

Spectral and statistical properties of radiation in a quantum SASE FEL

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We have solved numerically the 1D quantum SASE model

$$\frac{\partial c_n}{\partial \bar{z}} = -i \frac{n^2}{2\bar{\rho}^{3/2}} c_n - (A c_{n-1} - A^* c_{n+1})$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial \bar{z}_1} = \sum_{n=-\infty}^{\infty} c_n c_{n-1}^* + i\delta A$$

$$\bar{z} = \frac{z}{L_g}, \quad z_1 = \frac{z-vt}{L_c}, \quad \bar{\omega} = L_c(\omega - \omega_s), \quad \delta = \frac{\gamma_r - \gamma_0}{\gamma_r \rho}$$

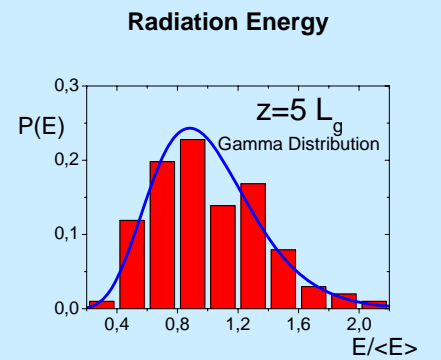
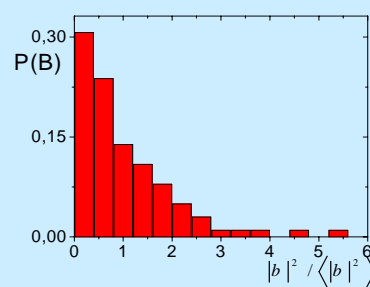
$$L_g = \frac{\lambda_w}{4\pi\rho\sqrt{\bar{\rho}}}, \quad L_c = \frac{\lambda_r}{\lambda_w} L_g,$$

$$\Psi(\theta, \bar{z}, z_1) = \sum_{n=-\infty}^{\infty} c_n(\bar{z}, z_1) e^{in\theta}$$

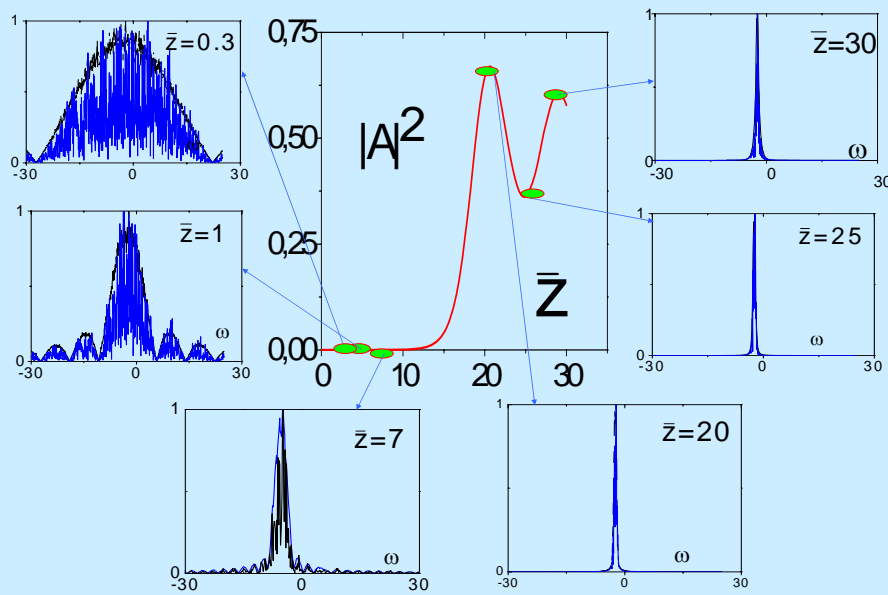
$$|A|^2 = \frac{\langle N_{ph} \rangle}{N_e}$$

$$\bar{\rho} = \left(\frac{mc\gamma_r}{\hbar k} \right) \rho,$$

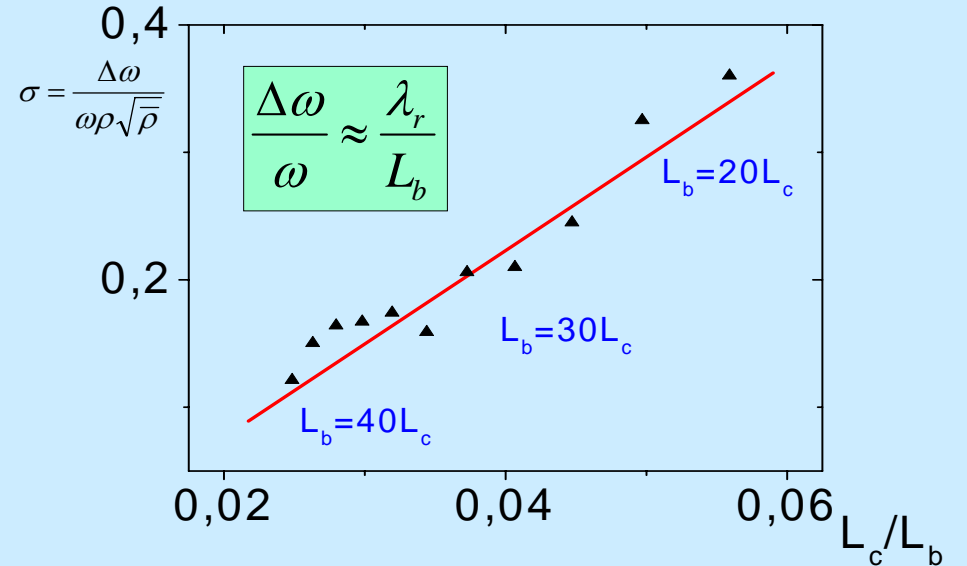
$$B = \sum_{n=-\infty}^{\infty} c_n c_{n-1}^* = |b_0| e^{i\phi_0}$$



Spectral properties of the radiation



Single-shot spectral width

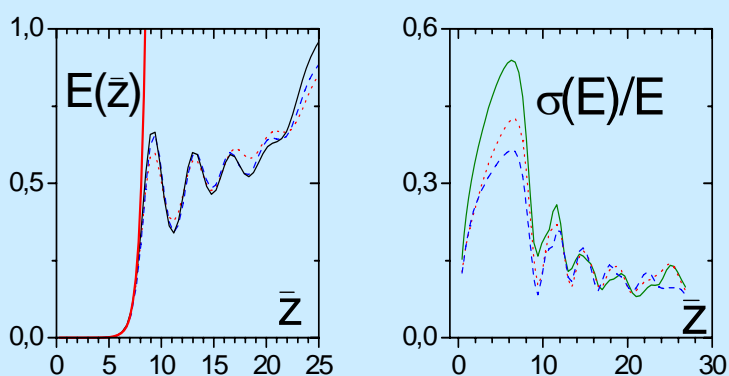


Average Energy of SASE radiation

$$E(\bar{z}) = \frac{1}{L_b} \int_0^{L_b + \bar{z}} dz_1 |A(\bar{z}, z_1)|^2 \approx \frac{\sqrt{\pi}}{2N_c \sqrt{\bar{z}}} e^{2\bar{z}}$$

$$\bar{\rho} = 0.2$$

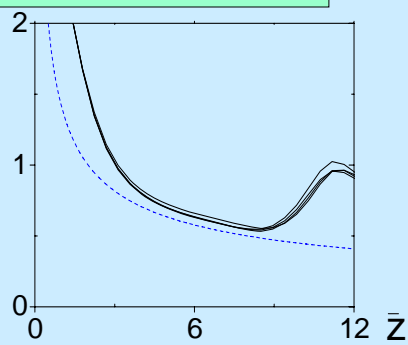
$$N_c = N_\lambda(\lambda/L_c)$$



$$L_b / L_c = 13, 18, 22,$$

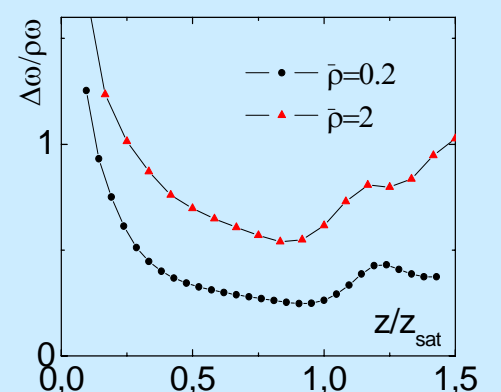
Average Spectral width

$$\sigma \equiv \frac{\Delta\omega}{\rho\sqrt{\bar{\rho}}\omega} \approx \sqrt{\frac{2}{\bar{z}}} \quad \bar{\rho} = 0.2$$



$$L_b / L_c = 13, 18, 22, 27,$$

Classical (red) and quantum (black) average spectral width



CONCLUSIONS

• We have numerically proved that in the Quantum SASE the FEL radiation spectrum is composed by a **single spike** with $\Delta\omega/\omega \sim \lambda_r/L_b$, a factor 10^3 smaller than in classical SASE spectrum (composed by $N=2\pi(L_b/L_c)$ spikes).

• Average spectrum, energy and fluctuations have been also determined in the quantum regime.

REFERENCE

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