$$V(\bar{r}) = \frac{f_\pi^2}{m_\pi^2} (\sigma_1 \nabla)(\sigma_2 \nabla) \frac{e^{-m_\pi r}}{r} (\bar{\tau}_1 \bar{\tau}_2)$$

[Eq (1.27)]

First, we re-write

$$(\sigma_1 \nabla)(\sigma_2 \nabla) = \left[(\sigma_1 \nabla)(\sigma_2 \nabla) - \frac{1}{3} (\sigma_1 \sigma_2) \nabla^2 \right] + \frac{1}{3} (\sigma_1 \sigma_2) \nabla^2$$

The advantage of this re-writing is that the term in square brackets does not receive contribution from diagonal terms.

$$\sum_{ij} (\sigma_{1i} \nabla_i)(\sigma_{2j} \nabla_j) - \frac{1}{3} \sum_i (\sigma_{1i} \sigma_{2i}) \nabla^2,$$

if we fix $j = i$, becomes

$$\sum_i (\sigma_{1i} \nabla_i)(\sigma_{2i} \nabla_i) - \frac{1}{3} \sum_i (\sigma_{1i} \sigma_{2i}) \nabla^2 =$$

$$= \sigma_{1x} \sigma_{2x} \nabla_x^2 + \sigma_{1y} \sigma_{2y} \nabla_y^2 + \sigma_{1z} \sigma_{2z} \nabla_z^2 +$$

$$- \frac{1}{3} (\sigma_{1x} \sigma_{2x} + \sigma_{1y} \sigma_{2y} + \sigma_{1z} \sigma_{2z}) (\nabla_x^2 + \nabla_y^2 + \nabla_z^2)$$

If we act on a function of r, $\nabla_x^2 = \nabla_y^2 = \nabla_z^2 = \frac{1}{3} \nabla^2$ and the above equation gives 0.

In addition to this property, we use other gradients

$$\nabla_i\, r = \nabla_i \sqrt{x^2+y^2+z^2} = \frac{1}{2}\,\frac{1}{r}\,2r_i = \frac{r_i}{r}$$

$$\nabla_i \frac{1}{r} = -\frac{1}{r^2}\,\frac{r_i}{r} = -\frac{r_i}{r^3}$$

$$\nabla_i \nabla_j \Big|_{i \neq j} \frac{1}{r} = 3\,\frac{r_i}{r^4}\,\frac{r_j}{r} = \frac{3 r_i r_j}{r^5}$$

$$\nabla_i^2 \frac{1}{r} = \frac{1}{3}\nabla^2 \frac{1}{r} = -\frac{4\pi}{3}\,\delta(r)$$

$$\nabla_i\, e^{-m_\pi r} = (-m_\pi)\, e^{-m_\pi r}\, \frac{r_i}{r}$$

$$\nabla_i \nabla_j \Big|_{i \neq j} e^{-m_\pi r} = (-m_\pi)\, e^{-m_\pi r}\, r_i \left[(-m_\pi)\frac{r_j}{r^2} - \frac{r_j}{r^3} \right]$$

$$= e^{-m_\pi r}\left[m_\pi^2 \frac{r_i r_j}{r^2} + m_\pi \frac{r_i r_j}{r^3} \right]$$

$$\nabla_i^2\, e^{-m_\pi r} = (-m_\pi)\, e^{-m_\pi r}\left[(-m_\pi)\frac{r_i^2}{r^2} + \frac{1}{r} - \frac{r_i^2}{r^3} \right]$$

$$= e^{-m_\pi r}\left[m_\pi^2 \frac{r_i^2}{r^2} - m_\pi \frac{1}{r} + m_\pi \frac{r_i^2}{r^3} \right]$$

$$\left[(\sigma_1 \nabla)(\sigma_2 \nabla) - \frac{1}{3} (\sigma_1 \sigma_2) \nabla^2 \right] \frac{e^{-m_\pi r}}{r} =$$

$$= \sum_{ij} \sigma_{1i} \sigma_{2j} \nabla_i \nabla_j \frac{e^{-m_\pi r}}{r} = \sum_{ij} \sigma_{1i} \sigma_{2j} \times$$

$$\times \left[\left(\nabla_i \nabla_j e^{-m_\pi r} \right) \frac{1}{r} + \nabla_i \frac{1}{r} \nabla_j e^{-m_\pi r} + \right.$$

$$\left. + \nabla_i e^{-m_\pi r} \nabla_j \frac{1}{r} + e^{-m_\pi r} \nabla_i \nabla_j \frac{1}{r} \right] =$$

$$= \sum_{ij} \sigma_{1i} \sigma_{2j} \left[\frac{1}{r} e^{-m_\pi r} \left(m_\pi^2 \frac{r_i r_j}{r^2} + m_\pi \frac{r_i r_j}{r^3} \right) + \right.$$

$$\left. + \frac{r_i}{r^3} (-m_\pi) e^{-m_\pi r} \frac{r_j}{r} + (i \leftrightarrow j) + e^{-m_\pi r} \frac{3 r_i r_j}{r^5} \right]$$

$$= \sum_{ij} \sigma_{1i} \sigma_{2j} e^{-m_\pi r} r_i r_j \frac{1}{r^2} \left[\frac{m_\pi^2}{r} + 3 \frac{m_\pi}{r^2} + 3 \frac{1}{r^3} \right]$$

$$= \sum_{(i \neq j)\, ij} \sigma_{1i} \sigma_{2j} e^{-m_\pi r} r_i r_j \frac{1}{r^2} m_\pi^3 \left[\frac{1}{m_\pi r} + \frac{3}{(m_\pi r)^2} + \frac{3}{(m_\pi r)^3} \right]$$

$$\frac{1}{3} (\sigma_1 \sigma_2) \nabla^2 = \frac{1}{3} (\sigma_1 \sigma_2) \left(\nabla_x^2 + \nabla_y^2 + \nabla_z^2 \right) = \sum_i \frac{1}{3} (\sigma_1 \sigma_2) \nabla_i^2$$

$$\sum_i \left[\frac{1}{3} (\sigma_1 \sigma_2) \nabla_i^2 \right] \frac{e^{-m_\pi r}}{r} = \sum_i \frac{1}{3} (\sigma_1 \sigma_2) \left[\frac{1}{r} \nabla_i^2 e^{-m_\pi r} + 2 \nabla_i \frac{1}{r} \nabla_i e^{-m_\pi r} \right.$$

$$\left. + e^{-m_\pi r} \nabla_i^2 \frac{1}{r} \right] =$$

$$= \frac{\sum_i}{3} (\sigma_1 \sigma_2) \Big[\frac{1}{r} e^{-m_\pi r} \Big(m_\pi^2 \frac{r_i^2}{r^2} - m_\pi \frac{1}{r} + m_\pi \frac{r_i^2}{r^3} \Big)$$

$$+ 2 \Big(- \frac{r_i}{r^3} \Big) (-m_\pi) e^{-m_\pi r} \frac{r_i}{r} - e^{-m_\pi r} \frac{4\pi}{3} \delta(r) \Big]$$

$$= (\sigma_1 \sigma_2) \Big(e^{-m_\pi r} \Big[m_\pi^2 \frac{1}{3r} - \cancel{m_\pi \frac{1}{r^2}} + \cancel{m_\pi \frac{1}{3r^2}}$$

$$\cancel{+ 2 m_\pi \frac{1}{3r^2}} - \frac{4\pi}{3} \delta(r) \Big] = (\sigma_1 \sigma_2) m_\pi^3 e^{-m_\pi r} \frac{1}{3(m_\pi r)} - \frac{4\pi}{3} \sigma_1 \sigma_2 \delta$$

$$\sum_{i \neq j} = \sum_{ij} - \sum_{i=j} \quad *$$

$$V(\vec{r}) = f_\pi^2 m_\pi (\tau_1 \tau_2) \Big[\Big((\sigma_1 \hat{r})(\sigma_2 \hat{r}) - \sigma_1 \sigma_2 \frac{1}{3} \Big)$$

$$e^{-m_\pi r} \, 3 \Big(\frac{1}{(m_\pi r)^3} + \frac{1}{(m_\pi r)^2} + \frac{1}{3 m_\pi r} \Big) +$$

$$+ \frac{\sigma_1 \sigma_2}{3} \frac{e^{-m_\pi r}}{m_\pi r} - \frac{4\pi}{3 m_\pi^3} \sigma_1 \sigma_2 \delta(\vec{r}) \Big]$$

$$* \quad \sum_{i \neq j} \sigma_{1i} \sigma_{1j} r_i r_j = \sum_{ij} - \sum_{i=j} = \sum (\sigma_{1i} r_i)(\sigma_{1j} r_j) +$$

$$- \Big(\sigma_{1x} \sigma_{2x} \frac{x}{r^3} + \sigma_{1y} \sigma_{2y} \frac{y}{r^3} + \sigma_{1z} \sigma_{2z} \frac{z}{r^3} \Big)$$