 200 GeV per nucleon) with targets of heavy nuclei is presently being explored experimentally.

Because heavy-ion collisions involve the interactions between relatively large pieces of nuclear matter in which the kinetic energy is shared among all the nucleons, they are particularly suitable for the study of the *macroscopic* features of nuclear behaviour. These are most easily described in terms more appropriate to the motions of liquid droplets rather than the motions of one or a few nucleons. For example, we find the concepts of viscosity and friction being invoked. The deep inelastic scattering already mentioned is one of these features. Another possibility is the formation of nuclear 'molecules' (Vogt and McManus, 1960) in which two nuclei make sufficient contact to be attracted to one another, but in which they do not completely fuse to form the usual kind of compound nucleus. This structure could be recognised by the large probability that it would decay by breaking up into the same two nuclei by which it was formed; it is not a compound nucleus in the usual sense because it 'remembers' the entrance channel and the independence hypothesis is not obeyed.

It has been suggested that nuclei with masses $A \sim 310$ may be stable although they have not yet been found in nature. One of the attractions of heavy-ion physics has been the possibility of forming these *super-heavy* elements by fusing two heavy ions. At the other extreme there is the possibility of using the fusion of two very light nuclei as a power source (Appendix D).

A thorough treatment of heavy-ion reactions is to be found in Bass (1980) and Bromley (1984).