(*) $k' = x$

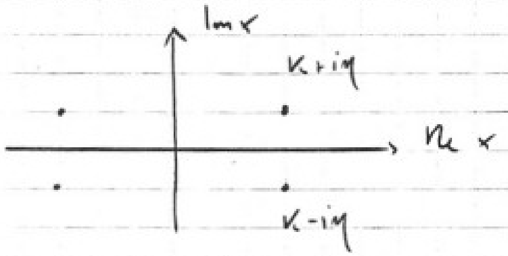
$|r-r'| = y$

$$I = 2\pi \int dx\, x^2 \frac{1}{k^2 - x^2 \pm i\eta} \underbrace{\int_0^\pi d\vartheta \sin\vartheta\, e^{ixy\cos\vartheta}}_{I'}$$

$$I' = \int_{-1}^{1} d\mu\, e^{ixy\mu} = \frac{1}{ixy}\left(e^{ixy} - e^{-ixy}\right)$$

$$I = \frac{-2\pi i}{y}\int_0^\infty dx\, x \frac{e^{ixy} - e^{-ixy}}{k^2-x^2\pm i\eta}$$

Poles $\quad x^2 = k^2 \pm i\eta \quad x^2 = k^2\left(1 \pm \frac{i\eta}{k^2}\right) \quad x = \pm k\left(1 \pm \frac{i\eta}{2k^2}\right) \sim \pm(k \pm i\eta)$



$$\text{Den} = \frac{1}{(k\pm i\eta - x)(k \pm i\eta + x)}$$

$I = I_1 + I_2$

$$I_1 = -\frac{2\pi i}{y}\, 2\pi i \,\text{Res}\,(x = k+i\eta) =$$

$$= \frac{4\pi^2}{y}(k+i\eta)\,(-1)\,\frac{e^{i(k+i\eta)y}}{2(k+i\eta)}$$ only contrib. to G^+

$$G_0^+(rr'E) = \frac{1}{8\pi^3}\frac{2m}{\hbar^2}\frac{4\pi^2}{y}\left(-\frac{1}{2}\right)e^{iky} = -\frac{m}{2\pi\hbar^2}\frac{e^{ik|r-r'|}}{|r-r'|}$$

$$I_2 = +\frac{2\pi i}{y}(-2\pi i)\,\text{Res}\,(x = k-i\eta) =$$

$$= \frac{4\pi^2}{y}(k-i\eta)\,(-1)\,\frac{e^{-i(k-i\eta)y}}{2(k-i\eta)}$$

$$G_0^-(rr'E) = \frac{1}{8\pi^3}\frac{2m}{\hbar^2}\frac{4\pi^2}{y}\left(-\frac{1}{2}\right)e^{-iky} = -\frac{m}{2\pi\hbar^2}\frac{e^{-ik|r-r'|}}{|r-r'|}$$