Equazione di Laplace

$$\nabla^2 V = 0$$

Equazione di Poisson con carica puntiforme

$$\nabla^2 V = -4\pi\rho = -4\pi e\, \delta(\vec{r})$$

Soluzione tramite FT

$$V(\vec{r}) = \frac{1}{(2\pi)^3} \int d^3k\, e^{i\vec{k}\vec{r}}\, \tilde{V}(k)$$

$$F(\delta) = \int d^3r\, e^{-i\vec{k}\vec{r}}\, \delta(\vec{r}) = 1$$

$$\frac{1}{(2\pi)^3} \int d^3k\, (-k^2)\, e^{i\vec{k}\vec{r}}\, \tilde{V} = -4\pi e \frac{1}{(2\pi)^3} \int d^3k\, e^{i\vec{k}\vec{r}}$$

$$-k^2 \tilde{V} = -4\pi e$$

$$\tilde{V} = \frac{4\pi e}{k^2} \qquad \frac{V}{e} = \frac{1}{(2\pi)^3} \int d^3k\, e^{i\vec{k}\vec{r}} \frac{4\pi}{k^2}$$

$$\frac{V}{e} = \frac{1}{8\pi^3} 8\pi^2 \int dk\, k^2 \int d\vartheta \overset{\sin\vartheta}{\vee} e^{ikr\cos\vartheta} \frac{1}{k^2}$$

$$= \frac{1}{\pi} \int dk \int_{-1}^{1} d\mu\, e^{ikr\mu} = \frac{1}{\pi} \int_0^\infty dk \frac{1}{ikr} \left(e^{ikr} - e^{-ikr} \right)$$

$$= \frac{2}{\pi} \frac{1}{2} \int_0^\infty dx \frac{\sin x}{x} = \frac{1}{2}$$

Se il ~~▭~~ mediatore ha massa,

$$(\Box^2 + \mu^2)\,\varphi = 0$$

sarebbe l'equazione del ~~pot~~ potenziale nel vuoto
Con una sorgente puntiforme nell'origine (il nucleone) e
nel caso indip. dal tempo

$$(-\nabla^2 + \mu^2)\,\varphi = g\,\delta(\bar{r})$$

$$(\nabla^2 - \mu^2)\,\varphi = -g\,\delta(\bar{r}) \quad \text{è la } (1.\cancel{23}) \; 24$$

FT

$$\varphi(\bar{r}) = \frac{1}{(2\pi)^3}\int d^3k\; e^{i\bar{k}\bar{r}}\;\tilde{\varphi}(\bar{k})$$

$$\frac{1}{(2\pi)^3}\int d^3k\; e^{i\bar{k}\bar{r}}\,(-k^2-\mu^2)\,\tilde{\varphi}(\bar{k}) = -g\,\frac{1}{(2\pi)^3}\int d^3k \times e^{i\bar{k}\bar{r}}$$

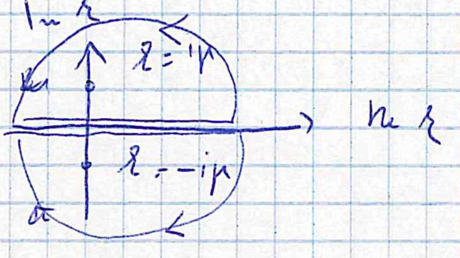
$$\tilde{\varphi} = \frac{g}{k^2+\mu^2}$$

$$\varphi = \frac{1}{(2\pi)^3}\int d^3k\; e^{i\bar{k}\bar{r}}\;\frac{g}{k^2+\mu^2}$$

$$\varphi = g\,\frac{1}{8\pi^3}\;2\pi\int\frac{dk\,k^2}{k^2+\mu^2}\int d\vartheta\,\sin\vartheta\;e^{ikr\cos\vartheta}$$

$$= g\,\frac{1}{4\pi^2}\int dk\;\frac{k^2}{k^2+\mu^2}\;\underbrace{\int_{-1}^{1} d\mu\; e^{ikr\mu}}_{\left(e^{ikr}-e^{-ikr}\right)\times\frac{1}{ikr}}$$

Applica il teorema dei residui



$$\frac{1}{ir}\int_0^\infty dk\;\frac{k}{k^2+\mu^2}\left(e^{ikr}-e^{-ikr}\right)$$

The first integral (second) must be done along the upper (lower) contour

$k^2+\mu^2=(k+i\mu)(k-i\mu)$

$$= \frac{1}{ir}\;\pi i\left(+\frac{+i\mu}{+2i\mu}\,e^{-\mu r}+\frac{-i\mu}{-2i\mu}\,e^{-\mu r}\right)$$

$$= \frac{\pi}{r}\,e^{-\mu r}$$

$$\boxed{\varphi = \frac{g}{4\pi}\,\frac{e^{-\mu r}}{r}}$$

4π is a convention

$\varphi \sim \frac{e^{-\mu r}}{r}$ cf $V \sim \frac{1}{r}$

p-n $\quad \sigma_1 k \quad \sigma_2 k \quad \tau_1\tau_2$