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Self-distributed feedback lasing in a system of cold atoms

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Abstract. – We present a semiclassical analysis of lasing with an injected signal in a system of cold two-level atoms where the effects of atomic recoil are included self-consistently. This analysis predicts that the atoms may bunch under the effects of dipole forces to produce a density grating. This grating produces distributed feedback which can induce lasing action even when the gain of the atomic medium is less than the cavity losses.

Introduction. – In general, a laser can be considered as an optical gain medium with feedback. The simplest method of feedback is to contain the gain medium within an optical cavity consisting of two or more mirrors. Feedback can also arise by periodically modulating the refractive index of the active medium. This so-called distributed feedback is due to a Bragg scattering of the radiation fields from the refractive index modulation. If this feedback is sufficient, the gain medium may lase in the absence of any cavity mirrors. Such a system has been called a distributed feedback laser [1]. Where the gain medium consists of active particles or passive scatterers immersed in an active medium, the random spatial distribution of the particles also gives rise to a random distributed feedback. This forms the basis of the randomly distributed feedback laser (RDFL) [2] or random laser [3].

In most theoretical analyses of lasers consisting of active particles which have translational degrees of freedom, the motion of the particles is generally included only through Doppler broadening terms and the effects of the radiation upon the particle motion is neglected. However, it is well known that light can play a significant role in the evolution of particle dynamics. For example light is used to control particle motion in atomic cooling [4]. Most analyses here, however, neglect the effects of the atomic interaction on the radiation fields.

Recent work has described the interaction between radiation and a passive atomic system using a semiclassical model in which the internal atomic degrees of freedom, the atomic centre-of-mass motion and the radiation fields evolve self-consistently [6–8]. This model displays a rich variety of phenomena, many of which involve the formation of an atomic density grating with a period of half the radiation wavelength when the atomic system is sufficiently cold.

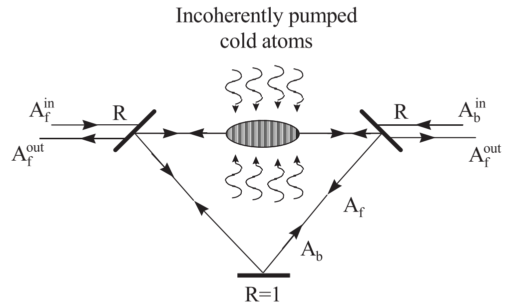

Fig. 1

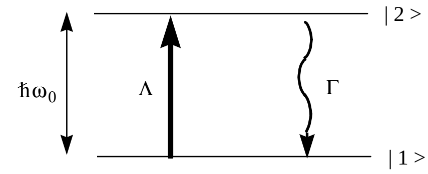

Fig. 2

Fig. 1 – A schematic diagram showing a cold atomic sample enclosed in a bidirectional ring cavity. Note that $A^{\rm eq}_{\rm b,f} = A^{\rm in}_{\rm b,f}/\sqrt{1-R}$ and $A^{\rm out}_{\rm b,f} = A_{\rm b,f}\sqrt{1-R}$, where R is the reflectivity of the output mirrors.

Fig. 2 – A schematic energy level diagram of the atoms.

If such a cold atomic system were to be incoherently pumped, it would be expected that with sufficient feedback the system would lase. If a self-consistent atomic bunching process were to occur, similar to that already predicted in passive atomic media, then self-distributed feedback (SDF) would occur. For sufficiently strong grating formation, this effect should be significantly greater than that in the RDFL. The prospect of reduced feedback requirements to obtain lasing in systems of sufficiently cold atoms is evident. In this letter the SDF mechanism is investigated analytically and numerically using the model described in [8], slightly modified to take account of incoherent excitation of the atomic medium.

Model. – The model described in [8] is one-dimensional and semiclassical. A schematic of the ring cavity system considered here is shown in fig. 1. We begin the analysis by defining the plane-wave radiation electric fields as follows:

$$\boldsymbol{E}(z,t) = (\mathcal{E}_{\rm f}(z,t)e^{i(k_{\rm f}z-\omega_{\rm f}t)} + \mathcal{E}_{\rm b}(z,t)e^{-i(k_{\rm b}z+\omega_{\rm b}t)} + {\rm c.c.})\hat{\boldsymbol{e}}\,,$$

where $\hat{\boldsymbol{e}}$ is a unit vector transverse to the propagation axis $\hat{\boldsymbol{z}}$ and subscipts f, b refer to the forward and backward propagating fields, respectively. The energy level diagram for each atom in the sample is shown in fig. 2. The evolution of the density matrix elements $\rho_{mn}, (m,n = 1,...,2)$ describe the internal evolution of each atom. The off-diagonal elements $(\rho_{21} = \rho^*_{12})$ describe the polarisation as induced by the forward and backward fields. The dipole moment is then given by

$$\boldsymbol{d} = \mu\,(\rho_{12} + \rho^*_{12})\,\hat{\boldsymbol{e}}\,. \tag{1}$$

where μ is the dipole matrix element. The off-diagonal elements in eq. (1) may be written conveniently as a sum of two polarisation waves:

$$\rho_{12} = S_{\rm f}(z,t)e^{i(k_{\rm f}z-\omega_{\rm f}t)} + S_{\rm b}(z,t)e^{-i(k_{\rm b}z+\omega_{\rm b}t)}\,. \tag{2}$$

We define D as half the population difference between the lower (1) and upper (2) states, so that

$$D = \frac{\rho_{11} - \rho_{22}}{2}$$

and the equilibrium value of D in the absence of the radiation fields, D^{eq}, is defined as (see fig. 2)

$$D^{\mathrm{eq}} = \frac{1}{2}\left(\frac{\Gamma_\parallel - \Lambda}{\Gamma_\parallel + \Lambda}\right). \tag{3}$$

Consequently, in the absence of excitation ($\Lambda = 0$), $D^{\mathrm{eq}} = 0.5$, the atoms relax to their ground state and the atomic medium is passive. In contrast, for strong excitation ($\Lambda \gg \Gamma_\parallel$) $D^{\mathrm{eq}} \approx -0.5$, the atoms relax to the upper state and the atomic system is inverted.

Use of the definitions above in the Bloch equations describing the two-level atomic system, the equation for the force on the j-th atom

$$F_{z_j} = \left(\boldsymbol{d}\cdot\frac{\partial \boldsymbol{E}}{\partial z}\right)\bigg|_{z=z_j},$$

and the Maxwell wave equation yield the following set of coupled scaled differential equations:

$$\frac{\mathrm{d}\tilde{S}_{\mathrm{f}_j}}{\mathrm{d}\bar{t}} = \left[-\Gamma_\perp + i\left(\Delta - \frac{p_j}{2}\right)\right]\tilde{S}_{\mathrm{f}_j} - 2\rho\tilde{A}_{\mathrm{f}}D_j\,, \tag{4}$$

$$\frac{\mathrm{d}S_{\mathrm{b}_j}}{\mathrm{d}\bar{t}} = \left[-\Gamma_\perp + i\left(\Delta + \frac{p_j}{2}\right)\right]S_{\mathrm{b}_j} - 2\rho A_{\mathrm{b}}D_j\,, \tag{5}$$

$$\frac{\mathrm{d}D_j}{\mathrm{d}\bar{t}} = -\left(\Gamma_\parallel + \Lambda\right)\left(D_j - D^{\mathrm{eq}}\right) + \rho\left[\tilde{S}_{\mathrm{f}_j}\left(\tilde{A}_{\mathrm{f}}^* + A_{\mathrm{b}}^* e^{i\theta_j}\right) + S_{\mathrm{b}_j}\left(\tilde{A}_{\mathrm{f}}^* e^{-i\theta_j} + A_{\mathrm{b}}^*\right) + \mathrm{c.c.}\right], \tag{6}$$

$$\frac{\mathrm{d}\theta_j}{\mathrm{d}\bar{t}} = p_j\,, \tag{7}$$

$$\frac{\mathrm{d}p_j}{\mathrm{d}\bar{t}} = -\left(\tilde{A}_{\mathrm{f}}\tilde{S}_{\mathrm{f}_j}^* - A_{\mathrm{b}}S_{\mathrm{b}_j}^* + \tilde{A}_{\mathrm{f}}S_{\mathrm{b}_j}^* e^{i\theta_j} - A_{\mathrm{b}}\tilde{S}_{\mathrm{f}_j}^* e^{-i\theta_j} + \mathrm{c.c.}\right), \tag{8}$$

$$\frac{\mathrm{d}\tilde{A}_{\mathrm{f}}}{\mathrm{d}\bar{t}} = \left\langle \tilde{S}_{\mathrm{f}}\right\rangle + \left\langle S_{\mathrm{b}}e^{-i\theta}\right\rangle + i\delta\tilde{A}_{\mathrm{f}} - \kappa_{\mathrm{f}}\left(\tilde{A}_{\mathrm{f}} - \tilde{A}_{\mathrm{f}}^{\mathrm{eq}}\right), \tag{9}$$

$$\frac{\mathrm{d}A_{\mathrm{b}}}{\mathrm{d}\bar{t}} = \left\langle S_{\mathrm{b}}\right\rangle + \left\langle \tilde{S}_{\mathrm{f}}e^{i\theta}\right\rangle - \kappa_{\mathrm{b}}\left(A_{\mathrm{b}} - A_{\mathrm{b}}^{\mathrm{eq}}\right), \tag{10}$$

where the general dependent variables $X \equiv X(\bar{t})$, $\tilde{X} \equiv X e^{i\delta\bar{t}}$ and

$$A_{\mathrm{f,b}} = -i\sqrt{\frac{2\epsilon_0}{n\hbar\omega\rho}}\mathcal{E}_{\mathrm{f,b}}\,, \qquad \rho = \left(\frac{\omega\mu^2 n}{2\epsilon_0\omega_r^2\hbar}\right)^{1/3},$$

$$p = \frac{M(v_z - \langle v_{z0}\rangle)}{\hbar k\rho}\,, \qquad \Delta = \frac{\omega_{\mathrm{b}} + k_{\mathrm{b}}\langle v_{z0}\rangle - \omega_0}{\omega_r\rho}\,,$$

$$\delta = \frac{2k\langle v_{z0}\rangle - (\omega_{\mathrm{f}} - \omega_{\mathrm{b}})}{\omega_r\rho}\,,$$

$\bar{t} = \omega_r\rho t$, $\theta = 2k(z - \langle v_{z0}\rangle t)$, $\omega_r = 2\hbar k^2/M$ is the single photon recoil frequency shift, $j = 1, ..., N$, $\langle ...\rangle = \frac{1}{N}\sum_{j=1}^{N}(...)$ and $\Gamma_{\perp,\parallel} = \gamma_{\perp,\parallel}/\omega_r\rho$ are the scaled decay rates of the polarisation and upper level population, respectively. In the above scaling, M is the atomic mass, ω_0 is the transition frequency, $n = n_{\mathrm{s}}L_{\mathrm{s}}/L_{\mathrm{cav}}$ is the "reduced" atomic density in the cavity, n_{s} is the atomic density of the sample, L_{s} is the sample length, L_{cav} is the cavity length and where appropriate we have assumed $k \approx (k_{\rm f} + k_{\rm b})/2$. Note that for an atomic sample which initially has zero mean velocity ($\langle v_{z0}\rangle = 0$), $\Delta \rightarrow (\omega_{\rm b} - \omega_0)/\omega_r\rho$ and $\delta \rightarrow (\omega_{\rm b} - \omega_{\rm f})/\omega_r\rho$. Hence Δ and δ describe the backward field-atom detuning and the backward-forward detuning respectively, scaled with respect to the "collective recoil bandwidth", $\omega_r\rho$ [6, 7]. In deriving eqs. (4)-(10), the fields are assumed to be average fields over intervals $\Delta z \sim \lambda$, the radiation wavelength, consistent with the slowly varying envelope approximation (SVEA) [9]. The mean, $\langle ...\rangle$, refers to the N atoms within that interval.

We have assumed that the mean field limit can be applied when describing the evolution of both fields. Cavity losses are assumed to be equal for both propagation directions, *i.e.* $\kappa_{\rm f} = \kappa_{\rm b} = \kappa$, where $\kappa = -c\ln(R)/\omega_r\rho L_{\rm cav}$ is the scaled cavity loss rate. Here $\tilde{A}_{\rm f}^{\rm eq}$ and $A_{\rm b}^{\rm eq}$ are the equilibrium fields in the cavity. It has been assumed that the backward field frequency coincides with that of a mode of the cavity, *i.e.* $\omega_{\rm b} = \omega^{(m)} = 2\pi m/\mathcal{L}$, where $\mathcal{L}$ is the cavity length and m is an integer. In this letter, we assume that the backward and forward fields have the same frequency and the mean atomic velocity is zero, so that $\delta = 0$, and the atoms are cold, *i.e.* $p_j = 0\ \forall\ j$. The atoms are considered to be strongly pumped, so that $\Lambda \gg \Gamma_\parallel$ and $D^{\rm eq} \approx -0.5$. Furthermore, we assume for simplicity that $\Gamma_\perp = \Lambda$. Relaxing this assumption does not change the results presented here qualitatively.

Results. – It is useful to first consider the case where the atomic variables $S_{\rm f,b}$ and D can be adiabatically eliminated and the atoms respond linearly to the fields, *i.e.* $S_{\rm f,b} = \alpha A_{\rm f,b}$, where $\alpha = \rho/(\Lambda - i\Delta)$ is the scaled atomic polarisability of the atoms. Substituting for this linearly dependent $S_{\rm f,b}$ into (9) and (10) we obtain

$$\frac{{\rm d}A_{\rm f}}{{\rm d}\bar{t}} = \alpha A_{\rm f} + \alpha b A_{\rm b} - \kappa\left(A_{\rm f} - A_{\rm f}^{\rm eq}\right), \tag{11}$$

$$\frac{{\rm d}A_{\rm b}}{{\rm d}\bar{t}} = \alpha A_{\rm b} + \alpha b^* A_{\rm f} - \kappa\left(A_{\rm b} - A_{\rm b}^{\rm eq}\right), \tag{12}$$

where $b = \langle \exp[-i\theta]\rangle$ is the bunching parameter which describes the spatial distribution of the atoms on the radiation wavelength scale. For uniformly distributed atoms, $b = 0$ and for perfectly bunched atoms where they all have the same value of θ, $|b| = 1$. In the absence of atomic bunching ($b = 0$), it can be seen from eqs. (11), (12) that the system decouples and each field evolves independently. Lasing will occur in the usual way when the gain is greater than the cavity losses, *i.e.* $\Re(\alpha) > \kappa$ [10].

In the presence of atomic bunching however, the system is coupled and the field evolution is more complicated, arising from a mutual scattering of the fields. This coupling can be considered as a distributed feedback of the fields within the atomic sample.

It is well known from studies of the mechanical effects of light on atoms that an atom is attracted to regions of either high or low intensity due to the dipole forces acting on the atom, depending upon the sign of the detuning Δ [4]. For $\Delta > 0$, an atom in the ground (excited) state will be attracted to regions of low (high) intensity and vice versa for $\Delta < 0$. The intensity modulation in a standing wave formed by two counterpropagating radiation waves will then tend to bunch atoms around either the intensity maxima or minima for non-zero Δ. The bunching parameter in these circumstances may become large $|b| \sim 1$ and from eqs. (11), (12) could be expected to play a significant role in the evolution of the fields [5]. It can be shown from eqs. (11), (12) that lasing can occur when the effects of the gain plus the distributed feedback arising from the bunched atoms is sufficient to overcome the cavity losses. The threshold value of $|b|$ above which this occurs can be shown to be

$$|b|_{\rm th} = \frac{\kappa}{\Re(\alpha)} - 1\,. \tag{13}$$

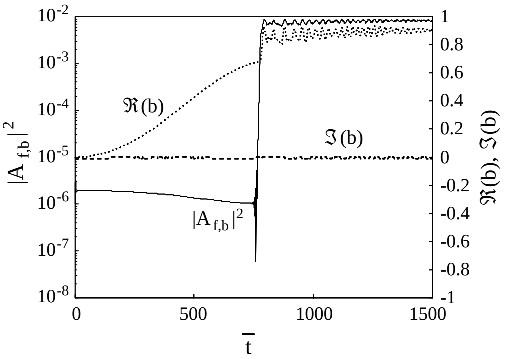

Fig. 3

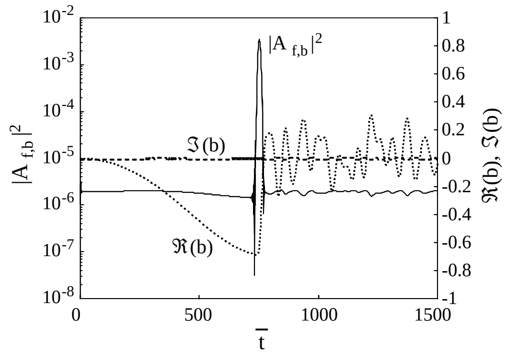

Fig. 4

Fig. 3 – Evolution of the scaled intensities $|A_{\mathrm{f,b}}|^2$ and bunching parameter b in the cavity as a function of scaled time $\bar{t}$ for $A_{\mathrm{f,b}}^{\mathrm{eq}} = 0.001$, $\rho = 20$, $\kappa = 1.7$, $\Lambda = 10$ and $\Delta = 10$.

Fig. 4 – Evolution of the scaled intensities $|A_{\mathrm{f,b}}|^2$ and bunching parameter b in the cavity as a function of scaled time $\bar{t}$ for $A_{\mathrm{f,b}}^{\mathrm{eq}} = 0.001$, $\rho = 20$, $\kappa = 1.7$, $\Lambda = 10$ and $\Delta = -10$.

In this way the cavity threshold required for lasing may be reduced so long as $|b|_{\mathrm{th}} < 1$, the maximum possible value for $|b|$. Hence to obtain lasing, the cavity losses as defined by κ can be up to a factor of two larger if compensated for by the atomic bunching.

The full system of eqs. (4)-(10) including the effects of atomic centre-of-mass motion are now solved numerically to demonstrate this effect. An atom-field detuning of $\Delta > 0$ is chosen so that the dipole forces acting on the atoms will tend to bunch the atoms at the intensity maxima of the standing wave formed by the interference of A_{f} and A_{b}. If the phases of the complex fields A_{f} and A_{b} initially evolve identically (as will be shown later) then bunching at the intensity maxima corresponds to the atoms bunching about $\theta = 0$, so that b will be real and positive. The results of the numerical solution are shown in fig. 3 where we plot the intensities of the forward and backward fields and the real and imaginary parts of the bunching parameter. It can be seen that at the beginning of the interaction the atoms are uniformly distributed ($|b| = 0$) and the laser is below threshold so that $A_{\mathrm{f,b}} \approx A_{\mathrm{f,b}}^{\mathrm{eq}}$. As the interaction progresses, the atoms bunch about $\theta = 0$. When $|b|$ reaches the threshold value of $|b|_{\mathrm{th}} = 0.7$ at $\bar{t} \approx 750$, lasing commences and the intercavity field intensities become much greater than those injected. The good agreement between the numerical value of $|b|$ and $|b|_{\mathrm{th}} = 0.7$ required for lasing results from the linear response of the atoms to the relatively weak injected fields.

An identical set of parameters to that used in fig. 3 is now considered with the exception that $\Delta < 0$. In this case, the dipole forces acting on the atoms will tend to bunch the atoms at the intensity minima of the standing wave. This corresponds to the atoms bunching about $\theta = \pi$, so that b will be real and negative. The results of the numerical solution are shown in fig. 4. As with the case for $\Delta > 0$, lasing commences once $|b| > |b|_{\mathrm{th}} = 0.7$. Note that in contrast to fig. 3, the bunching of the atoms is rapidly destroyed, reducing the distributed feedback within the atomic sample, which after a short period causes lasing to cease. The reason for the destruction of the atomic bunching and the associated switch-off of the laser may be understood from a linear analysis of the time-dependent phase difference, $\zeta_{\mathrm{b}} - \zeta_{\mathrm{f}}$, between the forward and backward wave envelopes as defined from $A_{\mathrm{f,b}} = |A_{\mathrm{f,b}}| \exp[i\zeta_{\mathrm{f,b}}]$. This analysis predicts that the phase difference is exponentially unstable when $|b|_{\mathrm{th}} + \Re(b) < 0$. As described above, the atoms bunch so that b is real and negative and also $|b| > |b|_{\mathrm{th}}$ on lasing. Hence $|b|_{\mathrm{th}} + \Re(b) < 0$ after lasing and the phase difference is unstable. Stability is regained when $\zeta_{\mathrm{b}} - \zeta_{\mathrm{f}} = \pi$. This phase change corresponds to an exchange in position of the intensity minima and maxima. The atoms now find themselves bunched around an intensity maximum but are still attracted to the intensity minima. This causes the atoms to rapidly debunch and is responsible for the subsequent switch-off of the laser. Note that for the case of fig. 3 where $\Delta > 0$, the atoms bunch around the intensity maxima so that $\Re(b) > 0$ and the phase difference is always stable. In addition, as a necessary condition for the phase difference between the fields to become unstable is that the laser must be above threshold ($|b| > |b|_{\mathrm{th}}$), and we only consider cases where the laser is below threshold at $t = 0$, our assumption that the phase evolution of the fields is initially identical, *i.e.* that the phase difference between the fields is initially constant, is justified.

Conclusions. – We have demonstrated that in a ring laser with an injected signal where the active medium consists of a collection of cold atoms, when atomic recoil is included self-consistently in the description of the system, the atoms may bunch to form a density grating at the scale of the radiation wavelength. This atomic density grating provides distributed feedback within the atomic sample and can induce a lasing action where the system would otherwise not lase. The principle of this lasing mechanism was demonstrated analytically in the regime of an adiabatic and linear response of the atoms to the field. The work presented here is restricted to the mean field limit, which generally requires mirror reflectivities which are relatively high. The relaxation of the mean field limit will require the effects of propagation to be included in the model. This may lead to the atomic bunching producing a more dramatic reduction in the mirror reflectivities required for lasing to occur.

Finally, we note that the principle of SDF as described in this letter may also be applicable to collections of free active nanoparticles much larger than the atomic scale but much smaller than the radiation wavelength.

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