Chapter 11

NUCLEAR ASTROPHYSICS

11.1 Standard stellar evolution

11.1.1 Hydrogen and CNO cycles

The energy production in the stars is a well known process. The initial energy which ignites the process arises from the gravitational contraction of a mass of gas. The contraction increases the pressure, temperature, and density, at the center of the star until values able to start the thermonuclear reactions (see Supplement 11.A), initiating the star lifetime. The energy liberated in these reactions yield a pressure in the plasma, which opposes compression due to gravitation. Thus, an equilibrium is reached for the energy which is produced, the energy which is liberated by radiation, the temperature, and the pressure.

The Sun is a star in its initial phase of evolution. The temperature in its surface is 6000° C, while in its interior the temperature reaches 1.5×10^7 K, with a pressure given by 6×10^{11} atm and density 150 g/cm^3. The present mass of the Sun is $M_\odot = 2 \times 10^{33}$ g and its main composition is hydrogen (70%), helium (29%) and less than 1% of more heavy elements, like carbon, oxygen, etc.

What are the nuclear processes which originate the huge thermonuclear energy of the Sun, and that has last 4.6×10^9 years (the assumed age of the Sun)? It cannot be the simple fusion of two protons, or of α-particles, or even the fusion of protons with α-particles, since neither ${}^2_2$He, ${}^8_4$Be, or ${}^5_3$Li, are not stable. The only possibility is the proton-proton fusion in the form

$$\mathrm{p} + \mathrm{p} \longrightarrow \mathrm{d} + \mathrm{e}^+ + \nu_e, \qquad (11.1)$$

which occurs via the β-decay, i.e., due to the weak-interaction. The cross section for this reaction for protons of energy around 1 MeV is very small, of the order of 10^{-23} b. The average lifetime of protons in the Sun due to the transformation to deuterons by means of Eq. 11.1 is about 10^{10} y. This explains why the energy radiated from the Sun is approximately constant in time, and not by means of an explosive process.

The deuteron produced in the above reaction is consumed almost immediately in the process

$$\mathrm{d} + \mathrm{p} \longrightarrow {}^3_2\mathrm{He} + \gamma. \qquad (11.2)$$

The resulting ${}^3_2$He reacts by means of

$${}^3_2\mathrm{He} + {}^3_2\mathrm{He} \longrightarrow {}^4_2\mathrm{He} + 2\mathrm{p}, \qquad (11.3)$$

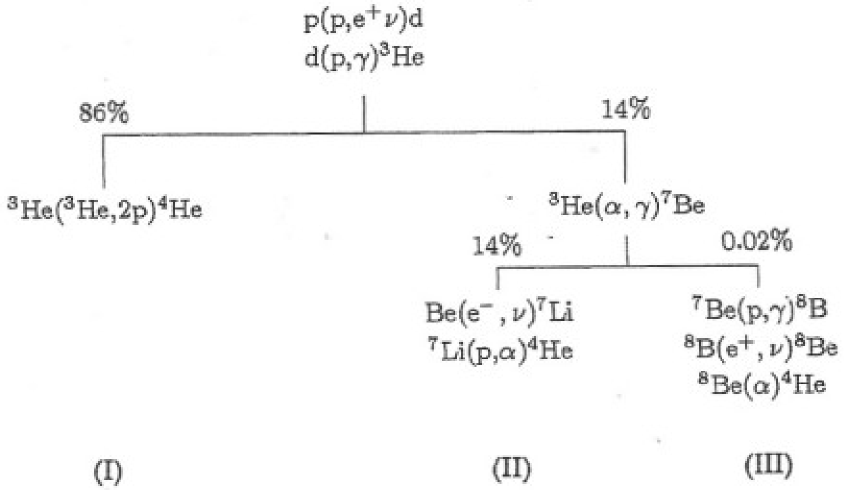

Figure 1 The chain reaction (p-p cycle). The percentage for the several branches are calculated in the center of the sun [1].

which produces the stable nucleus ${}^4_2\mathrm{He}$ with a great energy gain, or by means of the reaction

$$ {}^3_2\mathrm{He} + {}^4_2\mathrm{He} \longrightarrow {}^7_4\mathrm{Be} + \gamma. \tag{11.4}$$

In the second case, a chain reaction follows as

$$ {}^7_4\mathrm{Be} + e^- \longrightarrow {}^7_3\mathrm{Li} + \nu_e, \qquad {}^7_3\mathrm{Li} + p \longrightarrow 2\left({}^4_2\mathrm{He}\right), \tag{11.5}$$

or

$$ {}^7_4\mathrm{Be} + p \longrightarrow {}^8_5\mathrm{B} + \gamma, \qquad {}^8_5\mathrm{B} \longrightarrow 2\left({}^4_2\mathrm{He}\right) + e^+ + \nu_e. \tag{11.6}$$

The chain reaction 11.1-11.6 is called the hydrogen cycle. The result of this cycle is the transformation of four protons in an α-particle, with an energy gain of 26,7 MeV, about 20% of which are carried away by the neutrinos (see Fig. 1).

If the gas which gives birth to the star contains heavier elements, another cycle can occur; the *carbon cycle*, or *CNO cycle*. In this cycle the carbon, oxygen and nitrogen nuclei are catalyzers of nuclear processes, with the end product also in the form $4p \longrightarrow {}^4_2\mathrm{He}$. Fig. 2(a) describes the CNO cycle. Due to the larger Coulomb repulsion between the carbon nuclei, it occurs at higher temperatures (larger relative energy between the participant nuclei), up to 1.4×10^7 K. In the Sun the hydrogen cycle prevails. But, in stars with larger temperatures the CNO cycle is more important. Fig. 2(b) compares the energy production in stars for the hydrogen and for the CNO cycle as a function of the temperature at their center. For the Sun temperature, $T_\odot$, we see that the pp cycle is more efficient.

After the protons are transformed into helium at the center of a star like our Sun, the fusion reactions start to consume protons at the surface of the star. At the

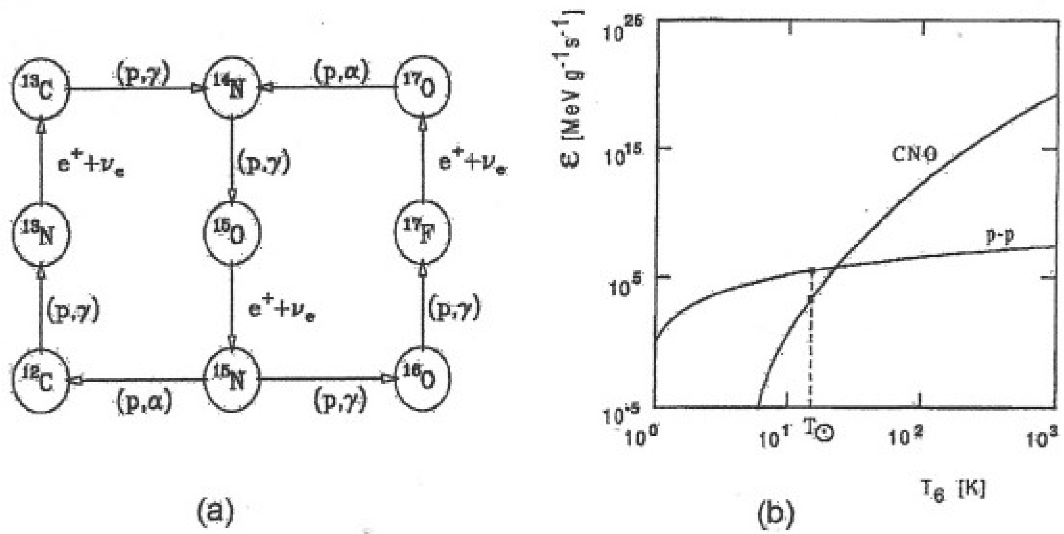

Figure 2 (a) The CNO cycle. (b) Comparison of the energy production in the pp and in the CNO cycle as a function of the star temperature [2].

stage the star starts to become a *red giant*. The energy generated by fusion increases the temperature and expands the surface of the star. The star luminosity increases. The red giant contracts again after the hydrogen fuel is burned.

Other thermonuclear processes start. The first is the helium burning when the temperature reaches 10^8 K and the density becomes 10^6 g.cm^{-3} Helium burning starts with the triple capture reaction

$$3\,({}^{4}_{2}\mathrm{He}) \longrightarrow \ {}^{12}_{6}\mathrm{C} + 7.65 \text{ MeV}, \tag{11.7}$$

followed by the formation of oxygen via the reaction

$${}^{12}_{6}\mathrm{C} + \ {}^{4}_{2}\mathrm{He} \longrightarrow \ {}^{16}_{8}\mathrm{O} + \gamma. \tag{11.8}$$

For a star with the Sun mass, helium burning occurs in about 10^7 y. For a much heavier star the temperature can reach 10^9 K. The compression process followed by the burning of heavier elements can lead to the formation of iron. After that the thermonuclear reactions are no more energetic and the star stops producing nuclear energy.

11.1.2 White dwarfs and neutron stars

If the thermonuclear processes in massive stars achieve the production of iron, there are the following possibilities for the star evolution.

(a) For stars with masses $< 1,2\ M_\odot$ the internal pressure of the degenerated electron gas (i.e., when the electrons occupy all states allowed by the Pauli principle) does not allow the star compression due to the gravitational attraction to continue indefinitely. For a free electron gas at temperature $T = 0$ (lowest energy state), the electrons occupy all energy states up to the Fermi energy. The total density of the star can be calculated adding up the individual electronic energies. Since each phase-space cell $d^3p \cdot V$ (where V is the volume occupied by the electrons) contains $d^3p \cdot V/(2\pi\hbar)^3$ states, we get

$$\begin{aligned} \frac{E}{V} &= 2\int_0^{p_F} \frac{d^3p}{(2\pi\hbar)^3}\, E(p) = 2\int_0^{p_F} \frac{d^3p}{(2\pi\hbar)^3} \sqrt{p^2c^2 + m_e^2c^4} = n_0 m_e c^2 x^3 \epsilon(x), \\ \epsilon(x) &= \frac{3}{8x^3}\left\{x(1+2x^2)(1+x^2)^{1/2} - \log[x + (1+x)^{1/2}]\right\}, \end{aligned} \tag{11.9}$$

where the factor 2 is due to the electron spin, and

$$x = \frac{p_F c}{m_e c^2} = \left(\frac{n}{n_0}\right)^{1/3} = \left(\frac{\rho}{\rho_0}\right)^{1/3}, \tag{11.10}$$

where

$$n_0 = \frac{me^3c^3}{\hbar^3} \quad \text{and} \quad \rho_0 = \frac{m_N n_0}{Y_e} = 9,79 \times 10^5\ Y_e^{-1}\ \text{g/cm}^3. \tag{11.11}$$

In the above relations p_F is the Fermi momentum of the electrons, m_e (m_N) is the electron (nucleon) mass, n is the density of electrons, and ρ is the mass density in the star. Y_e is the number of electrons per nucleon.

The variable x characterizes the electron density in terms of

$$n_0 = 5,89 \times 10^{29}\ \text{cm}^{-3}. \tag{11.12}$$

At this density the Fermi momentum is equal to the inverse of the Compton wavelength of the electron.

Using traditional methods of thermodynamics, the pressure is related to the energy variation by

$$P = -\frac{\partial E}{\partial V} = -\frac{\partial E}{\partial x}\frac{\partial x}{\partial V} = -\frac{\partial E}{\partial x}\left(-\frac{x}{3V}\right) = \frac{1}{3}\, n_0 m_e c^2 x^4 \frac{d\epsilon}{dx}. \tag{11.13}$$

This model allows us to calculate the pressure in the electron gas in a very simple form. Since the pressure increases with the electron density, which increases with the decreasing volume of the star, we expect that the gravitational collapse stops when the electronic pressure equals the gravitational pressure. When this occurs the star cools slowly and its luminosity decreases. The star becomes a *white dwarf* and in some cases its diameter can become smaller than that of the Moon.

(b) For stars with masses in the interval 1.2 - 1.6 $M_\odot$, the electron pressure is not sufficient to balance the gravitational attraction. The density increases to 2×10^{14}g.cm^{-3} and the matter "neutronizes". This occurs via the electron capture by the nuclei (inverse beta decay), transforming protons into neutrons. The final product is a *neutron star*, with a small radius. For example, if it were possible to form a neutron star from the Sun it would have a radius given by

$$\left(\frac{M_{\odot}}{\frac{4\pi}{3}\rho}\right)^{1/3} = \left(\frac{2\times10^{33}\ \mathrm{g}}{\frac{4\pi}{3}\times 2\times10^{14}\ \mathrm{g\,cm^{-3}}}\right)^{1/3} \simeq 14\ \mathrm{km}.$$

The process of transformation of iron nuclei into neutron matter occurs as following: for densities of the order of 1.15×10^{9} g.cm^{-3} the Fermi energy of the electron gas is larger than the upper energy of the energy spectrum for the β-decay of the isotope $^{56}_{25}$Mn. The decay of this isotope can be inverted and two neutron-rich isotopes of $^{56}_{25}$Mn are formed, i.e.,

$$^{56}_{26}\mathrm{Fe} + \ e^- \longrightarrow \ ^{56}_{25}\mathrm{Mn} + \nu_e, \tag{11.14}$$

These nuclei transform in $^{56}_{24}$Cr by means of the reaction

$$^{56}_{25}\mathrm{Mn} + \ e^- \longrightarrow \ ^{56}_{24}\mathrm{Cr} + \nu_e. \tag{11.15}$$

With the increasing of the pressure more isotopes can be formed, until neutrons start being emitted:

$$^{A}_{Z}\mathrm{X} + \ e^- \longrightarrow \ ^{A-1}_{Z-1}\mathrm{X} + \mathrm{n} + \nu_e. \tag{11.16}$$

For $^{56}_{26}$Fe this reaction network starts to occur at an energy of 22 MeV, which corresponds to a density of 4×10^{11} g$\cdot$cm^{-3}. With increasing density, the number of free neutrons increases and, when the density reaches 2×10^{14} g$\cdot$cm^{-3}, the density of free neutrons is 100 times larger than the density of the remaining electrons.

11.1.3 Synthesis of elements

In Figure 3(a) we show the relative distribution of elements in our galaxy. It has two distinct regions: in the region $A < 100$ it decreases with A approximately like an exponential, whereas for $A > 100$ it is approximately constant, except for the peaks in the region of the magic numbers $Z = 50$ e $N = 50$, 82, 126.

The thermonuclear processes 11.1-11.8 can explain the relative abundance of $^{4}_{2}$He, $^{12}_{6}$C and $^{16}_{8}$O. The processes occurring after $^{4}_{2}$He burning mainly form isotopes of $^{20}_{10}$Ne, $^{24}_{12}$Mg and $^{28}_{14}$Si. We can understand the small abundance of the elements Li, Be and B as due to the small velocity with which they are formed via the reaction 11.4 and the first equation of 11.5, while they are rapidly consumed by the second reaction in 11.5 and the first reaction in 11.6.

The synthesis of elements heavier than oxygen occur when, after the helium burn, a new compression and heating of the star rises the temperature to values higher than 6×10^{8} K. This situation ignites an intense carbon burning:

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \ \longrightarrow \ ^{20}_{10}\mathrm{Ne} + \ ^{4}_{2}\mathrm{He} \longrightarrow \ ^{23}_{11}\mathrm{Na} + \ \mathrm{p} \longrightarrow \ ^{23}_{12}\mathrm{Mg} + \ \mathrm{n} \longrightarrow \ ^{24}_{12}\mathrm{Mg} + \ \gamma. \tag{11.17}$$

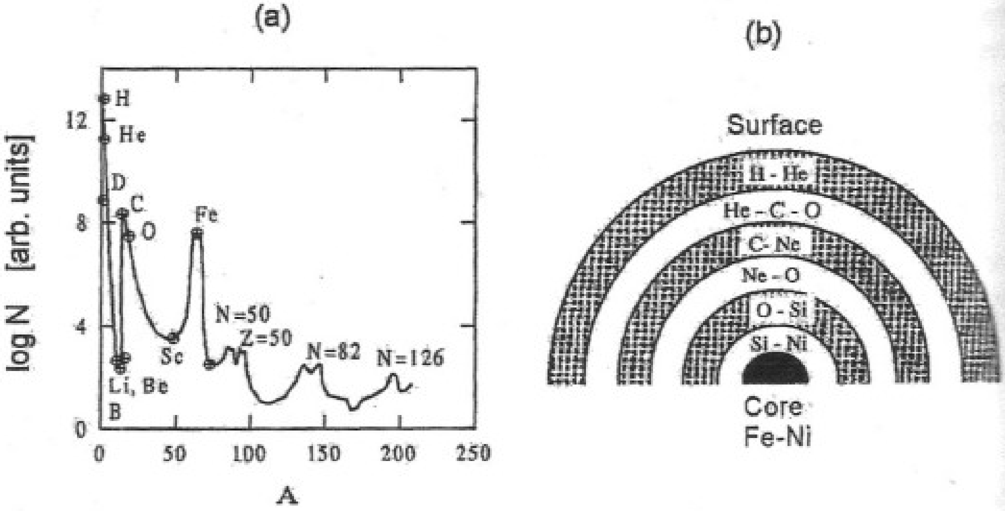

Figure 3 (a) Relative distribution of elements in our galaxy. (b) The "onion" structure of a supernova.

Carbon and oxygen can also burn simultaneously:

$$^{12}_{6}\mathrm{C} + \,^{16}_{8}\mathrm{O} \longrightarrow \,^{24}_{12}\mathrm{Mg} + \,^{4}_{2}\mathrm{He}, \text{ etc}, \qquad ^{16}_{8}\mathrm{O} + \,^{16}_{8}\mathrm{O} \longrightarrow \,^{28}_{14}\mathrm{Si} + \,^{4}_{2}\mathrm{He}, \text{ etc.} \tag{11.18}$$

For temperatures above 3×10^{9} K more photo-nuclear processes appear. These yield more nuclei to be burned and heavier nuclei are produced:

$$\gamma + \,^{28}_{14}\mathrm{Si} \longrightarrow \,^{24}_{12}\mathrm{Mg} + \,^{4}_{2}\mathrm{He}, \qquad ^{4}_{2}\mathrm{He} + \,^{28}_{14}\mathrm{O} \longrightarrow \,^{32}_{16}\mathrm{S} + \gamma, \text{ etc.} \tag{11.19}$$

Due to the large number of free neutrons, many (n,γ)-reactions (radiative neutron capture) elements in the mass range $A = 28, \ldots, 57$ are formed. This leads to a large abundance of elements in the iron mass region, which have the largest binding energy per nucleon. For elements heavier than iron the nuclear fusion processes do not generate energy.

For $A > 100$ the distribution of nuclei cannot be explained in terms of fusion reactions with charged particles. They are formed by the successive capture of slow neutrons and of β^{-}-decay. The maxima of the element distribution in $N = 50, 82,$ 126 are due to the small capture cross sections corresponding to the magic numbers. This yields a trash of isotopes at the observed element distribution.

11.1.4 Supernovae explosions

It has long been observed that, occasionally, a new star appears in the sky, increases its brightness to a maximum value, and decays afterwards until its visual disappearance. Such stars were called by *novae*. Among the novae some stars present an exceptional variation in their brightness and are called by *supernovae*.

Schematically a pre-supernova has the onion structure presented in figure 3(b). Starting from the center of the star, we first find a core of iron, the remnant of silicon burning. After that we pass by successive regions where ^{28}Si, ^{16}O, ^{12}C, ^{4}He, and ^{1}H form the dominant fraction. In the interfaces, the nuclear burning continues to happen.

The silicon burning exhausts the nuclear fuel. As we mentioned previously, the gravitational collapse of the iron core cannot be hold by means of pressure heat from nuclear reactions. However, Chandrasekhar [3] showed that a total collapse can be avoided by the electronic pressure. In this situation, the core is stabilized due to the pressure of the degenerated electron gas, $P(r)$, and the inward gravitational pressure. This means that for a given point inside the star,

$$\begin{aligned} -\frac{Gm(r)}{r^2}\,\rho(r) &= \frac{dP(r)}{dr} = \frac{d\rho}{dr}\,\frac{dP}{d\rho}, \\ \frac{dm}{dr} &= 4\pi r^2 \rho(r), \\ \frac{dP}{d\rho} &= Y_e\,\frac{m_e}{M_N}\,\frac{x^2}{3\sqrt{1+x^2}}. \end{aligned} \tag{11.20}$$

where m_e and M_N are defined following Eq. 11.9.

This model is appropriate for a non-rotating white dwarf. With the boundary conditions $m(r=0)=0$ and $\rho(r=0)=\rho_c$ (the central density), these equations can be solved easily [4]. For a given Y_e, the model is totally determined by ρ_c. Figure 4 shows the mass density of a white dwarf. We observe that the total mass of a white dwarf (of the order of a solar mass, $M_\odot = 1.98\times10^{33}$ g), increases with ρ_c. Nonetheless, and perhaps the most important, it cannot exceed the finite value of

$$M \le M_{Ch} \simeq 1.45\,(2Y_e)^2\,M_\odot, \tag{11.21}$$

which is know as the *Chandrasekhar mass* [3]. Applying these results to the nucleus of a star with any mass, we get from Eq. 11.20 that stars with mass $M > M_{Ch}$ cannot be stable against the gravitational collapse by the pressure of the degenerate electron gas. The collapse occurs inevitably for a massive star, since the silicon burning adds more and more material to the stellar core.

At the beginning of the collapse the temperature and density are of the order of $T\sim10^{10}$ K and $\rho\sim3\times10^{9}$ g/cm^3. The core is made of ^{56}Fe and of electrons. There are two possibilities, both accelerating the collapse:

1. At conditions present in the collapse the strong reactions and the electromagnetic reactions between the nuclei are in equilibrium with their inverse, i.e.,

$$\gamma + {}^{56}_{26}\mathrm{Fe} \Longleftrightarrow 13(^4\mathrm{He}) + 4\mathrm{n} - 124\ \mathrm{MeV}. \tag{11.22}$$

For example, with $\rho = 3\times10^9$ g/cm^3 and $T = 11\times10^9$ K, half of ^{56}Fe is dissociated. This dissociation takes energy from the core and causes pressure loss. The collapse is thus accelerated.

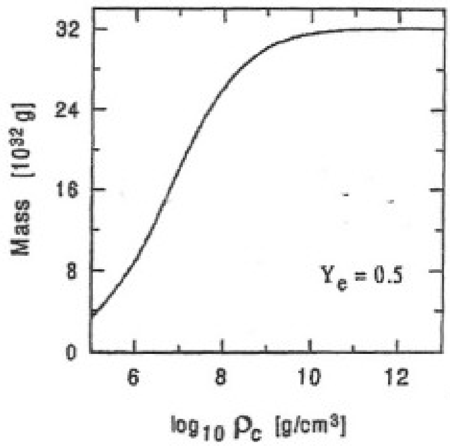

Figure 4 Masses of white dwarfs calculated as a function of ρ_c, the central density. With increasing ρ_c, the mass reaches a limiting value, the Chandrasekhar mass.

2. If the mass of the core exceeds M_{Ch}, electrons are captured by the nuclei to avoid the violation of the Pauli principle:

$$e^- + (Z, A) \longrightarrow (Z-1, A) + \nu_e. \qquad (11.23)$$

The neutrinos can escape the core, taking away energy. This is again accompanied by a pressure loss due to the decrease of the free electrons (this also decreases M_{Ch}). The collapse is again accelerated.

The gravitational contraction increases the temperature and density of the core. An important change in the physics of the collapse occurs when the density reaches $\rho_{\rm trap} \simeq 4 \times 10^{11}$ g/cm^3. The neutrinos become essentially confined to the core, since their diffusion time in the core is larger than the collapse time. After the neutrino confinement no energy is taken out of the core. Also, all reactions are in equilibrium, including the capture process 11.23. The degeneracy of the neutrino Fermi gas avoids a complete neutronization, directing the reaction 11.23 to the left. As a consequence, Y_e remains large during the collapse ($Y_e \approx$ 0.3-0.4 [5]). To equilibrate the charge, the number of protons must also be large. To reach $Z/A = Y_e \approx$ 0,3-0,4, the protons must be inside heavy nuclei which will therefore survive the collapse.

Two consequence follows:

1. The pressure is given by the degenerate electron gas that controls the whole collapse; the collapse is thus adiabatic, with the important consequence that the collapse of the most internal part of the core is *homologous*, i.e., the position $r(t)$ and the velocity $v(t)$ of a given element of mass of the core are related by

$$r(t) = \alpha(t) r_0; \qquad v(t) = \frac{\dot{\alpha}}{\alpha} r(t), \qquad (11.24)$$

where r_0 is the initial position.

2. Since the nuclei remain in the core of the star, the collapse has a reasonably large order and the entropy remains small during the collapse [5]. ($S \approx 1.5\ k$ per nucleon, where k is the Boltzmann constant).

The collapse continues homologously until nuclear densities of the order of $\rho_N \approx 10^{14}$ g/cm^3 are reached, when the matter can be thought as approximately a degenerate Fermi gas of nucleons. Since the nuclear matter has a finite compressibility, the homologous core deccelerate and starts to increase again as a response to the increase of the nuclear matter. This eventually leads to a *shock wave* which propagates to the external core (i.e., the iron core outside the homologous core) which, during the collapse time, continued to contract reaching the supersonic velocity. The collapse break followed by the shock wave is the mechanism which breads the supernova explosion. Nonetheless, several ingredients of this scenario are still unknown, including the equation of state of the nuclear matter. The compressibility influences the available energy for the shock wave, which must be of the order of 10^{51} erg.

The exact mechanism for the explosion of a supernova is still controversial.

1. In the *direct mechanism*, the shock wave is not only strong enough to stop the collapse, but also to explode the exterior stellar shells.

2. If the energy in the shock wave is insufficient for a direct explosion, the wave will deposit its energy in the exterior of the core, e.g., by excitation of the nuclei, what is frequently followed by electronic capture and emission of neutrinos (*neutrino eruption*). Additionally, neutrinos of the all three species are generated by the production of pairs in the hot environment. A new shock wave can be generated by the outward diffusion of neutrinos, what indeed carries the most part of the energy liberated in the gravitational collapse of the core ($\approx 10^{53}$ erg). If about 1% of the energy of the neutrinos is converted into kinetic energy due to the coherent neutrino-nucleus scattering, a new shock wave arises. This will be strong enough to explode the star. This process is know as the *retarded mechanism* for supernova explosion.

To know which of the above mechanism is responsible for the supernova explosion one needs to know the rate of electron capture, the nuclear compressibility, and the way neutrinos are transported. The iron core, remnant of the explosion (the homologous core and part of the external core) will not explode and will become either a neutron star, and possibly later a *pulsar* (rotating neutron star), or a *black-hole*, as in the case of more massive stars, with $M \geq 25 - 35 M_{\odot}$.

Type-II supernovae are defined as those showing H-lines in their spectra. It is likely that most, if not all, of the exploding massive stars still have some H-envelope left, and thus exhibit such a feature. In contrast, Type-I supernovae lack H in their ejecta.