Nilsson model

$$H_0 = \frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2} m\omega_0^2 r^2 + c\,\vec{l}\cdot\vec{s} + d\left(\vec{l}^2 - \langle \vec{l}^2\rangle\right)$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2} m\left[\omega_\perp^2 (x^2+y^2) + \omega_z^2 z^2\right] + (\dots)$$

$$\omega_z = \omega_0(\varepsilon)\left(1 - \frac{2}{3}\varepsilon\right)$$

$$\omega_\perp = \omega_0(\varepsilon)\left(1 + \frac{1}{3}\varepsilon\right)$$

$$\omega_\perp^2 \omega_z \stackrel{!}{=} \omega_0^3 = \left[\omega_0(\varepsilon)\right]^3\left(1 - \frac{2}{3}\varepsilon\right)\left(1 + \frac{2}{3}\varepsilon + \frac{1}{9}\varepsilon^2\right) =$$

$$= \left[\omega_0(\varepsilon)\right]^3\left(1 - \frac{1}{3}\varepsilon^2 + \dots\right)$$

$$\omega_0(\varepsilon) = \omega_0\left(1 - \frac{1}{3}\varepsilon^2 + \dots\right)^{-1/3} \approx \omega_0\left(1 + \frac{1}{9}\varepsilon^2\right)$$

$$H = H_0 + \varepsilon h^1 \quad \text{behandle wie } H_1$$

$$\varepsilon H_1 = \frac{1}{2} m\omega_0^2\left[\left(1 + \frac{2}{3}\varepsilon\right)(x^2+y^2) + \left(1 - \frac{4}{3}\varepsilon\right)z^2 - r^2\right] =$$

$$= \frac{1}{\cancel{2}} m\omega_0^2 \frac{\cancel{2}}{3}\varepsilon\left(x^2+y^2-2z^2\right)$$

$$Y_{20} = \frac{1}{4}\sqrt{\frac{5}{\pi}}\,\frac{2z^2-x^2-y^2}{r^2} \Rightarrow -\sqrt{\frac{16\pi}{5}}\, r^2 Y_{20} = x^2+y^2-2z^2$$

$$\varepsilon H_1 = -\frac{1}{3}\sqrt{\frac{16\pi}{5}}\, m\omega_0^2\, \varepsilon\, r^2 Y_{20}$$

$$\left[\; Y_{20} = \sqrt{\frac{5}{4\pi}}\, P_2 \quad \Rightarrow \quad \varepsilon H_1 = -\frac{2}{3} m\omega_0^2\, \varepsilon\, r^2 P_2 \;\right]$$

$$\langle n l j m | \varepsilon H_1 | n l j m \rangle = \int dr\, r^2\, u_{nlj}^2\, r^2 \quad \left(\varepsilon m \omega_0^2\right)\left(-\frac{1}{3}\sqrt{\frac{16\pi}{5}}\right)$$

$$\times \underbrace{\langle l j m | Y_{20} | l j m \rangle}_{A}$$

$$\langle n l j m | \varepsilon H_1 | n l j m \rangle = -\frac{1}{3}\sqrt{\frac{16\pi}{5}}\ \varepsilon m \omega_0^2 \int dr\, u_{nlj}^2\, r^4$$

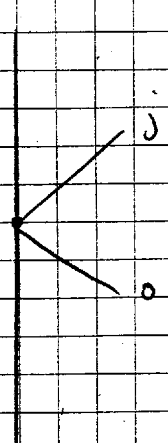
$$\langle l j m | Y_{20} | l j m \rangle =$$

$$= +\frac{1}{3}\sqrt{\frac{16\pi}{5}}\ \varepsilon m \omega_0^2 \int dr\, u^2 r^4 \sqrt{\frac{5}{64\pi}}\ \frac{3m^2 - j(j+1)}{j(j+1)}$$

$$= \frac{1}{3}\cancel{\sqrt{\frac{5}{16\pi}}}\ \frac{\varepsilon m \omega_0^2}{2} \underbrace{\int dr\, u^2 r^4}_{\langle r^2 \rangle} \frac{3m^2 - j(j+1)}{j(j+1)}$$

Check with Mathematica OK

$$\frac{d}{d\varepsilon}\Delta E \sim \frac{3m^2 - j(j+1)}{j(j+1)}$$



a) $m = j$ (orbit on the plane)

$$j^2 - j = j(j-1) > 0$$

(consistent with the picture (the potential is STEEPER on the xy plane)

b) $m = 0$ $\quad \frac{d}{d\varepsilon}\Delta E < 0$ (id)