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ISAPP 2003<sup>1</sup>  
Madonna di  
Campiglio

## NEUTRINO PROPERTIES

1<sup>st</sup> lecture

Chronological overview of  $\nu$  physics

Mass terms in field theory and  $\nu$ 's

Flavor oscillations

2<sup>nd</sup> lecture

Solar neutrinos

Atmospheric neutrinos

Long baseline experiments

3<sup>rd</sup> lecture

$\theta_{13}$  and CP phase

$\beta\beta$  decays

## NEUTRINOS. HISTORICAL SURVEY

1899. Rutherford discovers  $\beta$  radioactivity

1914. Chadwick finds a continuous spectrum for the emitted electron

- Several  $e^-$ 's are emitted?

Mosley (1912) electrometer }  $\Rightarrow$  NO  
Emelius (1924) Geiger counter }

- $e^-$ 's are a secondary radiation (L. Meitner)

1927: Calorimetric experiment

$\Rightarrow$  All the visible energy (measured in a real calorimeter) is carried away by the electron

$\rightarrow$  Bohr: Energy is not conserved in microscopic processes

$\rightarrow$  Pauli: Missing energy carried away by a new particle!

# La conception du neutrino

Lettre de Pauli du 4 Decembre 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li<sup>6</sup> nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

I agree that my remedy could seem incredible because one should have seen those neutrons very earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: "Oh, It's well better not to think to this at all, like new taxes". From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge. Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant

W. Pauli

1934 - Fermi presents his theory of  
 $\beta$  decay  $n \rightarrow p e^- \nu$

### Attempts to detect the neutrino

1935 - Nahmias tries to measure the ionisation  
produced by neutrinos due to their  
possible magnetic moment

First  
underground  
expt!

$$\Rightarrow \mu_\nu < 2 \cdot 10^{-4} \mu_B$$

$\Rightarrow$  Bethe: If  $\mu_\nu = 0$ ,  $\sigma_\nu \sim 10^{-44} \text{ cm}^2$   
(from Fermi constant)

1937 - Crane tries to transform Cl into S  
(then  $\text{Cl}^{35} + \nu \rightarrow \text{S}^{35} + e^+$ )

using a 1 mCi  $\text{Ra}^{228}$  source

$\Rightarrow$  Extracts S chemically

Tries to count  $\text{S}^{35}$  decays in a  
ionization chamber

First radiochemical expt!

# 1956: Discovery of the $\nu$

- Idea = Detect  $\nu$ 's through inverse  $\beta$  decay reaction



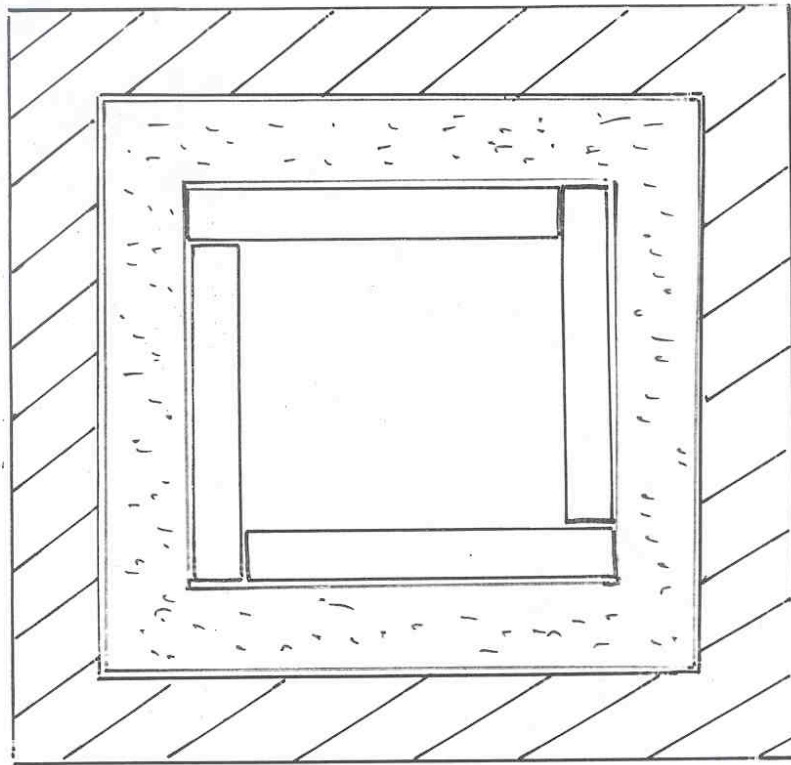
- Source = Nuclear reactor

$$[2 \cdot 10^{20} \bar{\nu}_e / \text{s for 1 Thermal GW}]$$


- Main problem for detection:  
The background!


Detector has to be

- close to the reactor
- well shielded against  $n, \gamma,$   
cosmics
- The event signature has to  
be very discriminating  
 $\Rightarrow$  sign BOTH  $e^+$  and  $n$

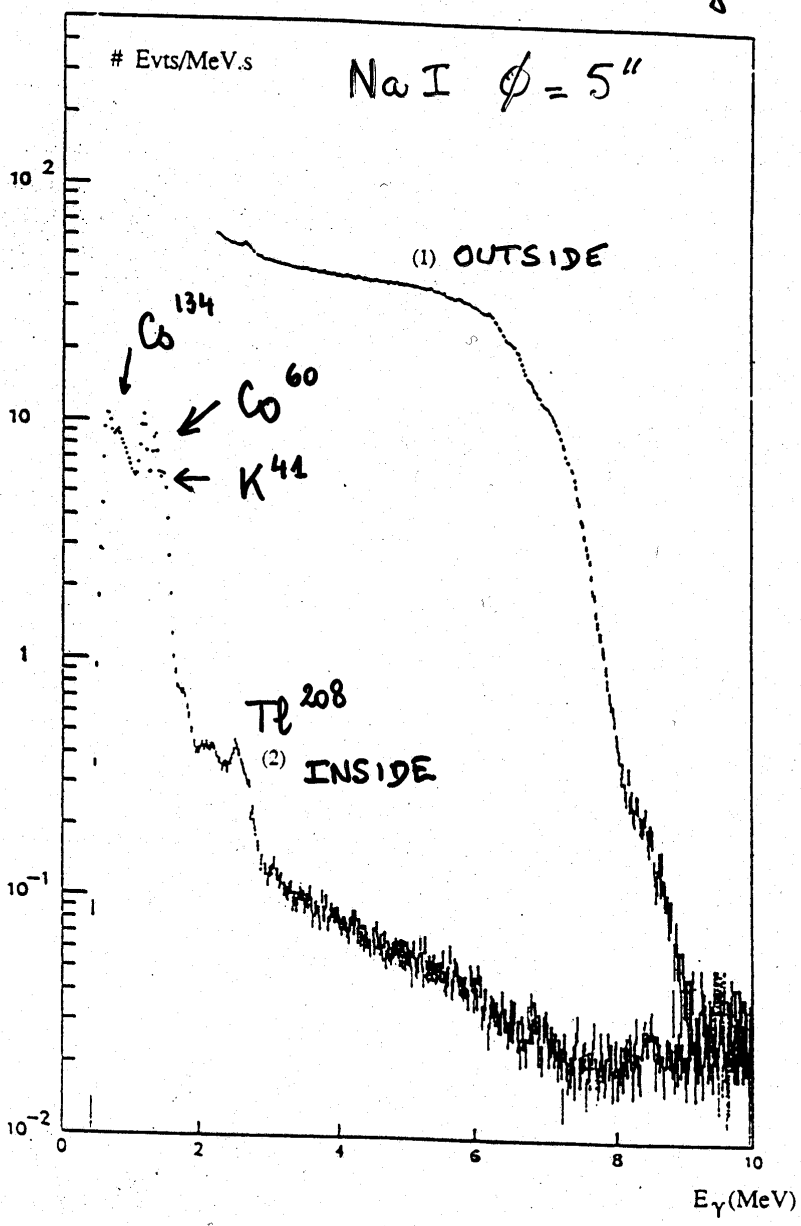


 Plomb, acier : Arrête les  $\gamma$

  $(H_2O) + Bore =$  Thermalisation  
(C, H<sub>2</sub>) et capture des n

 Scintillateur, chambres =  
(liq. ou solide) Veto actif contre les cosmiques

# Activité $\gamma$ / $\gamma$ activity

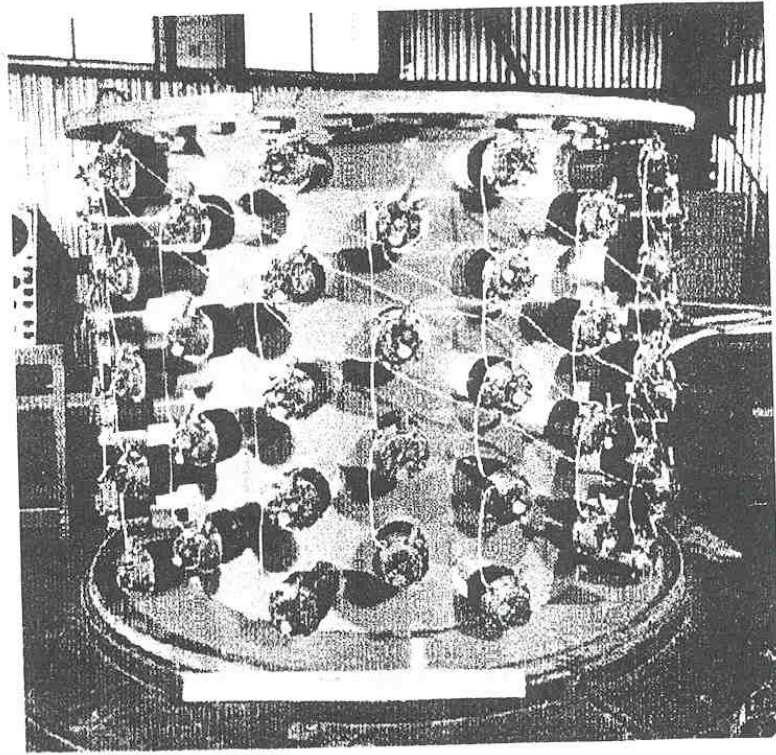


- 1. EXTERIEUR DU BLINDAGE
- 2. INTERIEUR DU BLINDAGE

$\gamma$  activity inside shielding compared to outside activity

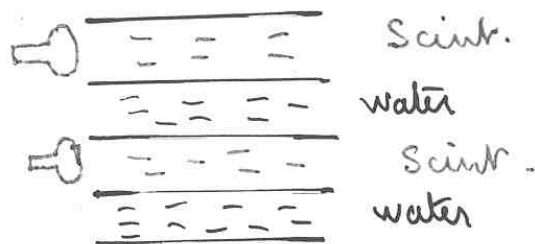
## SOME NUCLEI FOR n SIGNATURE

Nucleus	$\sigma$ (Barn)	Signature
P	0.3	$n+p \rightarrow d + \gamma$ (2.2 MeV)
Cd	2450	Several $\gamma$ 's $E_{\text{tot}} = 9$ MeV
B	767	$B^{10} + n \rightarrow \alpha + Li^7 + 2.8$ MeV (+ $\gamma$ at 0.48 MeV)
Gd	50000	3 $\gamma$ 's, $E_{\text{tot}} = 8$ MeV
He <sup>3</sup>	5350	$n + He^3 \rightarrow p + H^3 + 765$ keV
Li <sup>6</sup>	940	$n + Li^6 \rightarrow He^4 + He^3 + 4.8$ MeV



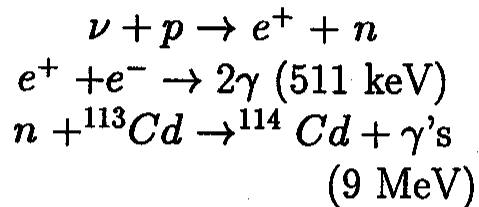
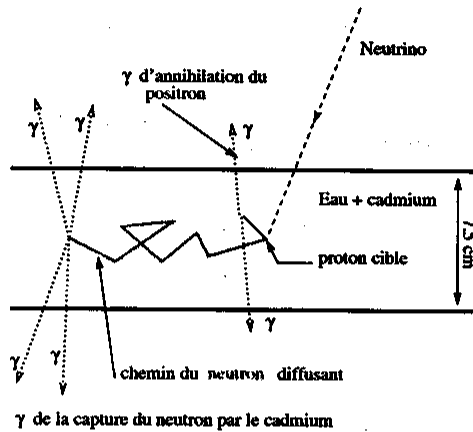
Detector used by Cowan and Reines  
to discover the neutrino.

Sandwich of water and of scintillator



# La naissance du neutrino

## Le principe de détection



Coincidence retardée (qq  $\mu\text{s}$ )

Efficacité d'environ 1 %

$$E_S \sim 1\%$$

## Le résultat

- Réacteur arrêté:  $\simeq 1$  evt/heure Reactor OFF
- Réacteur en marche:  $\simeq 4$  evt/heure Reactor ON
- Signal:  $2.88 \pm 0.22$  evt/heure  $\sigma \sim 10^{-43} \text{ cm}^2$

Detection principe:

Delayed coincidence (several  $\mu\text{s}$ )

between  $e^+$  pulse and

n capture pulse.

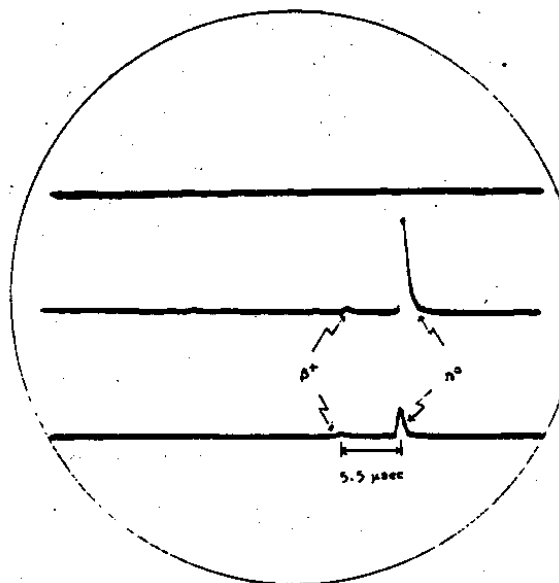
20 July 1956, Volume 124, Number 3212

# SCIENCE

## Detection of the Free Neutrino: a Confirmation

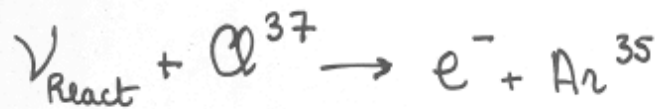
C. L. Cowan, Jr., F. Reines, F. B. Harrison,  
H. W. Kruse, A. D. McGuire

NATURE September 1, 1956 VOL. 178



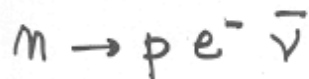
# 1955 - Difference between $\nu$ and $\bar{\nu}$

Davis experiment at Savannah River



NOT SEEN!

Reactor  $\nu$ 's are labelled  $\bar{\nu}$



→ The leptonic number is conserved

$\nu$  carry a charge ⇒ They are Dirac particles

$$\nu \neq \bar{\nu}$$

# 1956 - Parity violation in $\beta$ decays

→ Measured by Miss Wu -  
The violation is maximal

Lee and Yang modify the Fermi theory

$$\mathcal{H}_{\text{Fermi}} = G \hbar^\mu l_\mu \quad \text{Current-current interaction}$$

$$\begin{cases} l^\mu = \bar{\psi}_e \gamma^\mu \psi_\nu \\ h^\mu = \bar{\psi}_p \gamma^\mu \psi_n \end{cases} \quad \text{Parity is conserved}$$

$$\Rightarrow l^\mu = \bar{\psi}^e \gamma^\mu (a + b \gamma^5) \psi_\nu$$

violates parity

Maximal violation  $|a| = |b| = 1$

$$l^\mu = \bar{\psi}^e \gamma^\mu (1 - \gamma^5) \psi_\nu \quad \text{V-A}$$

Only left-handed  $\nu$ 's interact  
Right-handed  $\nu$ 's are sterile and do not even exist

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### Question of Parity Conservation in Weak Interactions\*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in  $\beta$  decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

### Experimental Test of Parity Conservation in Beta Decay\*

C. S. WU, *Columbia University, New York, New York*

AND

**R. ADLER, R. W. HAYWARD, D. D. HOPPE, AND R. P. HUDSON,**  
*National Bureau of Standards, Washington, D. C.*

(Received January 15, 1957)

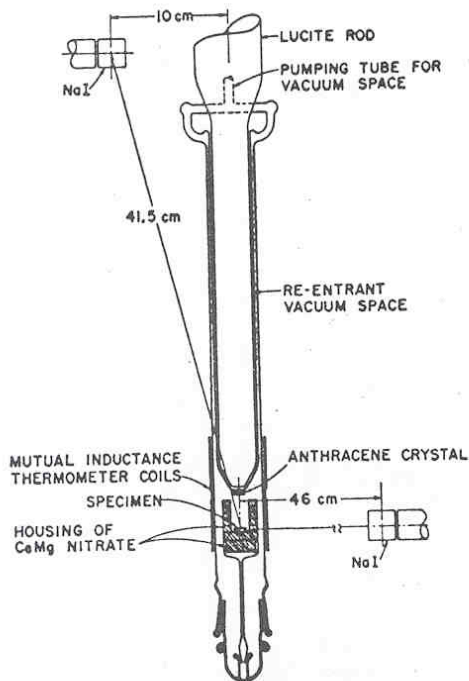


FIG. 1. Schematic drawing of the lower part of the cryostat.

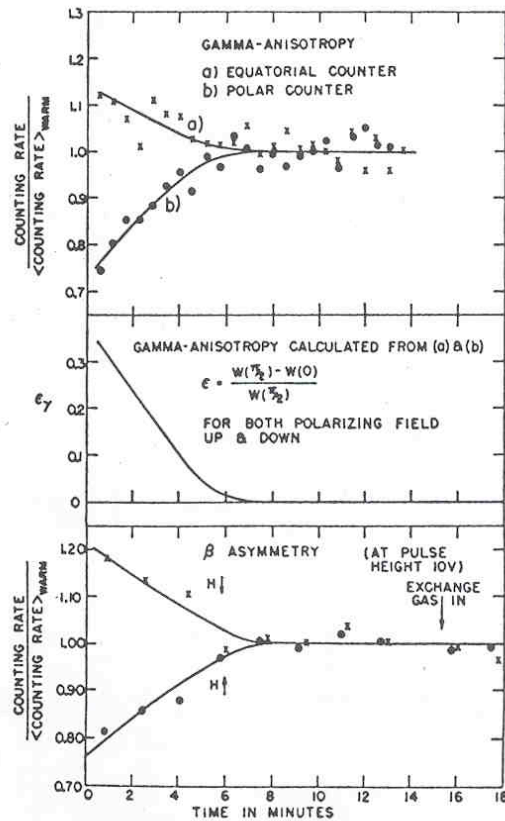
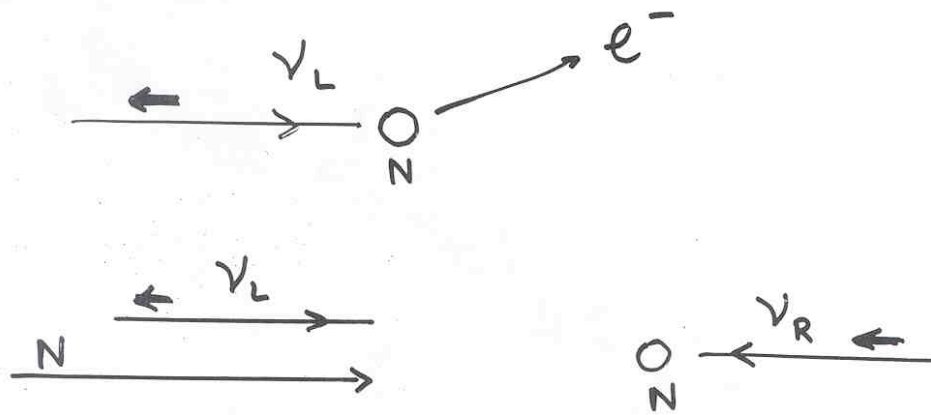


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

## Neutrinos: Dirac or Majorana particles?

- If  $\psi \neq \bar{\psi}$  Dirac
  - $\psi_L$  and  $\bar{\psi}_R$  interact
  - $\bar{\psi}_L$  and  $\psi_R$  do not exist
- But another equivalent possibility  
Majorana:  $\psi \equiv \bar{\psi}$  (no charge)
  - $\psi_L$  produces  $e^-$
  - $\psi_R$  produces  $e^+$  (and is called  $\bar{\nu}$ )
- \* If  $\psi$ 's are massless, perfect equivalence (Weyl neutrino)
- \* If  $\psi$ 's are massive, one should be able to decide.



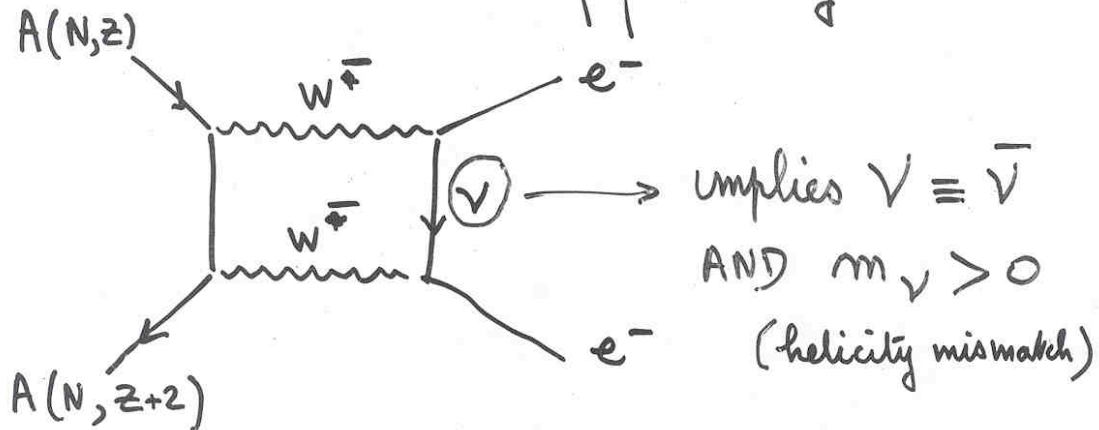
If Dirac : no interaction

If Majorana :  $e^+$  production

**IMPOSSIBLE IN PRACTICE**

Only realistic test:

Neutrinoless  $\beta\beta$  decay



Amplitude  $\propto m_\nu$

This reaction would prove that  $\nu$ 's are massive Majorana particles

## 1962 - 2000 3 neutrino families

$$\pi \rightarrow \mu \nu$$

Is this neutrino the same neutrino produced in  $\beta$  decays?

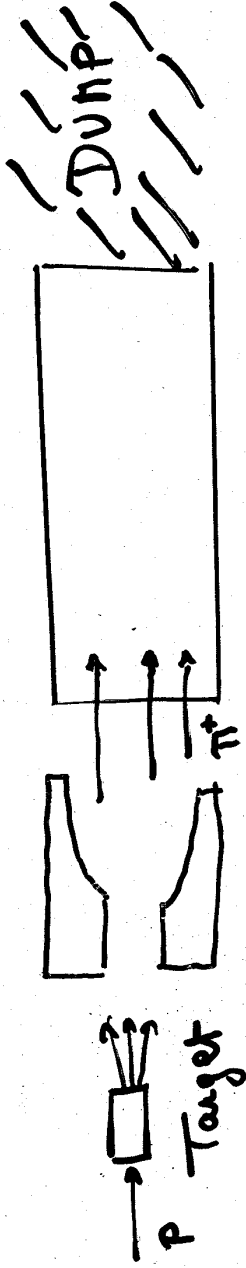
- If yes, this  $\nu$  should produce  $e$  and  $\mu$  equally when interacting with matter
- If no, this  $\nu$  should produce only  $\mu$ 's (and  $\beta$  decay neutrinos only  $e$ 's, but it is implied by energy)

1962: proton accelerators ( $\sim 1\text{GeV}$ )

Secondary pion beams

$\Rightarrow \nu_{\pi}$  beams by  $\pi$  decay

# Typical $\nu$ beam



Horn:

focalises  $\pi^+$

defocalises  $\pi^-$

Decay channel

$\pi^+ \rightarrow \mu^+ \nu_\mu$

Beam mostly  $\nu_\mu$

Contamination in  $\nu_e$ :  $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$

$K^+, K^0$  decays

Contamination in  $\bar{\nu}_\mu$  ( $\pi^-, K^-$ )

1962 -  $\nu_\mu \neq \nu_e$

(38)

OBSERVATION OF HIGH-ENERGY NEUTRINO REACTIONS AND THE EXISTENCE OF TWO KINDS OF NEUTRINOS\*

G. Danby, J-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz,† and J. Steinberger†

Columbia University, New York, New York and Brookhaven National Laboratory, Upton, New York (Received June 15, 1962)

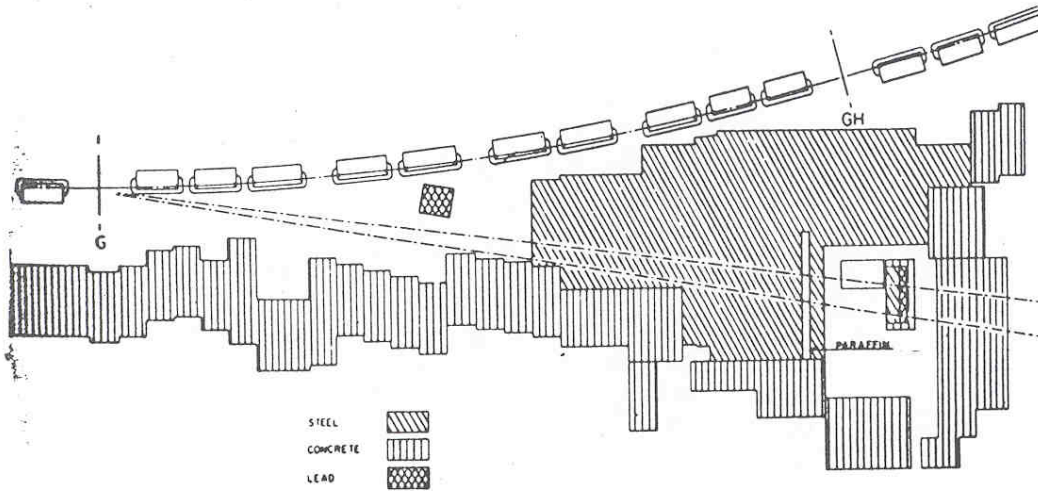


FIG. 1. Plan view of AGS neutrino experiment.

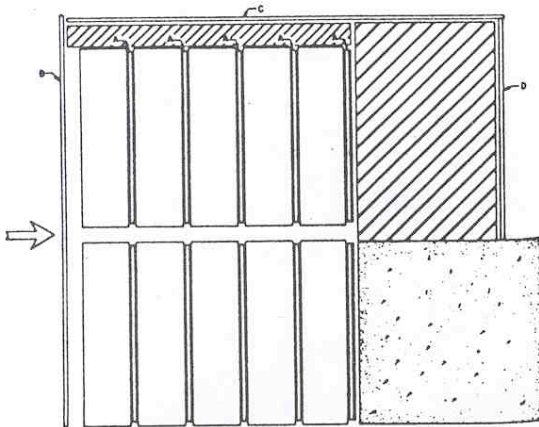


FIG. 3. Spark chamber and counter arrangement. A are the triggering slabs; B, C, and D are anticoincidence slabs. This is the front view seen by the four-camera stereo system.

34 "single  $\mu$ "  
(Bruit de fond 5)

6 "showers" ( $e, \delta$ )

$\Rightarrow \nu_\mu \neq \nu_e$

$\nu_\pi \rightarrow \mu$  } exclus car  
 $\nu_K \rightarrow e$  }  $E_{\text{shower}}$  faible

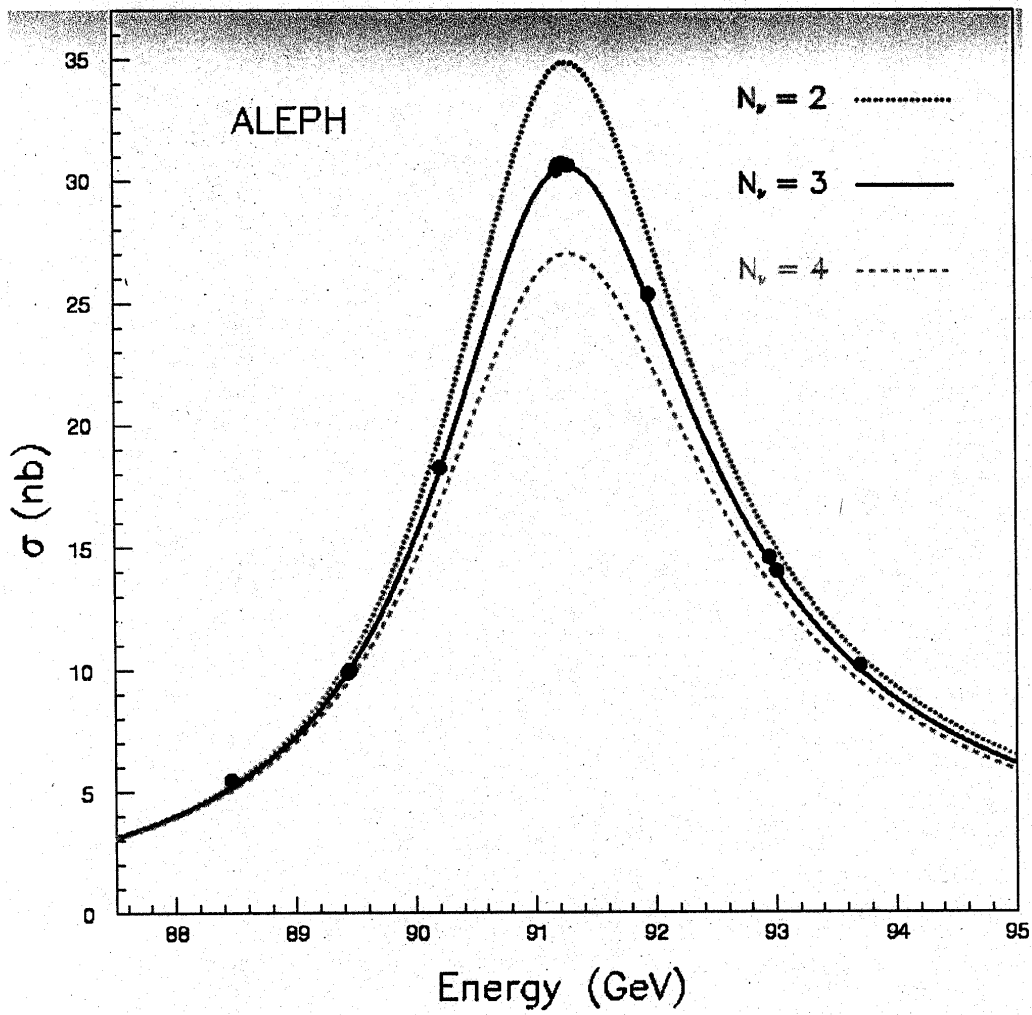
1975 :  $\tau$  discovery 3 charged leptons

Are there 3  $\neq$  neutrinos

$$\nu_e, \nu_\mu, \nu_\tau ?$$

1990 : LEP shows that 3 different light neutrinos are necessary to explain the  $Z^0$  width

2000 : DONUT experiment observes the production of  $\tau$ 's by  $\nu_\tau$  (Beam dump expt)



$\Gamma_\nu = 110$  MeV for each neutrino flavor



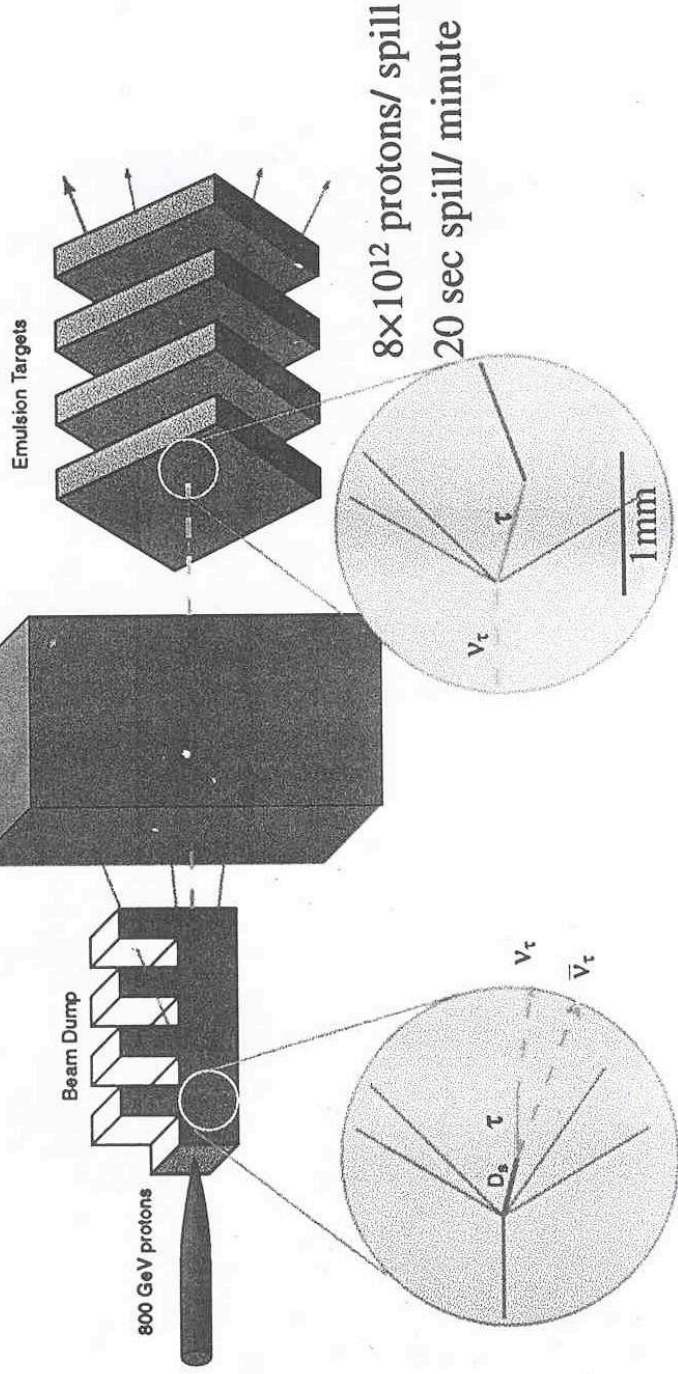
W&C 21 July 2000

# Prompt Neutrino Beam

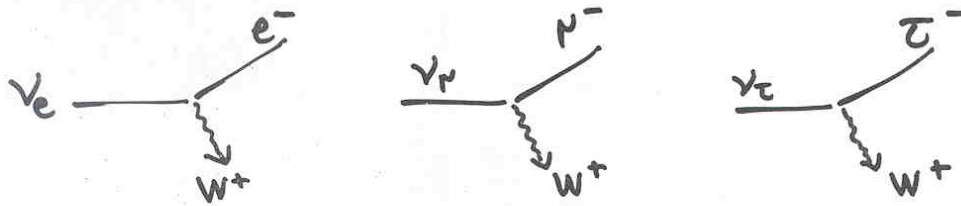
E-872

Making  $\nu_\tau$  interactions  
from protons

Shielding



## Back to neutrino interactions



→ Charged current interactions mediated by  $W$

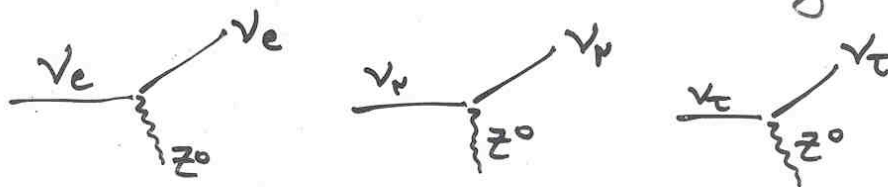
1973 = The bubble chamber Gargamelle observes

$$\nu_\mu e^- \rightarrow \nu_\mu e^-$$

Impossible to explain by C.C.

(only  $\nu_\mu e^- \rightarrow \nu_e \mu^-$  possible)

⇒ New type of  $\nu$  interaction,  
neutral currents mediated by  $Z^0$



(Flavor conserving)

Fundamental result for the  
elaboration of our standard  
electroweak model.

# GARGAMELLE 1973

Chambre à bulles à fréon (CF<sub>3</sub>Br)

L = 4.8 m    V = 6.2 m<sup>3</sup>    λ<sub>R</sub> = 11 cm

360 000  $\bar{\nu}$  pictures    1 candidat

(fond 0.03 ± 0.02)

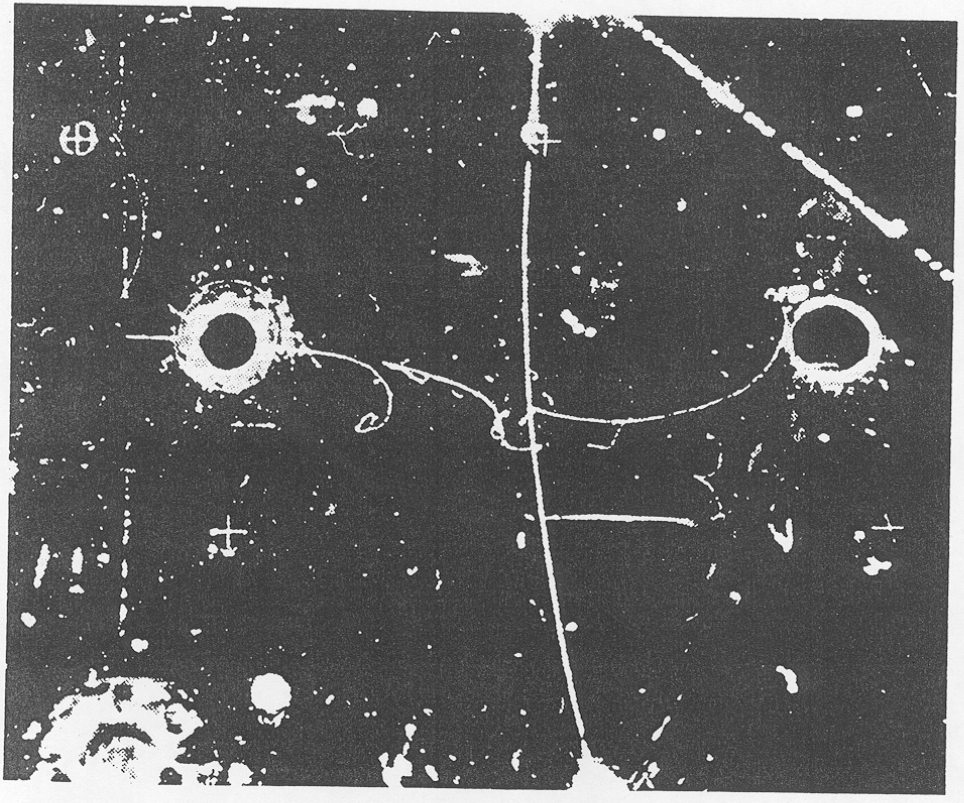


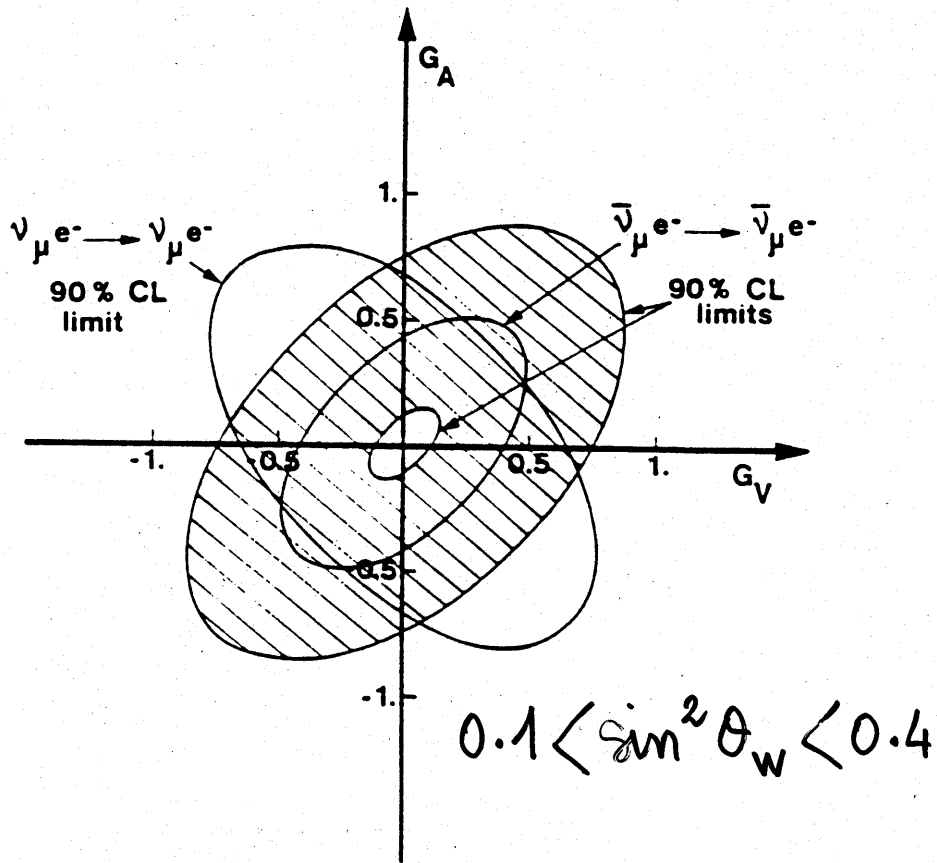
Fig. 10. First event of  $\bar{\nu}_\mu e$  scattering observed in Gargamelle [1]

1976 = 3 événements vis.

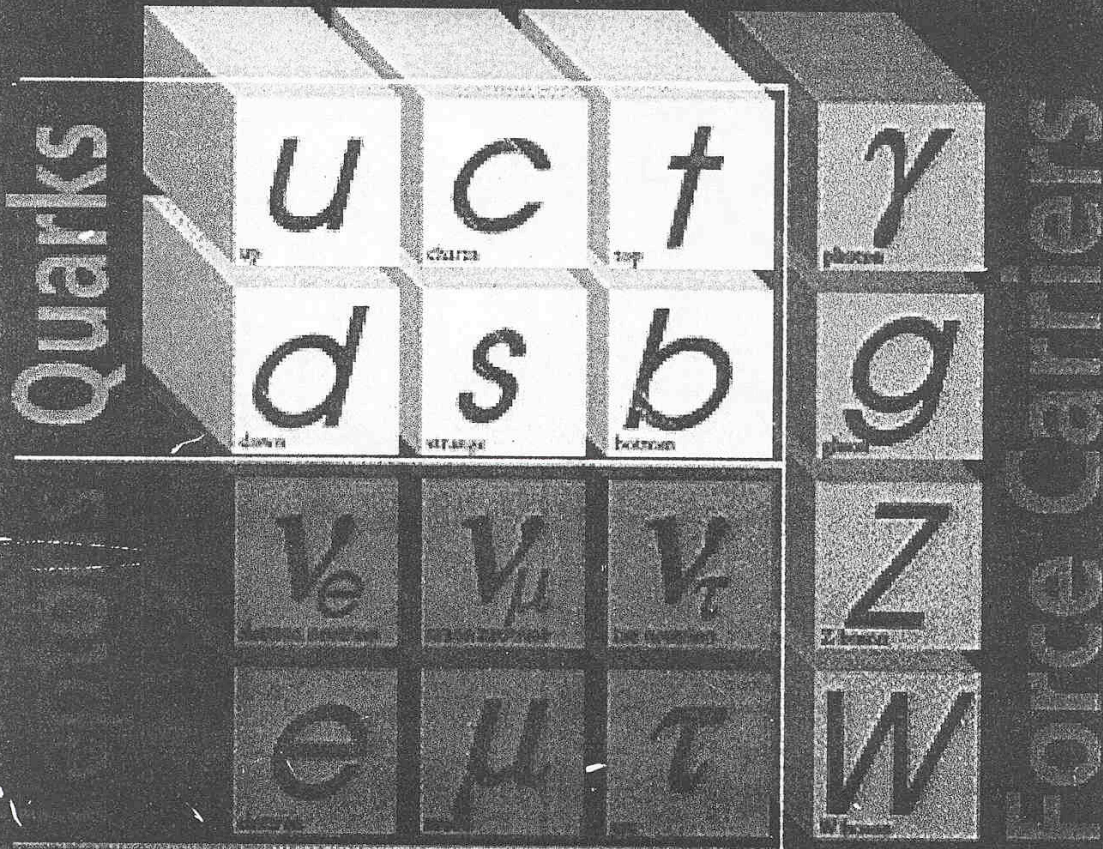
*J. Blietschau et al. /  $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$*

**EVIDENCE FOR THE LEPTONIC NEUTRAL CURRENT REACTION**  
 $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$

In the Gargamelle neutrino experiment, three unambiguous candidates for the reaction  $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$  have been observed corresponding to a cross section for a recoil electron energy within the range  $0.3 < E_{e^-} < 2.0$  GeV of  $0.06 \times 10^{-41} E_{\bar{\nu}} (\text{GeV}) \text{ cm}^2 / \text{electron}$ . The calculated background is  $0.44 \pm 0.13$  events and the probability that all three candidates could be due to this background is 1%.



# ELEMENTARY PARTICLES



I II III  
Three Generations of Matter

## Standard model of electroweak interactions

Gauge group  $SU(2) \times U(1)$

Gauge vector bosons:  $W^\pm, Z^0, \gamma$

$V$ 's are massless and interact through  $V-A$ : Maximal parity violation is a natural consequence.

---

But as soon as 1968, one is led to challenge this simple picture

- What prevents  $V$ 's from acquiring a mass like charged leptons and quarks?
- And how to explain the newly discovered solar neutrino deficit?

⇒ Would the neutrinos be massive?!

# MASS TERMS IN FIELD THEORY

- A field  $\Psi$  obeys Dirac equation

$$i(\gamma^\mu \partial_\mu - m)\Psi = 0$$

- Chiral fields:

$$\Psi = \frac{1}{2}(1 + \gamma_5)\Psi + \frac{1}{2}(1 - \gamma_5)\Psi = \Psi_R + \Psi_L$$

Chirality conserved in Lorentz transformation

$$\Psi \rightarrow \hat{\Psi} \quad \Psi_R \rightarrow (\hat{\Psi})_R \quad \Psi_L \rightarrow (\hat{\Psi})_L$$

- Helicity "measurement of spin along  $\vec{p}$ "

$$\Psi_R \begin{cases} \text{Big component } +\frac{1}{2} \\ \text{Small component } -\frac{1}{2} \text{ order } \frac{m}{E} \end{cases}$$

$\Psi_L$  the opposite

When  $m=0$ , chirality  $\equiv$  helicity

## Charge conjugation

$$\psi^c = C \bar{\psi}^T \quad \begin{cases} \bar{\psi} = \psi^\dagger \gamma^0 \\ C = i\gamma^2 \gamma^0 \end{cases}$$

- $i(\gamma^\mu \partial_\mu - m) \psi = 0 \Rightarrow i(\gamma^\mu \partial_\mu - m) \psi^c = 0$
- $(\psi_L)^c = (\psi_R^c)$
- $\psi$  and  $\psi^c$  obey the same Lorentz transformations

$$\Rightarrow \begin{cases} \bar{\psi} \psi \text{ is Lorentz invariant} \\ \bar{\psi} \psi^c \text{ is Lorentz invariant} \end{cases}$$

- Transformation under  $U(1)$  symmetry:

$$\left. \begin{aligned} \psi &\rightarrow e^{i\theta} \psi \\ \psi^c &\rightarrow e^{-i\theta} \psi^c \end{aligned} \right\} \begin{array}{l} \text{OPPOSITE} \\ \text{CHARGES} \end{array}$$

$$\begin{cases} \bar{\psi} \psi \text{ is } U(1) \text{ invariant} \\ \bar{\psi} \psi^c \text{ is NOT } U(1) \text{ invariant} \end{cases}$$

$\Rightarrow$  No  $\bar{\psi} \psi^c$  term in Lagrangian for charged particles

$\bar{\psi} \psi^c$  term OK for NEUTRAL particles

## $\nu$ LAGRANGIAN WITH CHIRAL FIELDS

• Interactions: Only  $\psi_{\nu L}$ , no  $\psi_{\nu R}$  (V-A)

• kinetic energy:

$$\bar{\psi} \gamma^\mu \partial_\mu \psi = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R$$

$\Rightarrow$  Conserves chirality

• mass terms:

1) DIRAC

$$m \bar{\psi} \psi = m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

• chirality violation

• No  $U(1)$  violation

$\Rightarrow$  If lepton number conservation, this is the only authorized term for  $\nu$ 's (Dirac neutrinos)

• Electrically charged particles are Dirac particles.

## 2) MAJORANA

$$m \bar{\Psi} \Psi^c = m (\bar{\Psi}_L \Psi_R^c + \bar{\Psi}_R \Psi_L^c)$$

- Chirality violation
- $U(1)$  violation  $\Rightarrow$  no conserved charge

### General case

$m$  left handed fields  $\Psi_L$

$m$  right handed fields  $\Psi_R^c$

$$L_{\text{mass}} = \bar{\Psi}_j M_{jk}^D \Psi_k^c$$

$$+ \bar{\Psi}_j M_{jk}^L \Psi_k^c + \bar{\Psi}_j M_{jk}^R \Psi_k^c + \text{h.c.}$$

$M^D$ : Dirac mass matrix ( $m \times m$ )

$M^L, M^R$ : Majorana mass matrices  $\begin{cases} m \times m \\ m \times m \end{cases}$

$$L_{\text{mass}} = \bar{N} \begin{pmatrix} M^L & M^D \\ (M^D)^T & M^R \end{pmatrix} N^c + \text{h.c.}$$

$$N = \begin{pmatrix} \psi_1^c \\ \psi_2^c \\ \vdots \\ \psi_1 \\ \vdots \\ \psi_m \end{pmatrix}$$

•  $M^L$  and  $M^R$  are symmetric

• Exercises:

$$\begin{cases} \bar{\psi} M^D \psi = \bar{\psi}^c (M^D)^T \psi^c \\ \bar{\psi}_i \psi_j^c = \bar{\psi}_j \psi_i^c \end{cases}$$

Propagation eigenstates are the eigenvectors of the mass matrix (states with definite mass)

The mass eigenstates are

$m+n$  autoconjugate fields

(i.e.  $\psi^c \equiv \psi$ ) called

Majorana neutrinos

## Demonstration

$$L = \sum_{\mathbf{k}} \lambda_{\mathbf{k}} \bar{\chi}_{\mathbf{k}} \chi_{\mathbf{k}}^c + \text{h.c.}$$

$$= \sum_{\mathbf{k}} \lambda_{\mathbf{k}} \bar{\chi}_{\mathbf{k}} \chi_{\mathbf{k}}^c + \lambda_{\mathbf{k}}^* \bar{\chi}_{\mathbf{k}}^c \chi_{\mathbf{k}}$$

$$\lambda_{\mathbf{k}} = m_{\mathbf{k}} e^{2i\psi} \Rightarrow \chi_{\mathbf{k}}' = e^{i\psi} \chi_{\mathbf{k}}$$

$$L = \sum_{\mathbf{k}} m_{\mathbf{k}} \left( \bar{\chi}_{\mathbf{k}}' \chi_{\mathbf{k}}'^c + \bar{\chi}_{\mathbf{k}}'^c \chi_{\mathbf{k}}' \right)$$

using  $\bar{\psi}_L \psi_L \equiv 0$  and  $\bar{\psi}_R \psi_R \equiv 0$

$$L = \sum_{\mathbf{k}} m_{\mathbf{k}} \left( \bar{\chi}_{\mathbf{k}}' + \bar{\chi}_{\mathbf{k}}'^c \right) \left( \chi_{\mathbf{k}}' + \chi_{\mathbf{k}}'^c \right)$$

$$N_{\mathbf{k}} = \chi_{\mathbf{k}}' + \chi_{\mathbf{k}}'^c \quad (N_{\mathbf{k}}^c = N_{\mathbf{k}})$$

$$L = \sum_{\mathbf{k}} m_{\mathbf{k}} \bar{N}_{\mathbf{k}} N_{\mathbf{k}}$$

$\Rightarrow m+n$  autoconjugate fields with definite mass

Q.E.D.

What happens for a Dirac neutrino?

$$M = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}$$

eigenvalues  $+m$   $(1, 1)$   
 $-m$   $(1, -1)$

→ 2 Majorana neutrinos  $\psi_1$  and  $\psi_2$   
with same mass  $m$ .

$$L = i(\bar{\psi}_1 \gamma^\mu \partial_\mu \psi_1 - m \bar{\psi}_1 \psi_1) \\ + i(\bar{\psi}_2 \gamma^\mu \partial_\mu \psi_2 - m \bar{\psi}_2 \psi_2)$$

Redefine  $\psi_D = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)$

$$L = i \bar{\psi}_D \gamma^\mu \partial_\mu \psi_D - m \bar{\psi}_D \psi_D$$

$\psi_D$  Dirac field

⇒ A Dirac field is equivalent to  
2 Majorana fields with same mass  
and opposite charge conjugation

## Application: see-saw mechanism

$$M = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \quad \begin{array}{l} m \text{ typical Dirac} \\ \text{mass} \sim m_q \sim m_l \end{array}$$

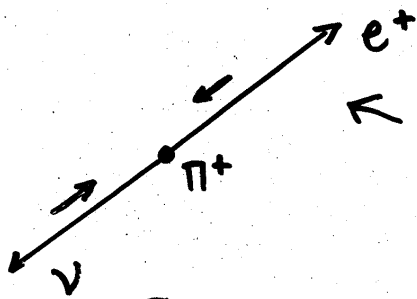
$M$  Majorana mass at unification scale  
for  $\nu$  singlets = Only  $M \nu_R \bar{\nu}_R$  can  
conserve  $(SU2)_w$

$$\lambda = \frac{1}{2} (M \pm \sqrt{M^2 + 4m^2})$$

$$M \gg m \Rightarrow \begin{cases} \lambda_+ \sim M \\ \lambda_- \sim \frac{m^2}{M} \ll m! \end{cases}$$

$\Rightarrow$  "natural" explanation for very light  
neutrino masses

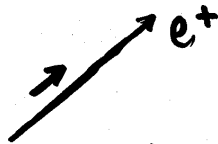
## V-A and $\pi$ decay



From  
angular momentum  
conservation

← left handed helicity

V-A demands  
that  $e^+$  is of  
right-handed chirality



At the limit  $m_e \rightarrow 0$ , decay is FORBIDDEN

Decay authorized only because of the  
difference between chirality and helicity  
when  $m_e \neq 0$ .

$$\Gamma_{\nu e} \propto m_e^2$$

$$\Gamma_{\nu \mu} \propto m_\mu^2$$

$$\left\{ \begin{array}{l} \pi \rightarrow \mu \nu \sim 100\% \\ \pi \rightarrow e \nu \sim 1.2 \cdot 10^{-4} \end{array} \right.$$

# NEUTRINO OSCILLATIONS

$$\mathcal{L} = \mathcal{L}_I + \mathcal{L}_M$$

$\mathcal{L}_I$  is diagonal in the flavor lepton basis

$$\nu_e, \nu_\mu, \nu_\tau$$

$\mathcal{L}_M$  in general is non-diagonal in this basis

Mass eigenstates  $\nu_1, \nu_2, \nu_3$  related to flavor eigenstates through a unitary matrix  $U$

$$\boxed{\begin{array}{l} \nu_l = U_{lm} \nu_m \\ l = e, \mu, \tau \qquad m = 1, 2, 3 \end{array}}$$

- Unitarity:  $\sum_m U_{lm} U_{l'm}^* = 1$
- CP conservation  $\equiv U$  is real

$U$  non diagonal  $\Rightarrow$  flavor transitions

Demonstration:

$t=0$  a  $\nu_e$  is produced

$$|\nu(t=0)\rangle = \sum_m U_{em} |\nu_m\rangle = \nu_e$$

Schrödinger:  $|\nu_m(t)\rangle = e^{-iE_m t} |\nu_m\rangle$

So

$$|\nu(t)\rangle = \sum_m U_{em} e^{-iE_m t} |\nu_m\rangle$$

$E_i = \sqrt{p^2 + m_i^2}$  3 different phases if  $m_1 \neq m_2 \neq m_3$

$$\langle \nu_e | \nu_e(t) \rangle = \sum_m U_{em}^* e^{-iE_m t} U_{em}$$

is in general  $\neq 0$

$\Rightarrow$  A neutrino born as a  $\nu_e$  develops components in other flavors during its propagation in vacuum

## Case of 2 flavors ( $\nu_e, \nu_\mu$ )

$$\nu_e = \cos\theta |\nu_1\rangle + \sin\theta |\nu_2\rangle$$

$$\nu_\mu = -\sin\theta |\nu_1\rangle + \cos\theta |\nu_2\rangle$$

$$\langle \nu_e | \nu_e(t) \rangle = \cos^2\theta e^{-iE_1 t} + \sin^2\theta e^{-iE_2 t}$$

$$\langle \nu_\mu | \nu_e(t) \rangle = -\sin\theta \cos\theta e^{-iE_1 t} + \sin\theta \cos\theta e^{-iE_2 t}$$

$$\begin{aligned} P_t(\nu_e \rightarrow \nu_\mu) &= 1 - P_t(\nu_e \rightarrow \nu_e) = |\langle \nu_\mu | \nu_e(t) \rangle|^2 \\ &= \sin^2\theta \cos^2\theta \left[ 2 - 2 \cos(E_1 - E_2)t \right] \end{aligned}$$

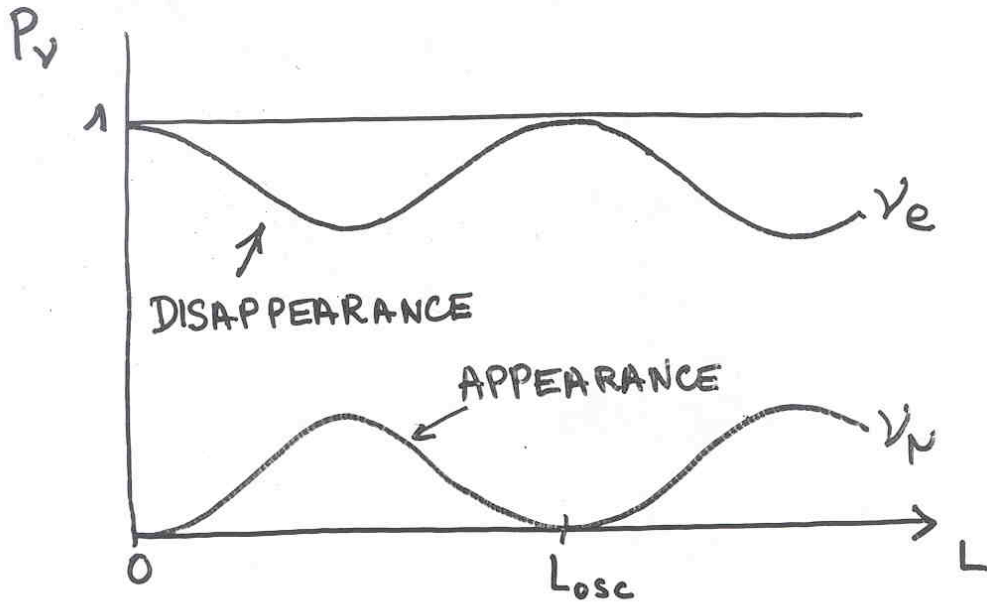
Relativistic approx.  $E_i = \sqrt{p^2 + m_i^2} \sim p + \frac{m_i^2}{2p}$

$$P_t(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4p} t\right)$$

$$P_{l=ct}(\nu_e \rightarrow \nu_\mu) = A \sin^2 \pi \frac{l}{l_{\text{osc}}} \quad \Delta m^2 = m_1^2 - m_2^2$$

Amplitude	$A = \sin^2 2\theta$
Oscillation length	$l_{\text{osc}} = \frac{4\pi p}{\Delta m^2}$

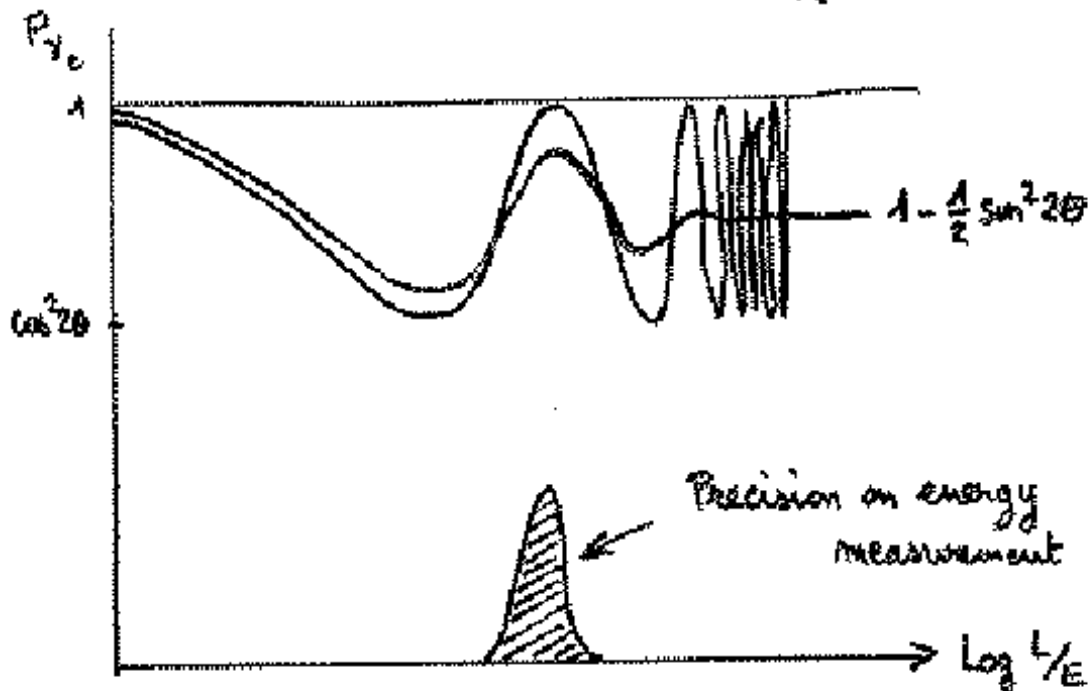
Point-source emitting monoenergetic  $\nu_e$ 's.



In natural units  $L_{osc} (m) = \frac{2.5 E_\nu (MeV)}{\Delta m^2 (eV^2)}$

SOURCE	ENERGY	DISTANCE	$\Delta m^2 (L=L_{osc})$
Accelerator	1 GeV	1 km	$2.5 eV^2$
Reactor	4 MeV	100 m	$0.1 eV^2$
Atmospheric $\nu$ 's	400 MeV	20 km	$0.05 eV^2$
		10 000 km.	$10^{-4} eV^2$
Sun	1 MeV	500 s	$10^{-11} eV^2$

Point source, spectrum, energy is measured



→  $1/E$  is the right variable to see the oscillation

→ The uncertainty  $\sigma_E$  on the energy measurement washes out the oscillation pattern

$$\text{When } L \gg L_{\text{osc}} \frac{E}{\sigma_E}$$

Experimental washing out comes before the quantum mechanical washing out

Oscillations  $\nu_\mu \leftrightarrow \nu_e$  and  $\mu \leftrightarrow e$

•  $\nu$  case:

$$M_b = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

Plug in corrections

$$\begin{pmatrix} m + \epsilon_\mu & \epsilon \\ \epsilon & m + \epsilon_e \end{pmatrix} \quad \delta m^2 \sim \epsilon$$
$$\tan 2\theta = \frac{2\epsilon}{\epsilon_\mu - \epsilon_e}$$

$\Rightarrow$  macroscopic oscillations

•  $e, \nu$  case

$$M_b = \begin{pmatrix} M & 0 \\ 0 & m \end{pmatrix} \rightarrow \begin{pmatrix} M + \epsilon_\mu & \epsilon \\ \epsilon & m + \epsilon_e \end{pmatrix}$$

$$\delta m^2 \sim M^2 - m^2 \quad t_{\text{osc}} \text{ is microscopic}$$

$$\tan 2\theta \approx \frac{2\epsilon}{M - m} \quad \text{tiny}$$

Oscillations cannot be seen

At most, very small lepton flavor violation ( $\nu \rightarrow e \gamma$ ).

## OSCILLATION EXPTS

Ideal case

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Source:

- Point-like
- flavor pure (or known and small contam.)
- Stable
- known intensity
- Monoenergetic, or known spectrum

Detector:

- known efficiency
- good energy measurement
- good meas<sup>r</sup> of interaction point
- LOW background

## Realistic cases:

- Source size: oscillations get slightly washed out
- Badly known intensity and spectrum:  
Do measurements at 2 distances and compare
- Source varies with time:  
Do simultaneously measurements at 2 distances
- Background  
Perform source-off background meas<sup>r</sup>
  - not always possible
  - Source related bckgd: Change shielding conditions
- And of course  
Necessity of calibrations  
(efficiency, energy scale)  
interpolated with monitoring

## Results : exclusion plots

Results presented in  $(\sin^2 2\theta, \Delta m^2)$  plots

Excluded / Accepted regions

If no oscillation seen:

$$P_{ee'} < \alpha$$

$$\sin^2 2\theta \cdot \left\langle \sin^2 \frac{\Delta m^2 L}{4E} \right\rangle < \alpha$$

High  $\Delta m^2$  ( $L_{osc} \ll L$ ):

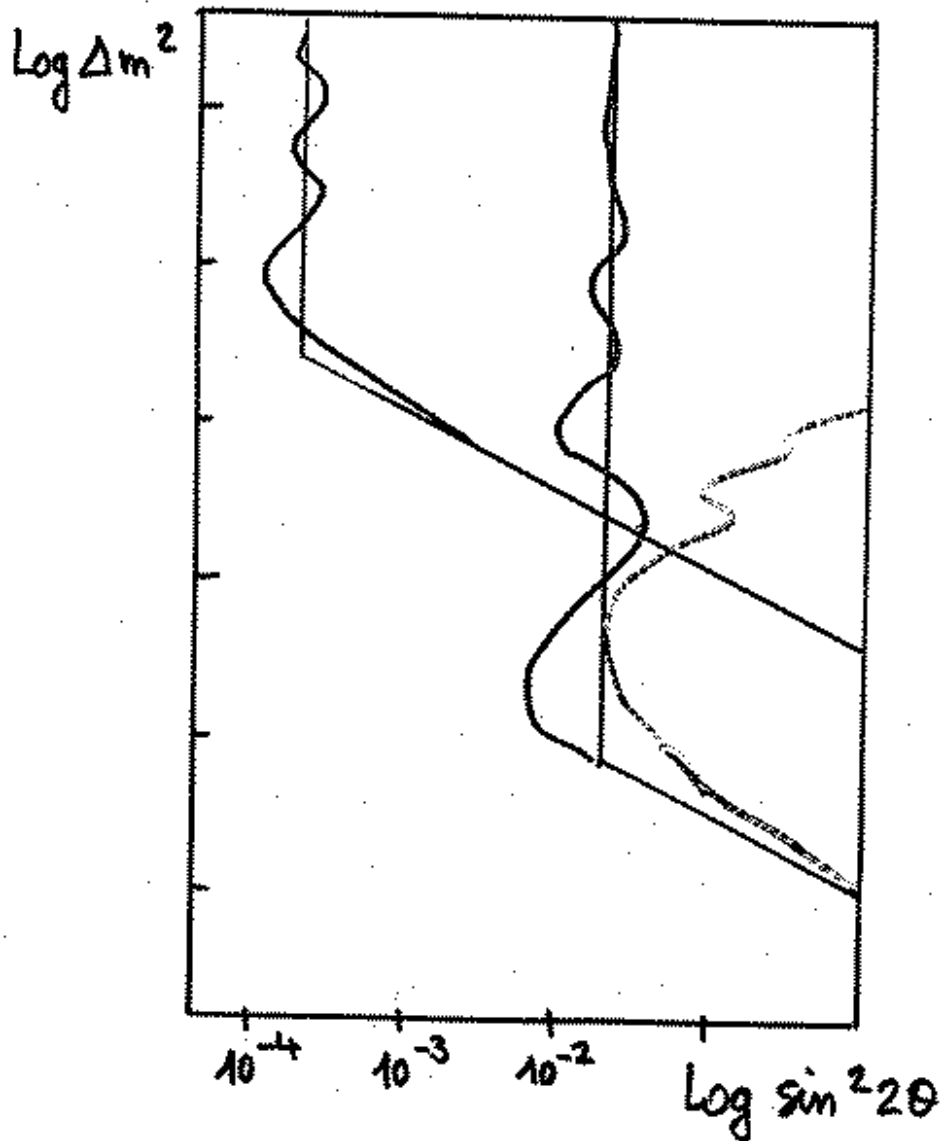
$$\frac{1}{2} \sin^2 2\theta < \alpha$$

Very low  $\Delta m^2$  ( $L_{osc} \gg L$ )

$$\sin^2 2\theta (\Delta m^2)^2 < \beta \sim \alpha \cdot \frac{4GE^2}{L^2}$$

Intermediate region:

Sensitivity depends on details of the experiment: Spectrum, energy resolution



- Appearance ( $\nu_\mu \rightarrow \nu_e$  accelerator)
- Disappearance ( $\bar{\nu}_e$  reactor)
- Disappearance = Comparison at 2 distances.