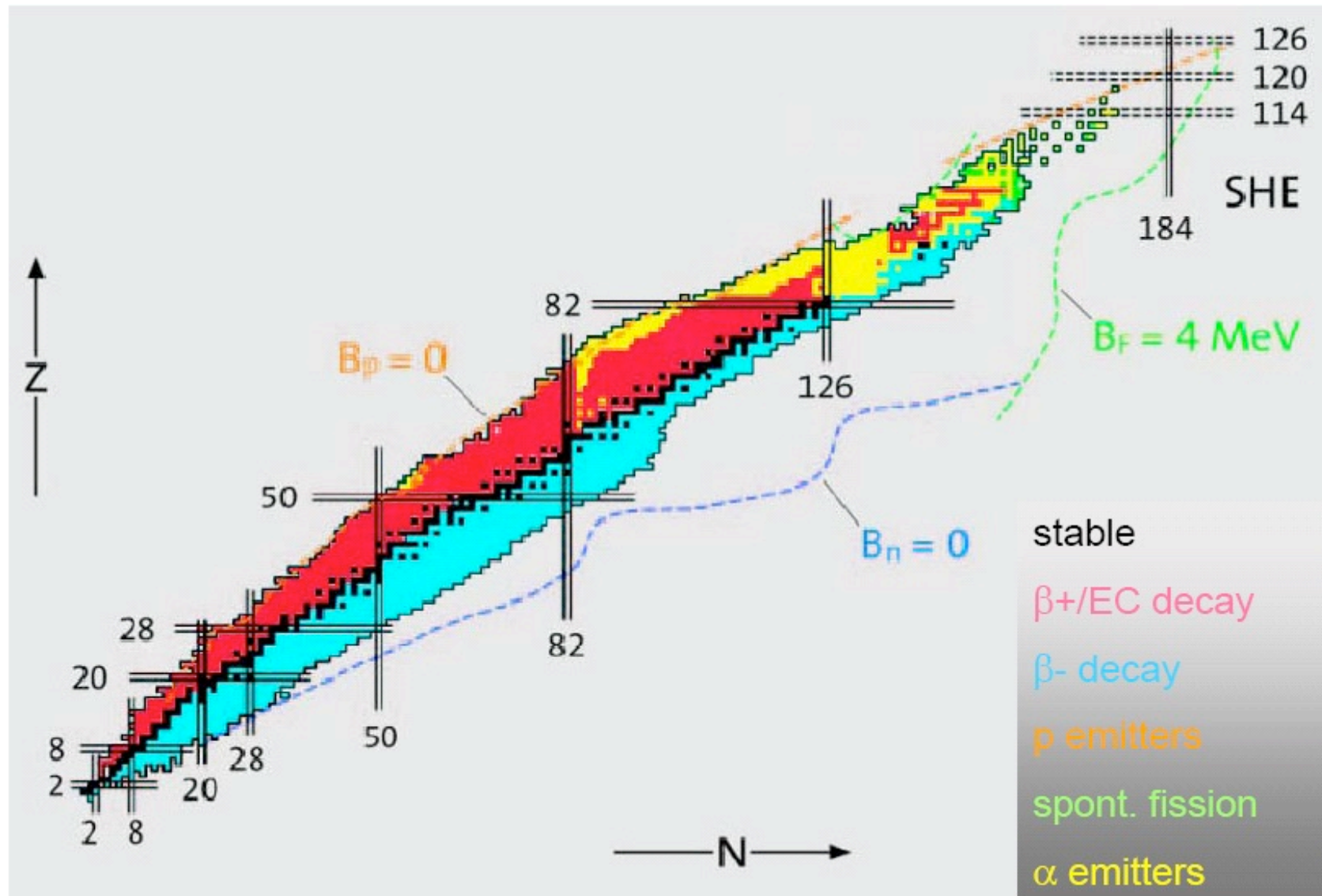
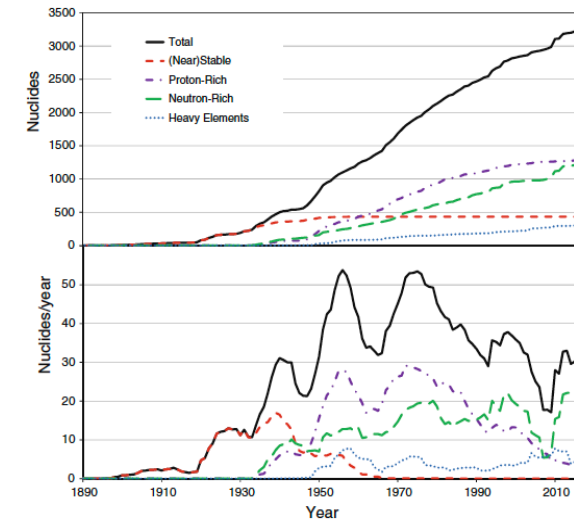
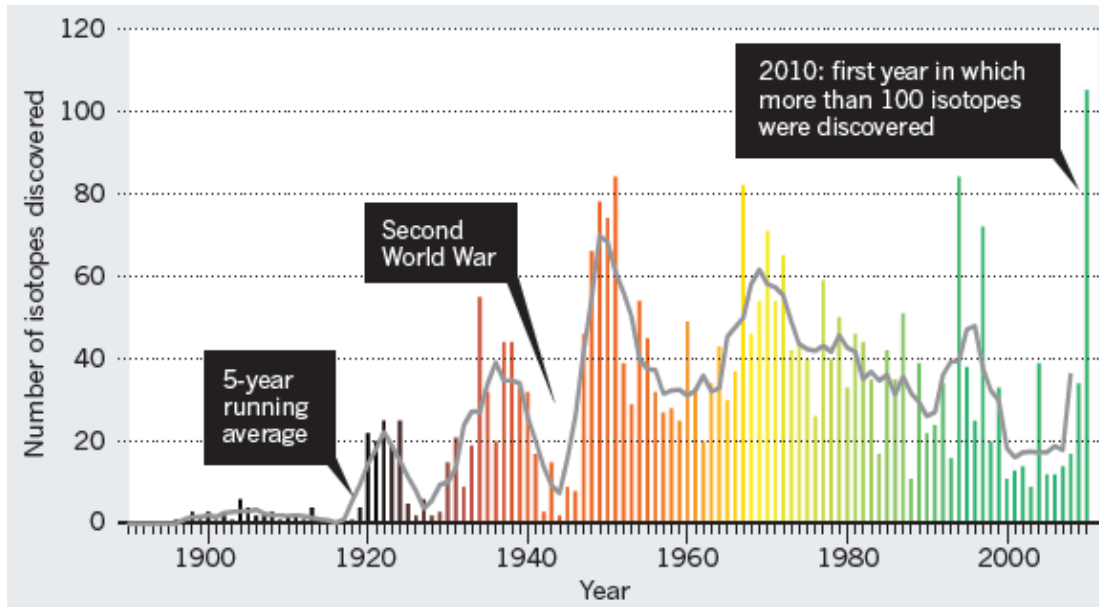


- Unstable vs. stable nuclei: neutron-rich and proton-rich systems
- Limit of the nuclear stability and definition of drip lines
- How to produce them ?
- Masses and density distributions of unstable nuclei
- Halo systems



From: Exotic Nuclei, J. Enders, TU Darmstadt, Summer 2003

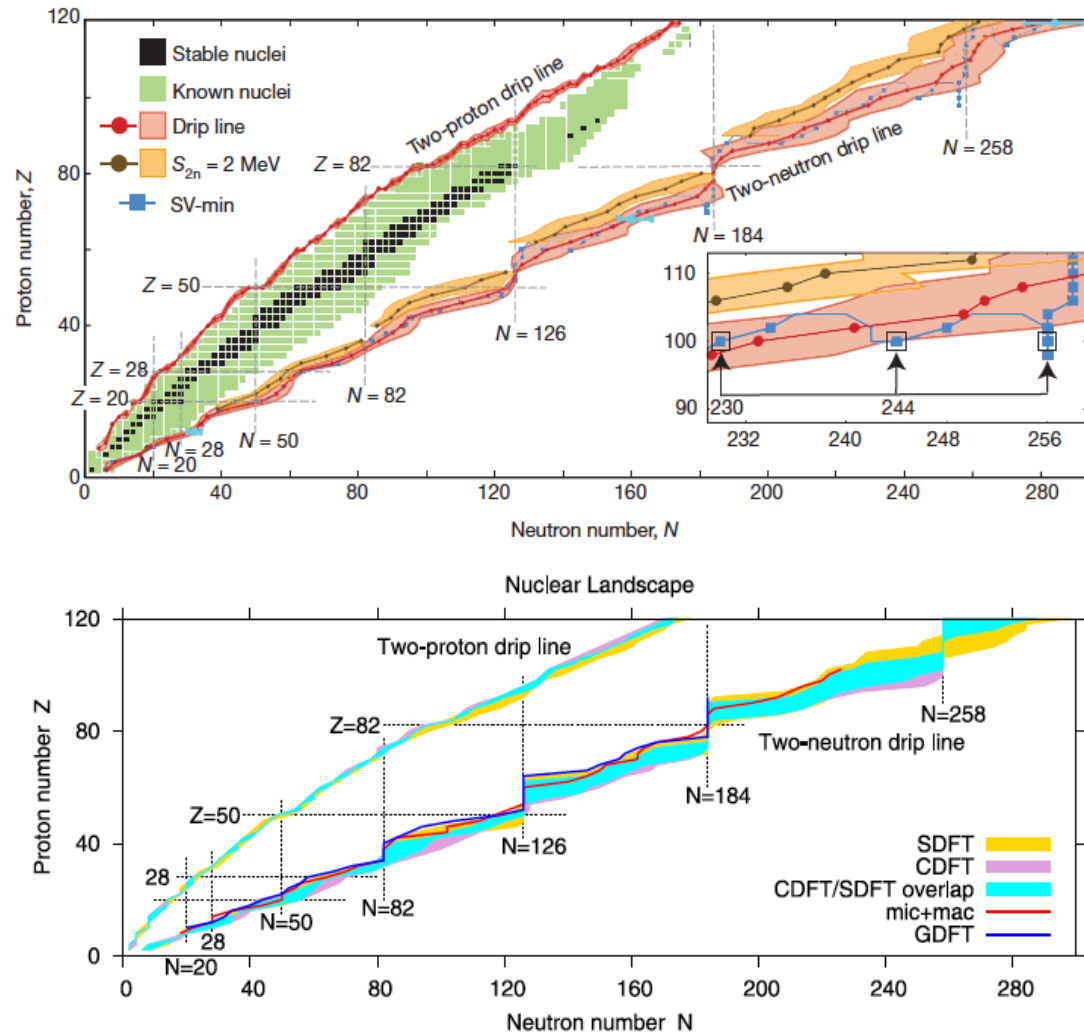
Discovering new exotic isotopes



M. Thoennessen, B. Sherrill, Nature 473, 25 (2011)

<http://www.nscl.msu.edu/~thoennes/isotopes/>.

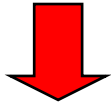
Different extrapolations to the drip line



J. Erler *et al.*, Nature 486, 509 (2012) - SEDF

A.V. Afanasjev *et al.*, Phys. Lett. B726, 680 (2013) - CEDF

Fig. 4. The comparison of the uncertainties in the definition of two-proton and two-neutron drip lines obtained in CDFT and SDFT. The shaded areas are defined by the extremes of the predictions of the corresponding drip lines obtained with different parametrizations. The blue shaded area shows the area where the CDFT and SDFT results overlap. Non-overlapping regions are shown by dark yellow and plum colors for SDFT and CDFT, respectively. The results of the SDFT calculations are taken from the supplement to Ref. [2]. The two-neutron drip lines obtained by microscopic + macroscopic (FRDM [3]) and Gogny D1S DFT [5] calculations are shown by red and blue lines, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)



In the previous figure:

black dots correspond to stable nuclei: i.e., infinite lifetime.

Stable nuclei can be found around the so-called stability line.

First problems: for each A (that is, for each isobaric chain), what is the nucleus with largest binding energy ? And how does this evolve if we move towards right (left) in the previous figure, that is, if we move increasing (decreasing) $(N-Z)$?

The solution of the first problem can be attempted by using the well-known Bethe-Weizsäcker mass formula. We take this problem from [Hey94].

$$\begin{aligned} M(A, Z)c^2 = & Zm_pc^2 + (A - Z)m_nc^2 \\ & - a_V A + a_SA^{2/3} + a_A(A - 2Z)^2 A^{-1} \\ & + a_C Z(Z - 1)A^{-1/3} + \\ & \text{pairing term : } 0, \pm\delta. \end{aligned} \quad (1)$$

If we wish, for each A , the nucleus with the lowest mass, or largest binding energy, we must re-write the above equation (neglecting the pairing term) as

$$M(A, Z)c^2 = f(A) + pZ + qZ^2, \quad (2)$$

where the constants p and q can be easily obtained, and then

$$\frac{\partial}{\partial Z} M c^2 = 0 \quad (3)$$

is solved for

$$Z_0 = \frac{-p}{2q}. \quad (4)$$

We can obtain Z_0 (value of Z corresponding to the lowest mass) by replacing the values of p and q and then multiplying the numerator and denominator by $A/8a_A$:

$$Z_0 = \frac{\frac{A}{2} + (m_n - m_p)c^2 \frac{A}{8a_A} + \frac{a_C A^{2/3}}{8a_A}}{1 + \frac{1}{4} \frac{a_C}{a_A} A^{2/3}}. \quad (1)$$

In the numerator, the second and third terms are negligible with respect to the first one. This leads to

$$Z_0 = \frac{\frac{A}{2}}{1 + 0.0077 A^{2/3}}. \quad (2)$$

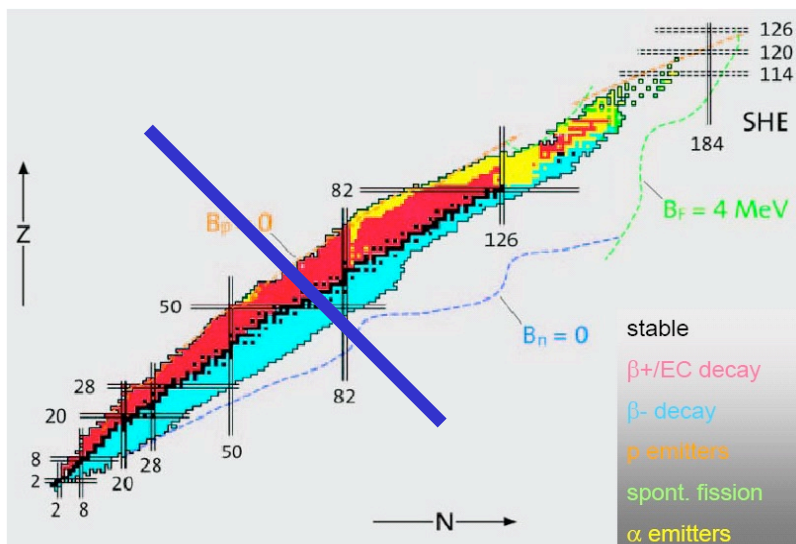
Values:

$$a_V = 15.85 \text{ MeV}$$

$$a_S = 18.34 \text{ MeV}$$

$$a_C = 0.71 \text{ MeV}$$

$$a_A = 23.21 \text{ MeV}$$



The blue line represents constant A : for $A=120$ we meet Z_0 close to 50 (i.e., Sn).

Around the “most stable” nucleus there are other stable systems. If we increase the number of neutrons with respect to the protons, we expect to go towards beta instability.

Why ?

Remember: the pp and nn force is active only in $T=1$ channel (on the average, not so much attractive) whereas the pn force is active also in the $T=0$ channel which provides attraction. This is known from the existence of the deuteron as bound systems and the non-existence of the di-neutron.

Therefore, neutrons feel more the proton attraction than the attraction of the other neutrons. In a system with increasing $N-Z$, the neutrons become less bound with respect to the protons. This leads to β^- instability.

For analogous reasons, if we decrease $N-Z$ we meet a region of β^+ instability.

Gamow-Teller beta decay and isospin impurity in nuclei near the proton drip line

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(Received 4 May 1993)

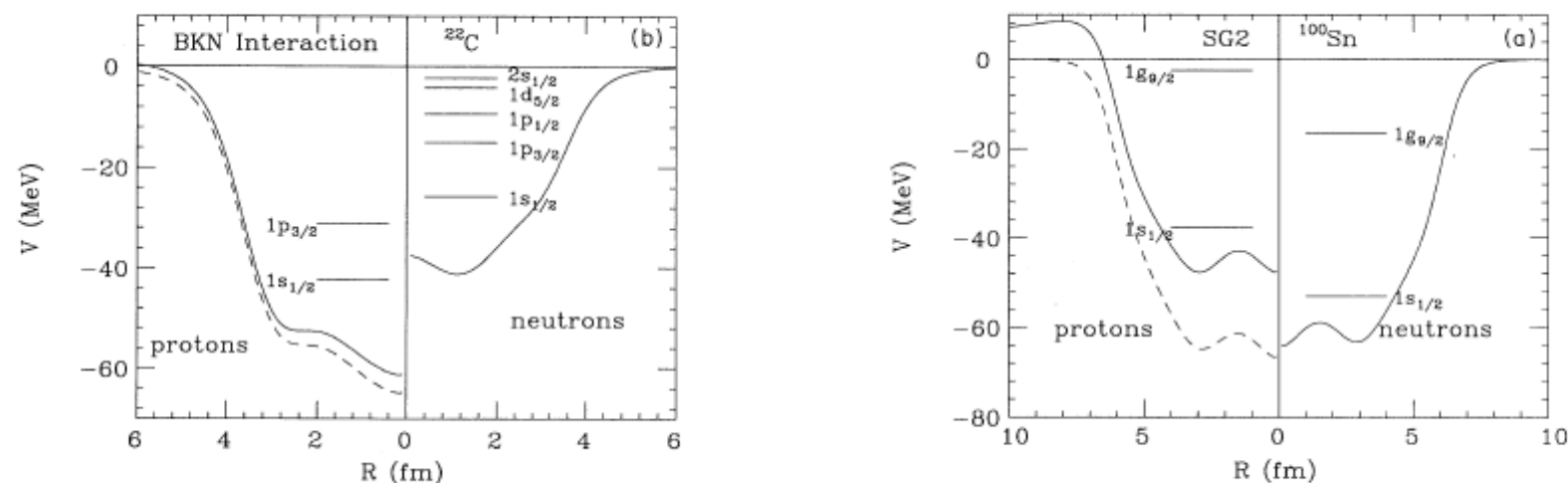
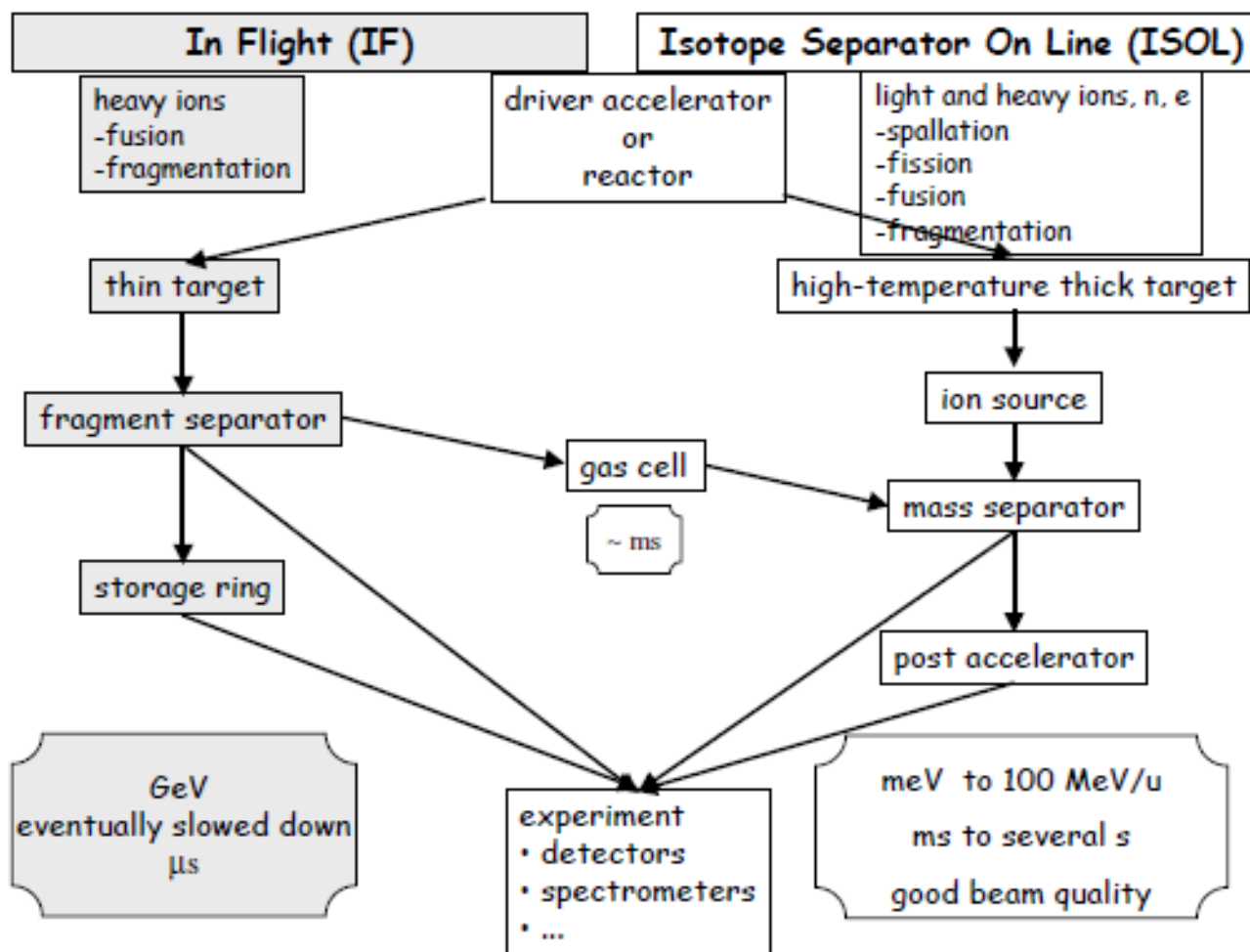


FIG. 1. HF potentials of ^{100}Sn (a) and ^{22}C (b). The Skyrme interactions, SG2 and BKN, are used for ^{100}Sn and ^{22}C , respectively. The dashed (solid) lines for protons show the HF potential without (with) the Coulomb potential. Since the HF calculation is performed with the Coulomb potential, there is a small difference between the dashed line for protons and the solid line for neutrons in ^{100}Sn . In ^{100}Sn only the lowest-lying one-particle level ($=1s_{1/2}$) and the highest occupied level ($=1g_{9/2}$) are shown, while in ^{22}C all occupied one-particle levels are denoted.

Lifetimes for beta-decay can be quite long and nuclei can nonetheless can be studied nowadays using RIB (Radioactive Isotope Beam) facilities.

We meet, by further increasing (or decreasing) $N-Z$ the neutron (proton) drip line. These are defined as the limits beyond which the systems are unstable against particle emission. In the case of neutrons, the one-neutron or two-neutron separation energies ($S_n = BE(N) - BE(N-1)$ or S_{2n}) become zero.

In certain cases, systems beyond the drip lines can be studied: for instance, if the lifetime is relatively long due to the fact that the extra neutron (or proton) has a resonant state available. But this is not the rule !



The Why and How of Radioactive-Beam Research

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Measurements of masses, sizes, densities of radioactive nuclei

- **Masses** (or equivalently binding energies) can be measured with good accuracy by means of mass spectrometry.
- **The difference between stable and unstable nuclei comes at the level of density measurements.** In the case of stable nuclei, electron scattering has been the main source of information. The electromagnetic interaction is known, and this has allowed to interpret the data since the differential elastic cross section for electron scattering is expected to be the Mott cross section (corresponding to the diffusion on a point charge) multiplied by the form factor $F(q^2)$ squared, that is, the Fourier transform of the nuclear charge density,

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(q^2)|^2 \\ \rho_{\text{charge}}(r) &= \frac{1}{2\pi^2 r} \int_0^\infty F(q^2) \sin(qr) q dq\end{aligned}$$

In the case of unstable nuclei, this is not possible. Sizes and densities have been measured using hadron scattering → with the associated uncertainties !

SIZES AND ENERGIES OF EXOTIC NUCLEI

3.1 Introduction

The first experiments with unstable nuclear beams were designed to measure the nuclear sizes, namely the matter distribution of protons and neutrons. For stable nuclei such experiments are best accomplished with electron beams, which probe the nuclear charge (proton) distribution. Electron scattering experiments with unstable beams can only be performed in an electron-nucleus collider. Such machines are not yet available. The easiest solution is to measure the *interaction cross section* in collisions of unstable beams with a fixed target nucleus.

The interaction cross section is defined as the cross section for the change of proton and/or neutron number in the incident nucleus. To extract the *interaction radii* of the radioactive secondary beam nuclei, one has assumed that it can be expressed as [1]

$$\sigma_I(P, T) = \pi [R_I(P) + R_I(T)]^2 \quad (3.1)$$

where $R_I(P)$ and $R_I(T)$ are the interaction radii of the projectile and the target nuclei, respectively. $R_I(T)$ can be obtained from σ_I in collisions between identical nuclei, while $R_I(P)$ can be obtained by measuring σ_I for different targets T [1].

The above equation assumes a separability of the projectile and target radius. This hypothesis has been tested by Tanihata and collaborators [1]. As an example, the interaction radii R_I for Li and Be isotopes have been obtained using three different targets. The results are shown in Fig. 1.

In Table 3.1 we show in the first column the interaction radii of several nuclei obtained with this technique [1]. In the last column the root mean charge radius of some nuclei obtained by electron scattering, R_{rms}^e , are also shown. One observes that R_{rms}^e is almost constant for $A \geq 6$, while R_I increases with A . One can show that this difference is due to the definitions of the two radii but not due to a difference between the charge and the matter distributions. To prove it we use an eikonal calculation for the cross sections. The *rms* radius of the matter density can be determined independently of the assumed model density functions. The eikonal approximation and its use in nuclear physics is presented in Chapters 1 and 2.

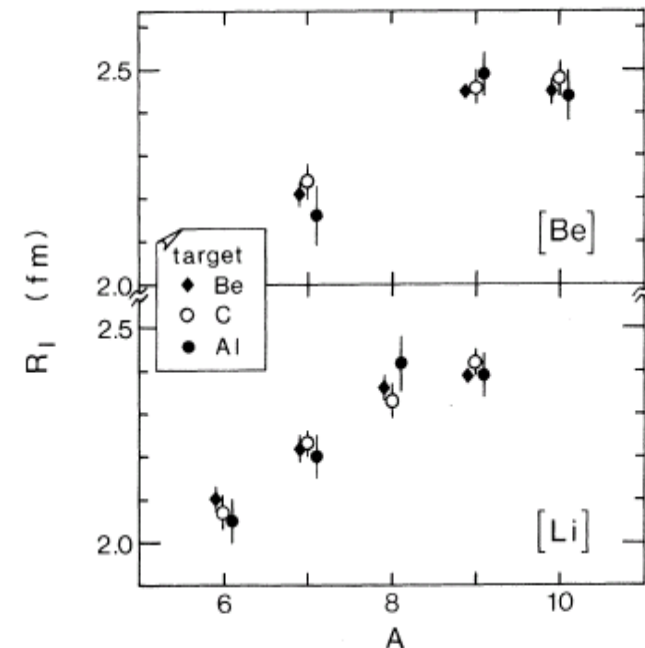


FIG. 1. R_I for Li and Be isotopes. The values obtained by three different targets agree with each other showing the separability of projectile and target R_I .

Table 3.1: cf. next page

TABLE II. Interaction nuclear radii and rms radii, in fermis.

		e scat.	Gaussian	Harmonic oscillator		
	R_I	R_{rms}^e	R_{rms}^G	$R_{\text{rms}}^m{}^a$	$R_{\text{rms}}^c{}^a$	$R_{\text{rms}}^n{}^a$
${}^4\text{He}$	1.41 ± 0.03	1.67 ± 0.01	1.72 ± 0.06	1.72 ± 0.06	1.72 ± 0.06	1.72 ± 0.06
${}^6\text{He}$	2.18 ± 0.02		2.75 ± 0.04	2.73 ± 0.04	2.46 ± 0.04	2.87 ± 0.04
${}^8\text{He}$	2.48 ± 0.03		2.70 ± 0.03	2.69 ± 0.03	2.33 ± 0.03	2.81 ± 0.03
${}^6\text{Li}$	2.09 ± 0.02	2.56 ± 0.10	2.54 ± 0.03	2.54 ± 0.03	2.54 ± 0.03	2.54 ± 0.03
${}^7\text{Li}$	2.23 ± 0.02	2.41 ± 0.10	2.50 ± 0.03	2.50 ± 0.03	2.43 ± 0.03	2.54 ± 0.03
${}^8\text{Li}$	2.36 ± 0.02		2.51 ± 0.03	2.51 ± 0.03	2.41 ± 0.03	2.57 ± 0.03
${}^9\text{Li}$	2.41 ± 0.02		2.43 ± 0.02	2.43 ± 0.02	2.30 ± 0.02	2.50 ± 0.02
${}^{11}\text{Li}$	3.14 ± 0.16		3.27 ± 0.24	3.27 ± 0.24	3.03 ± 0.24	3.36 ± 0.24
${}^7\text{Be}$	2.22 ± 0.02		2.48 ± 0.03	2.48 ± 0.03	2.52 ± 0.03	2.41 ± 0.03
${}^9\text{Be}$	2.45 ± 0.01	2.52 ± 0.01	2.49 ± 0.01	2.50 ± 0.01	2.47 ± 0.01	2.53 ± 0.01
${}^{10}\text{Be}$	2.46 ± 0.03		2.38 ± 0.02	2.39 ± 0.02	2.34 ± 0.02	2.43 ± 0.02
${}^{12}\text{C}$	2.61 ± 0.02	2.45 ± 0.01	2.40 ± 0.02	2.43 ± 0.02	2.43 ± 0.02	2.43 ± 0.02

^aSuperscripts m , c , and n indicate the nuclear matter, the charge, and the neutron matter distributions, respectively.

and

$$\frac{E}{\hbar\omega} = \sum_{\Lambda=0}^{\Lambda_0} 2N_{\Lambda} \left(\Lambda + \frac{3}{2} \right) \cong \frac{1}{2}(\Lambda_0 + 2)^4 - \frac{1}{3}(\Lambda_0 + 2)^3 + \dots \quad (3.35)$$

Eliminating $(\Lambda_0 + 2)$ from the above equations and retaining terms of the highest powers of $(\Lambda_0 + 2)$, we obtain

$$\frac{E}{\hbar\omega} \cong \frac{1}{2} \left(\frac{3}{2} A \right)^{4/3}. \quad (3.36)$$

Or, using 3.32 and 3.33,

$$\hbar\omega \cong 41 A^{-1/3} \text{ MeV}. \quad (3.37)$$

The giant dipole resonances in nuclei are excitations with $\Delta\ell = \pm 1$ do in fact vary with the nuclear mass as $A^{-1/3}$ and are a good example of application of 3.37.

3.3 Halo nuclei

In order to show that the rms radii obtained by a comparison of reaction cross section calculations with the experimentally determined σ_I are equal, Fig. 3 shows the calculated rms charge radii and those obtained by electron-scattering experiments for stable nuclei. Even the difference between the radii of ${}^6\text{Li}$ and ${}^7\text{Li}$ because of the occupation-number difference between protons and neutrons is reproduced by the harmonic-oscillator distribution (solid line). The rms radii obtained with Gaussian distributions (which are the same for protons as for neutrons) is shown by the dashed-line.

The calculations also show that R_I represents the radius where the matter density is about 0.05 fm^{-3} for $A \geq 6$ nuclei. Now we can understand why the rms radii and R_I behave differently with A . While the rms radii stay constant, the absolute density increases with A . Therefore R_I , which represents constant density, increases with A . These interesting results are presented in figure 5(a) where the rms radii of He, Li, Be, and C isotopes are shown [7]. The curves are guides to the eyes.

We observe a great increase in the rms radii for the neutron-rich isotopes ${}^6\text{He}$, ${}^8\text{He}$ and ${}^{11}\text{Li}$. Thus, the addition of the neutrons to ${}^4\text{He}$ and ${}^9\text{Li}$ nuclei increase their radii considerably. This might be understood in terms of the binding energy of the outer nucleons. The large matter radii of these nuclei have lead the experimentalists to call them by "*halo nuclei*". The binding energy of the last two neutrons in ${}^{11}\text{Li}$ is equal to $315 \pm 50 \text{ keV}$ [6]. In ${}^6\text{He}$ it is 0.97 MeV . These are very small values and should be compared with $S_n = 6 - 8 \text{ MeV}$ which is the average binding of nucleons in stable nuclei.

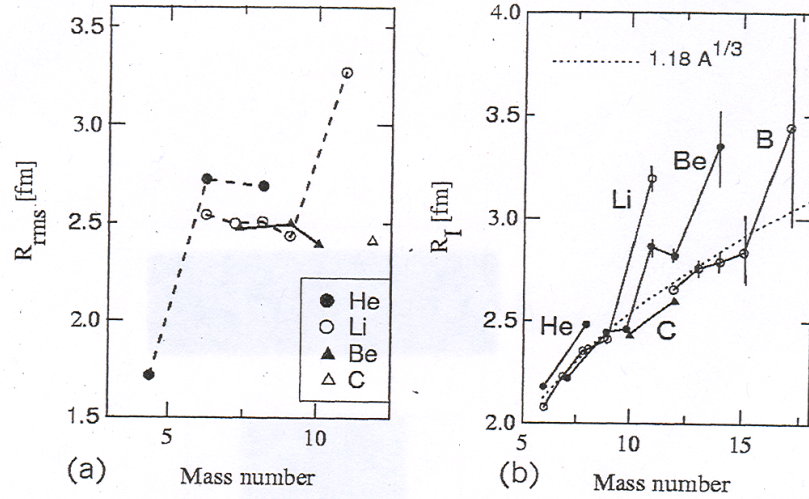


Figure 5 (a) Rms radii for the neutron-rich isotopes He, Li, Be, and C. (b) The matter density radii of several light nuclei compared to the trend $R \sim 1.18 A^{1/3}$ fm (dashed line) for normal nuclei. The solid lines are guide to the eyes.

The wavefunction of a loosely-bound nucleon (as in the case of the deuteron) extends far beyond the nuclear potential. For large distances the wavefunction behaves as an Yukawa function,

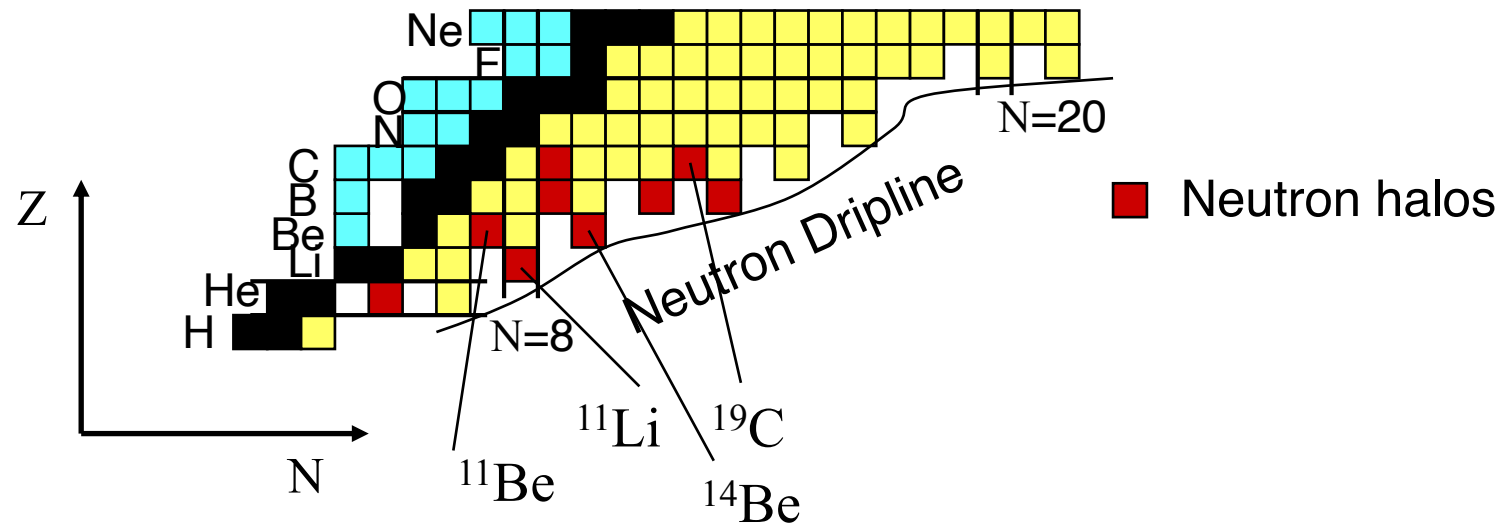
$$R(r)/r \sim \frac{e^{-\eta r}}{r} \quad (3.38)$$

where $(\hbar\eta)^2 = 2mB$, with B equal to the binding energy and m the nucleon mass. Thus, the smaller the value of B is, the more the wavefunction extends to larger r 's. Thus the "halo" in an exotic unstable nuclei, like ^{11}Li , is a simple manifestation of the weak binding energy of the last nucleons. What is not as trivial is to know why ^6He and ^{11}Li are bound while ^5He and ^{10}Li are not. We will continue this discussion later.

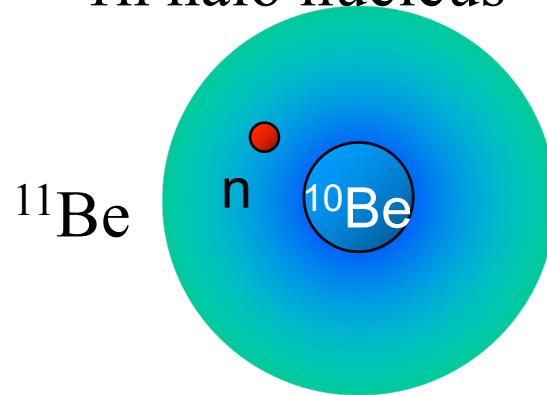
Abnormally large radii were also found for other light neutron-rich nuclei [7] as shown in figure 5(b).

The matter density radii of these nuclei do not follow the commonly observed trend $R \sim 1.18 A^{1/3}$ fm of normal nuclei. Thus the halo seems to be a common feature of loosely-bound neutron-rich nuclei. In Table 3.3 we list the spin, parities and mass number of some light neutron-rich nuclei. The separation energy of one neutron (S_n) and of two neutrons (S_{2n}) are also shown. One observes that the two-neutron separation energies of ^{11}Li , ^{14}Be and ^{17}B are very small and are responsible for large matter radii of these nuclei, as seen in figure 5(b). A nuclear chart with the halo nuclei is shown in Figure 6.

Examples: ^{11}Li and ^{11}Be (courtesy: T. Nakamura)

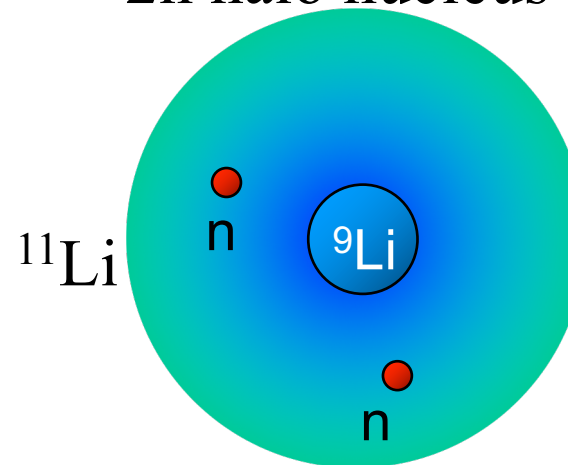


1n halo nucleus



$S_n = 504 \text{ keV}$

2n halo nucleus



$S_{2n} = 300 \text{ keV}$

In order to understand the correlation between the appearance of a halo and the small binding energy of a neutron, we use a simple model (square well). It is well known that in this case, namely

$$V = -V_0(r < r_0) = 0(r > r_0), \quad (1)$$

the radial Schrödinger equation takes the form

$$-\frac{\hbar^2}{2\mu} \frac{d^2 u}{dr^2} + V_{\text{eff}}(r)u = Eu, \quad (2)$$

where μ is the reduced mass and the effective potential includes the centrifugal term. For $\ell=0$, the centrifugal barrier is absent and the solution for negative energy is

$$\begin{aligned} u(r) &= A \sin(kr), \quad \text{for } r < r_0 \\ &= B e^{-\eta r}, \quad \text{for } r > r_0, \end{aligned} \quad (3)$$

where

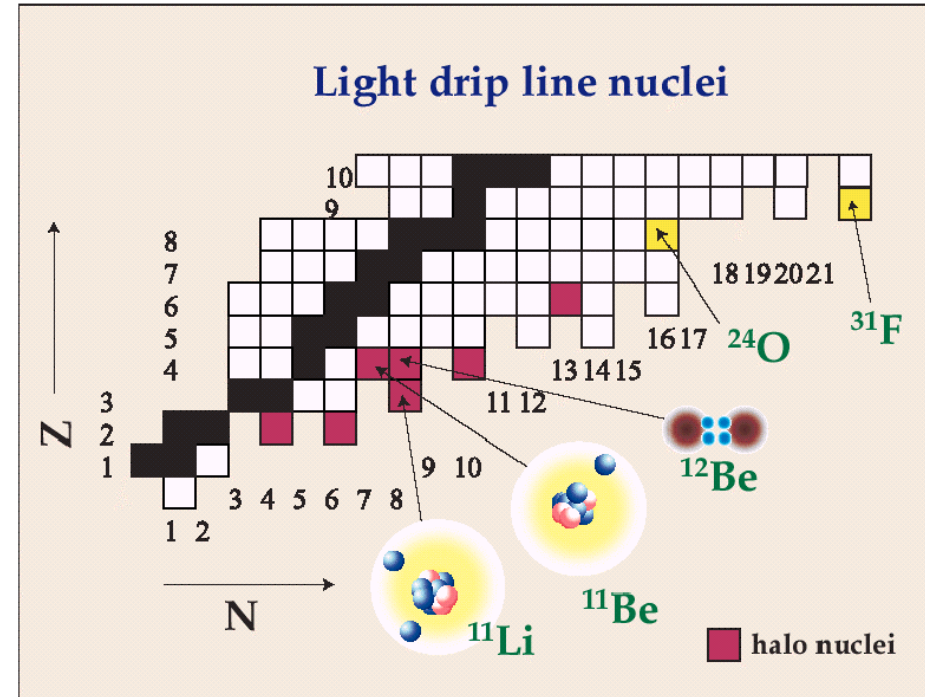
$$\begin{aligned} k &= \sqrt{\frac{2\mu}{\hbar^2}(V_0 + E)} \\ \eta &= \sqrt{\frac{2\mu|E|}{\hbar^2}}. \end{aligned} \quad (4)$$

Let us the model (oversimplified !) for the case of ^{11}Be , which is a one-neutron halo.

We know S_n is about 0.5 MeV, so that $\eta \sim 1/6 \text{ fm}^{-1}$.

The size of the neutron orbit in ^{11}Be is two times the core size

$$R_0 A^{1/3} = 2.67 \text{ fm}$$



Other evidences

- **Other experiments** which have been historically important, to make the character of halo nuclei evident, are: (i) momentum distributions of projectile fragments, and (ii) electromagnetic dissociation.

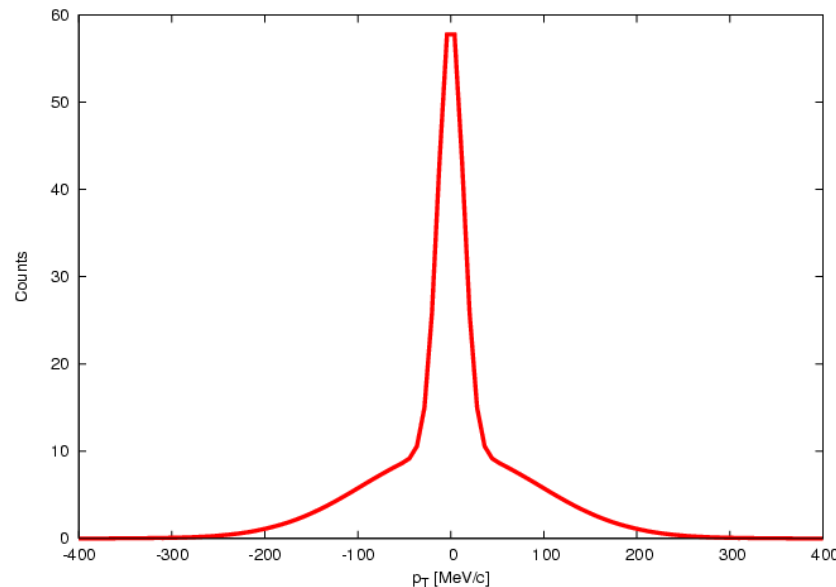
These are reviewed in papers e.g. by I. Tanihata.

The idea of the momentum distribution experiments is quite simple. If we hit, e.g., ^{11}Li on a C target at 800 MeV/u we can measure the transverse momentum of the fragments and we find a “double” distribution.

The component with “small width” has a Δp of about 19 MeV/c. From the uncertainty principle

$$\Delta x \sim hc/\Delta p \sim 12 \text{ fm},$$

that is, the narrow component is arising from the halo.



The best-fitted density distribution $\rho(r)$ for ^{11}Li is shown in Fig. 10. For the halo neutrons, 1p or 2s orbits were assumed, and the both fits gave essentially the same distribution as seen in the figure. A difference was seen in the orbital energy, which was ascribed to the centrifugal-barrier effect. It was noted here that the selection of orbital was more or less arbitrary and only the final density distribution had the practical meaning. The obtained density distribution gave the rms radii, $R_{\text{rms}}(^{11}\text{Li}) = 3.1 \pm 0.3$ fm, and $R_{\text{rms}}(2n) = 4.8 \pm 0.8$ fm for the ^{11}Li nucleus and the halo neutrons, respectively. The size of the ^9Li core ($R_{\text{rms}} \sim 2.6$ fm) turned out to be slightly larger than that of the ^9Li nucleus ($R_{\text{rms}} = 2.32 \pm 0.02$ fm), qualitatively consistent with the center-of-mass motion of ^9Li core in ^{11}Li .

Functional shapes other than the one described above were also investigated: a) Single Gaussian gave no good fit to the data and was definitely inappropriate. b) Gaussian core + Yukawa tail gave also a good fit and the obtained $\rho(r)$ showed essentially the same halo tail but with a slightly higher central density.

Intensive theoretical studies have been performed to understand the structure of these halos by various authors. Bertsch, Brown and Sagawa¹⁹ made a Hartree-Fock calculation by constraining the separation energy of the last occupied orbit. The calculated density distributions are shown in corresponding figures (Fig. 9 and Fig. 10). They show good agreement with the present semi-empirical result. An important implication of the HF calculation is that the long tail of the density distribution mainly arises from the weakly-bound last neutron.

A microscopic model of ^{11}Li was developed by Ikeda, Suzuki and their collaborators²⁰ by a hybrid model combining a cluster-orbital-shell model with an extended cluster model. The model gave a plausible explanation of the binding mechanism of the three-body system of $^9\text{Li}+n+n$, and both the rms radius and the last-neutron binding energy were consistently reproduced.

4.3 Momentum distributions

Studies of projectile fragmentation at high energy (≥ 400 MeV/nucleon) shows that the momentum distribution of nucleons inside a projectile nucleus can be determined from the momentum distribution of the projectile fragments. Extending the method used

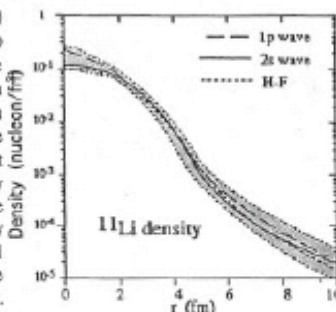


Fig. 10. Density distribution of ^{11}Li deduced from the interaction cross sections. The selection of orbitals, 1p or 1s for the halo neutrons, gave only a minor difference. The shaded area shows the fitting uncertainty. The dotted line shows the H-F model prediction constrained by the last neutron separation energy.

for stripping reactions to many nucleon removal, the width of the P_T distribution* of the projectile fragment is expressed by the separation energy of last neutrons,

$$\sigma^2 = 2u(E_s) \frac{A_p(A_p - A_f)}{A_p} \quad (4.5)$$

where u is the atomic mass unit and $\langle E_s \rangle$ is an average separation energy of the removed nucleons. It is essentially the same conclusion with the discussion in sec. 4.1.

The transverse momentum distributions of fragments of ^{11}Be and ^{11}Li by carbon target were firstly measured at 790 MeV/nucleon.³ Figure 11 presents a transverse momentum distribution of (a) ^{10}Be fragment from $^{11}\text{Be} + \text{C}$ reaction and (b) ^9Li fragment from $^{11}\text{Li} + \text{C}$ reaction both at 800.4 MeV. Both of the data show a very narrow peak on top of another wider peak. The fitting of the momentum distribution by two gaussians gives the width to be $\sigma_{\text{narrow}} = 25 \pm 4$ MeV/c and $\sigma_{\text{wide}} = 109 \pm 7$ MeV/c for ^{10}Be spectrum. In ^9Li , $\sigma_{\text{narrow}} = 21 \pm 3$ MeV/c and $\sigma_{\text{wide}} = 80 \pm 4$ MeV/c. The narrow peaks indicates an existence of neutrons with extremely small momentum fluctuation as expected in the neutron halo.

The σ_{wide} is consistent with the momentum fluctuations of usual nucleons.

* The P_T distribution ($d\sigma/dP_T$) here is defined as,

$$\frac{d\sigma}{dP_T} = \iint \frac{d^3\sigma}{dP_x dP_y dP_z} dP_x dP_y dP_z$$
 where (P_x, P_y, P_z) corresponds to the momentum vector in the Cartesian coordinate (x, y, z) with z being the direction of an incident beam.

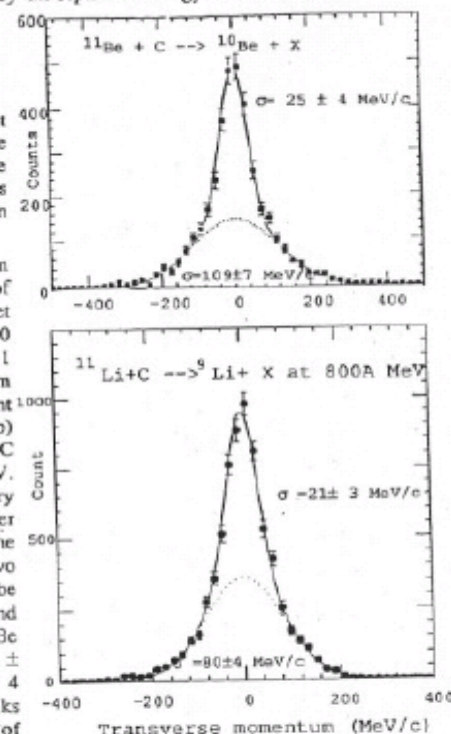


Fig. 11. Transverse momentum distribution of fragments from halo nuclei ^{11}Be and ^{11}Li