The region of abrupt changes in the nuclear structure are also often visible in a graph of E against I. Figure 5.26 illustrates the case of $^{160}_{70}$Yb. The ground state band has the lowest energy up to $I = 10$. Starting from there the *yrast* line follows the excited band that begins in 12^+. At 30^+ we have a new turn, with the beginning of a new band. The first turn is visible as a change in the inclination of the *yrast* line, but it appears in a much clearer way in the plot τ against ω^2 inserted in the figure. The second turn is visible only in this last graph.

An even-even nucleus at the ground state has all the nucleon pairs coupled with angular momentum zero and, in this situation, it is equivalent to a *superfluid*. The pairing broken by the centrifugal force corresponds to a transition of the superfluid phase to that of a normal fluid. In this aspect there is an analogy with a superconductor, where a magnetic field provokes the change to normal conduction phase (Meissner effect).

For very high rotation speeds the new forces that govern the nuclear balance can make the surface of the nucleus assume forms very different from the ground state form. In particular, rotational bands where the nucleus has a very large deformation have been discovered since 1985. Today we know more than 160 *superdeformed bands* distributed in four regions, of $A = 80$, 130, 150, and 190. In these bands the nucleus has a prolonged ellipsoidal form with a ratio between the axes of 2:1, 1.5:1, 2:1, and 1.7:1, respectively. Only a few superdeformed bands can have a well established connection with the *yrast* line.

5.10 Microscopic Theories

Microscopic theories for the nuclei start with a fundamental NN interaction and build the properties of the nuclei in a self-consistent way. There are numerous theories which have been developed for different purposes. We will describe only two of these theories whose goal is to describe the static properties of the nuclei, for example, their density profiles, binding energies, etc.

5.10.1 Hartree-Fock theory

Let us consider a system of particles with a central (mean-field) potential, U_0, and a two-body (particle-particle) potential, v, for instance, the atomic electron system

$$H = \sum_i^A \left[T_i + U_0(\mathbf{r}_i)\right] + \frac{1}{2}\sum_{ij} v(\mathbf{r}_i, \mathbf{r}_j), \tag{5.117}$$

where the factor $\frac{1}{2}$ prevents double-counting the two-body interaction energy (see figure 5.27a). The interaction v in (5.117) is the residual interaction of (5.37).

Neglecting $v(r_i, r_j)$, we have

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 + U_0(\mathbf{r})\right\}\psi_i(\mathbf{r}) = \epsilon_i\,\psi_i(\mathbf{r}). \tag{5.118}$$

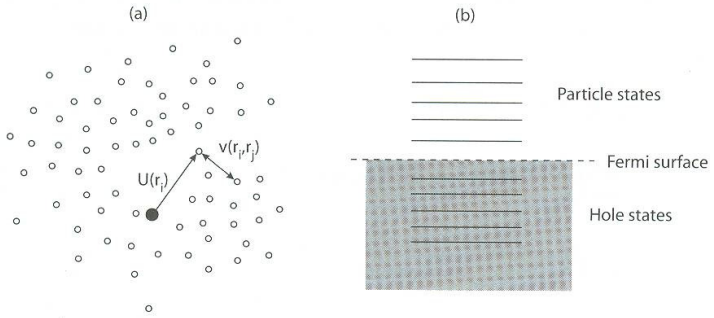

Figure 5.27 (a) Particles, i and j, interacting with a mean field $U_0(\mathbf{r})$ and among themselves with an interaction $v(\mathbf{r}_i, \mathbf{r}_j)$. (b) Labeling of states used in Hartree-Fock calculations.

If antisymmetrization is neglected, the total wavefunction is given by (5.40) and the average interaction felt by particle i due to all other particles is

$$U_1(\mathbf{r}_i) = \sum_{j \neq i}^{A} \int d^3 r_j |\psi_j(\mathbf{r}_j)|^2 v(\mathbf{r}_i, \mathbf{r}_j). \tag{5.119}$$

Now we add this potential to (5.118) and obtain a new wave equation:

$$\left\{ -\frac{\hbar}{2m} \nabla^2 + U_0(\mathbf{r}) + U_1(\mathbf{r}) \right\} \psi_i(\mathbf{r}) = \epsilon_i \psi_i(\mathbf{r}). \tag{5.120}$$

The *Hartree method* is now clear: one begins with the shell model Hamiltonian (5.118) and constructs the total wavefunction (5.40). From this wavefunction one calculates the average two-body interaction (5.119). One then solves the new wave equation (5.120). This sequence (5.40) $\rightarrow$ (5.119) $\rightarrow$ (5.120) is repeated (a process called iteration) until a stable solution is found. In general, only a few iterations are needed if $|U_0(r)| \gg |U_1(r)|$.

The *Hartree-Fock method* is easily derived from a variational method. The total wavefunction is now antisymmetrized as

$$\phi(1, 2, \ldots, A) = \mathcal{A}\, \psi_1(1)\psi_2(2) \ldots \psi_A(A), \tag{5.121}$$

where $\mathcal{A}\psi_1(1)\psi_2(2) \ldots \psi_A(A)$ is the Slater determinant of 5.42.

For a small variation $\delta\phi$, the energy expectation should be stationary. Thus,

$$\delta\langle\phi|H|\phi\rangle = \langle\delta\phi|H|\phi\rangle = 0. \tag{5.122}$$

The variation of $\delta\phi$ should preserve normalization of the wavefunctions:

$$\int |\psi_i(\mathbf{r})|^2 d^3 r = 1. \tag{5.123}$$

Now we solve the many-body Hamiltonian with two-body forces only, that is, with the Hamiltonian of (5.35). The expectation value with the wavefunction (5.121) is

$$\begin{aligned}\langle\phi|H|\phi\rangle = &-\frac{\hbar^2}{2m}\sum_i^A \int \psi_i^*(\mathbf{r})\nabla^2\psi_i(\mathbf{r})d^3r \\ &+\frac{1}{2}\sum_{ij}^A \iint \psi_i^*(\mathbf{r})\psi_j^*(\mathbf{r}')v(\mathbf{r},\mathbf{r}')\psi_i(\mathbf{r})\psi_j(\mathbf{r}')d^3rd^3r' \\ &-\frac{1}{2}\sum_{ij}^A \iint \psi_i^*(\mathbf{r})\psi_j^*(\mathbf{r}')v(\mathbf{r},\mathbf{r}')\psi_i(\mathbf{r}')\psi_j(\mathbf{r})d^3rd^3r'. \end{aligned} \tag{5.124}$$

The last term is a consequence of antisymmetrization. Applying the variation on $\psi_i(r)$ we obtain

$$\begin{aligned}-\frac{\hbar^2}{2m}\nabla^2\psi_i(\mathbf{r}) + &\sum_j^A \int d^3r'\psi_j^*(\mathbf{r}')v(\mathbf{r},\mathbf{r}')\psi_j(\mathbf{r}')\psi_i(\mathbf{r}) \\ &-\sum_j^A \int d^3r'\psi_j^*(\mathbf{r}')v(\mathbf{r},\mathbf{r}')\psi_j(\mathbf{r})\psi_i(\mathbf{r}') = \epsilon_i\psi_i(\mathbf{r}) \end{aligned} \tag{5.125}$$

where ϵ_i is seen as a Lagrange multiplier that enforces the constraint (5.123). It has the significance of a single particle energy.

We can rewrite (5.125) as

$$-\frac{\hbar^2}{2m}\nabla^2\psi_i(\mathbf{r}) + \int d^3r'\,U(\mathbf{r},\mathbf{r}')\psi_i(\mathbf{r}') = \epsilon_i\psi_i(\mathbf{r}), \tag{5.126}$$

where $U(r,r')$ is the self-consistent field

$$U(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}')\sum_j^A \int d^3r'\,v(\mathbf{r},\mathbf{r}')\psi_j(\mathbf{r}')\psi_j^*(\mathbf{r}') - \sum_j^A v(\mathbf{r},\mathbf{r}')\psi_j(\mathbf{r})\psi_j^*(\mathbf{r}'). \tag{5.127}$$

The first term is the direct term (*Hartree field*). The second is called the exchange interaction and is nonlocal. This nonlocality is closely related to the range of the two-body interaction. If we use a δ-force, then the Fock (or exchange) term is also local.

In constructing the HF (Hartree-Fock) determinant one naturally selects the A lowest energy wavefunctions. Thus the HF state corresponds to a Fermi sea of particles with a sharp Fermi surface.

As in the Hartree method, one guesses an initial $U^{(0)}$, solves (5.126), finds ψ_i, calculates (5.127), finds new $\psi_i's$, and so on, until the desired accuracy is achieved.

5.10.2 The Skyrme interaction

The most popular effective interaction used in Hartree-Fock calculations is the *Skyrme interaction*. It is based on the fact that a finite range interaction can be simulated by a momentum dependence. This can be shown by transforming an interaction $V(r)$, where