the β-stable nuclei (see Fig. 2-18). More exacting tests of the semi-empirical mass formula are encountered when one tries to predict the masses of nuclei far from the line of β stability (as is of special interest in connection with the theories of nucleogenesis; see Sec. 2-3b) and of nuclei much heavier than those so far studied. It is likely that for these purposes additional terms in the mass formula may be required, describing, for example, a dependence of the surface energy and of the average nuclear density on the charge symmetry parameter, $N-Z$.

A more detailed examination of the nuclear binding energies reveals systematic deviations from a smoothly varying function of Z and N, which may amount to about 10 MeV in the total binding, as indicated in Fig. 2-4. A quantitative treatment of these quantal effects requires a rather complete description of the nuclear structure, including detailed configuration assignments, evaluation of correlation effects, and so on. The largest of the quantal effects are associated with the shell structure and the nuclear deformations, as discussed in Sec. 9-1. For attempts to generalize the binding energy formula to include quantal effects in terms of a number of average parameters, see Zeldes *et al.* (1967); Myers and Swiatecki (1966). We consider in the next section a part of the quantal effect, the pairing energy, which has an especially simple structure.

2-1e Pairing Energy

The nuclear binding energies are found to exhibit a systematic variation depending on the evenness or oddness of Z and N

$$\delta\mathscr{B} = \begin{cases} \Delta & Z \text{ even} \quad N \text{ even} \\ 0 & A \text{ odd} \\ -\Delta & Z \text{ odd} \quad N \text{ odd} \end{cases} \tag{2-15}$$

From the Fermi gas model, we expect an odd-even difference (pairing energy) resulting from the fact that each orbit, $\mathbf{k}$, can be occupied by two protons and by two neutrons. Thus, we obtain an odd-even parameter Δ that is of the order of the spacing between the one-particle energies in the neighborhood of the Fermi energy

$$(\Delta)_{\text{kin}} \approx \frac{2}{3}\frac{\varepsilon_F}{A} \approx \frac{25}{A} \quad \text{MeV} \tag{2-16}$$

The observed pairing energies are shown in Fig. 2-5, p. 170, and are seen to be almost an order of magnitude larger than the estimate (2-16).

The large observed odd-even effect may be described in terms of a pairwise correlation of identical particles, which contributes an additional binding energy of 2Δ per pair (for nucleons near the top of the Fermi distribution).

most stable isobar for each even value of the mass number A is plotted in Fig. 2-4. The restriction to even A avoids the systematic odd-even variation which is illustrated in Fig. 2-5.

The smooth curve in Fig. 2-4 is calculated from the mass formula (2-12) with the constants

$$\begin{aligned} b_{\text{vol}} &= 15.56 \text{ MeV} \\ b_{\text{surf}} &= 17.23 \text{ MeV} \\ b_{\text{sym}} &= 46.57 \text{ MeV} \\ R_c &= 1.24\, A^{1/3} \text{ fm} \end{aligned} \tag{2-91}$$

Besides the general trends described by the mass formula (2-12), one can see, in the measured binding energies, a number of significant local variations. Thus, in the light nuclei, the binding energy is systematically greater for mass numbers $A = 4n$ than for $A = 4n + 2$ (n an integer). These "short periods" are associated with the fact that the nuclear interactions favor states that are spatially symmetric (see, for example, Chapter 7). In heavier nuclei, the binding energies exhibit local maxima, which are associated with the completion of major shells.

Pairing energies (Fig. 2-5)

The odd-even mass parameter Δ defined by Eq. (2-15) can be determined from the empirical masses of a sequence of isotopes or isotones. Assuming that the masses are a smooth function of Z and N except for the pairing effect, we can define a local average of the masses of odd-A nuclei, and by comparing this value with the observed masses of the even-even nuclei, we obtain Δ. Thus, for even N, we may define

$$\begin{aligned} \Delta_n &= \tfrac{1}{4}\{\mathscr{B}(N-2,Z) - 3\mathscr{B}(N-1,Z) + 3\mathscr{B}(N,Z) - \mathscr{B}(N+1,Z)\} \\ &= -\tfrac{1}{4}\{S_n(N-1,Z) - 2S_n(N,Z) + S_n(N+1,Z)\} \\ S_n(N,Z) &\equiv \mathscr{B}(N,Z) - \mathscr{B}(N-1,Z) \end{aligned} \tag{2-92}$$

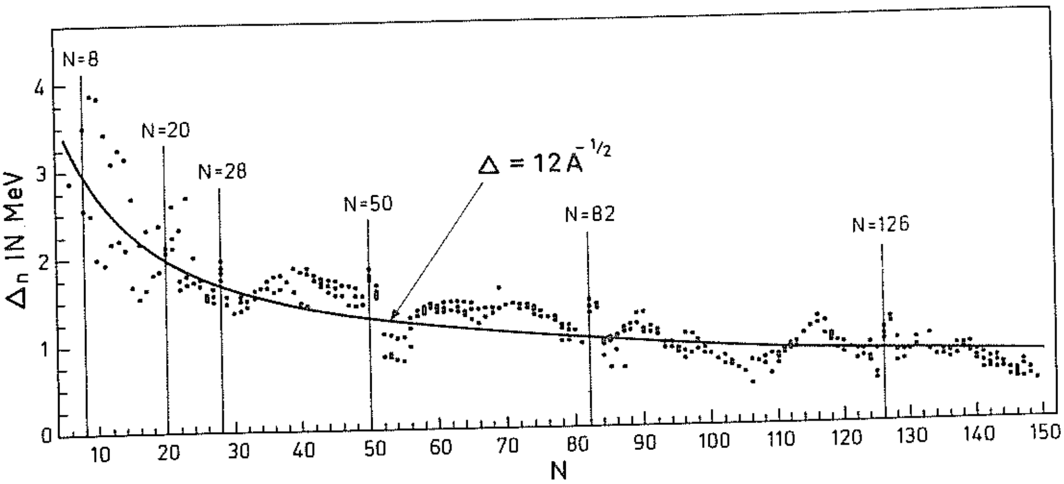
while, for odd N, the negative of the expression (2-92) is taken as the neutron pairing energy. Similarly, for even Z, we define the proton pairing energy

$$\begin{aligned} \Delta_p &= \tfrac{1}{4}\{\mathscr{B}(N,Z-2) - 3\mathscr{B}(N,Z-1) + 3\mathscr{B}(N,Z) - \mathscr{B}(N,Z+1)\} \\ &= -\tfrac{1}{4}\{S_p(N,Z-1) - 2S_p(N,Z) + S_p(N,Z+1)\} \\ S_p(N,Z) &\equiv \mathscr{B}(N,Z) - \mathscr{B}(N,Z-1) \end{aligned} \tag{2-93}$$

while, for odd Z, the negative of Eq. (2-93) is used. The pairing energies (2-92) and (2-93) obtained from the empirical masses are plotted in Fig. 2-5 as a function of the number of neutrons or protons in the nucleus. It is seen that the general trend in the observed pairing energies is fit by the simple expression

$$\Delta \approx \frac{12}{A^{1/2}} \text{ MeV} \tag{2-94}$$

although significant local variations occur and appear to be correlated with the



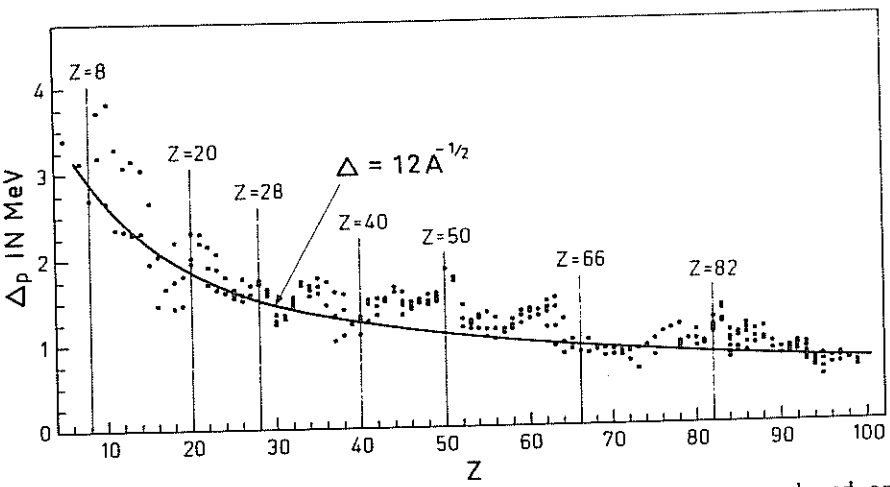


Figure 2-5 The odd-even mass differences for neutrons and protons are based on the analysis of N. Zeldes, A. Grill, and A. Simievic, *Mat. Fys. Skr. Dan. Vid. Selsk.* 3, No. 5 (1967).

shell structure. There is a slight tendency for Δ_p to exceed Δ_n. This fact leads to a predominance of odd-N nuclei among the β-stable species as compared with odd Z (53 odd-Z, compared to 68 odd-N β-stable nuclei with $A < 238$). In Chapter 8, we shall consider the origin and more detailed interpretation of the pairing energy.

The simple description of the pairing energy used in Eq. (2-15) implies that the extra energy of an odd-odd nucleus, as compared with an even-even configuration, is

$$\mathscr{E}_{\text{odd-odd}} - \mathscr{E}_{\text{even-even}} \approx \Delta_n + \Delta_p \approx 2\Delta \tag{2-95}$$

Comparison with the observed masses of odd-odd nuclei reveals that this relation is approximately fulfilled, but that there is a systematic tendency for the