Quantum effects in the Collective Atomic Recoil Lasing (CARL)

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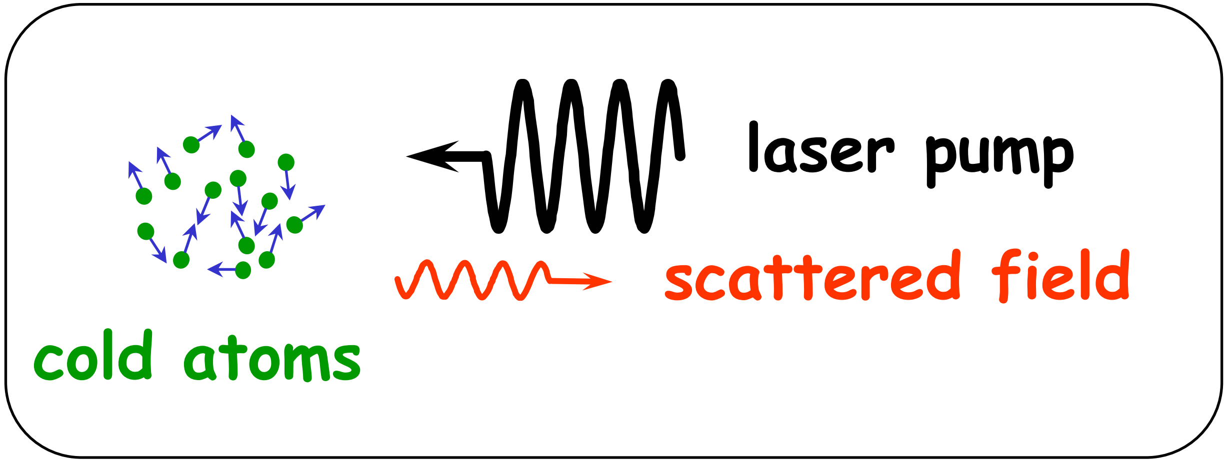
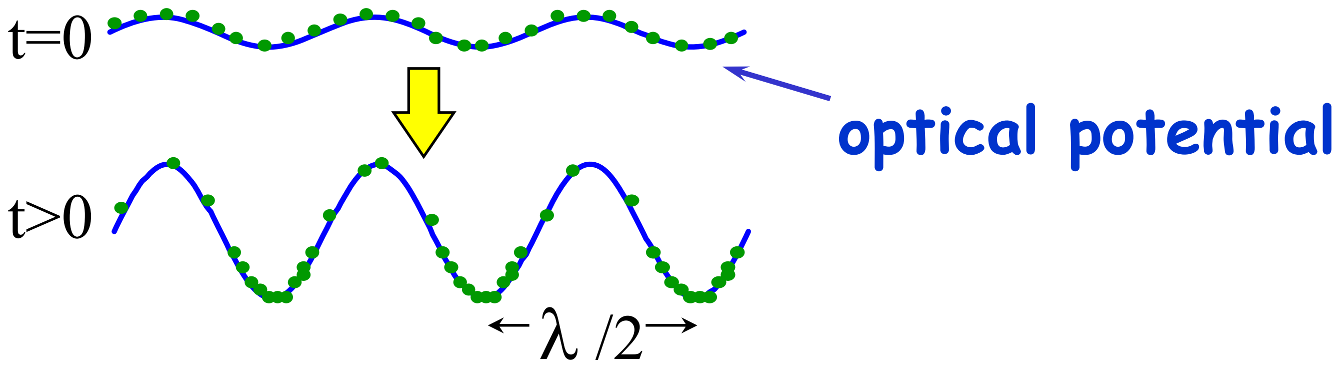
- Quantum regime of CARL & SR
- Transverse self-focusing force in CARL
- Atom-atom & atom-photon entanglement in CARL

Collective Atomic Recoil Lasing (CARL)



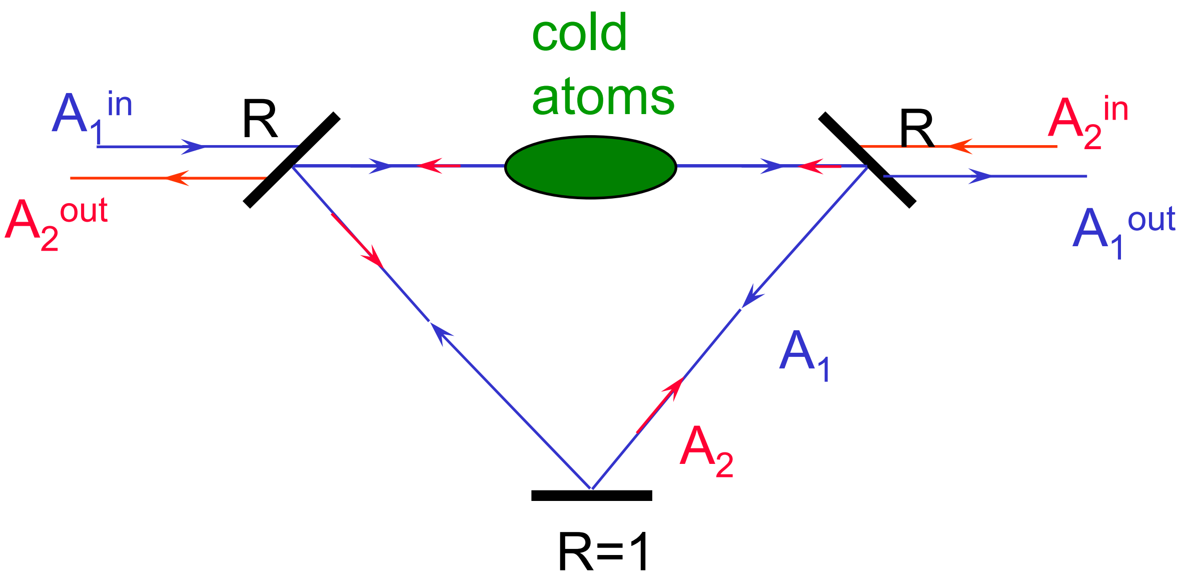
Particles **self-organize** to form compact bunches ~ $\lambda/2$ which radiate **coherently** as N^2.

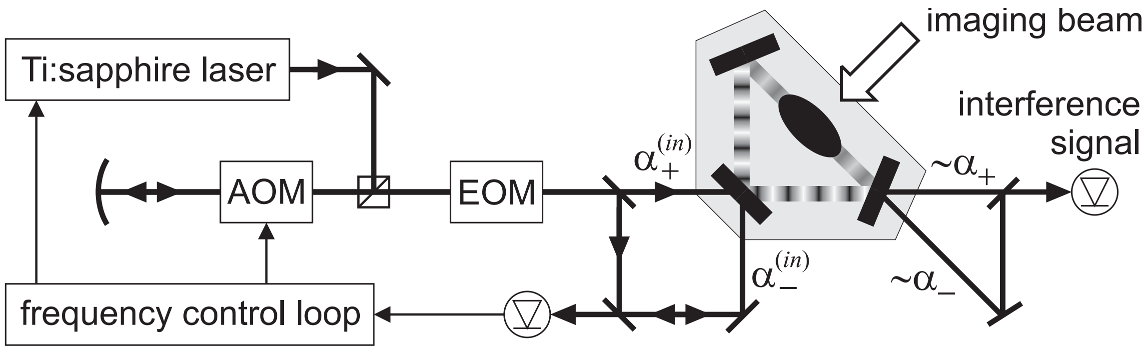


Collective Atomic Recoil Lasing = Optical gain + bunching

BIDIRECTIONAL RING CAVITY:

only one mode in each direction interacts with atoms





(Tubingen experiment)

CARL model:

- strong pump >> scattered field
- pump is far detuned from atomic resonance, ($\Delta_0 >> \gamma$) so atoms remain in their ground state
- adiabatically eliminate internal atomic degrees of freedom

$$H = \sum_{j=1}^{N} \left\{ \frac{p_{zj}^2}{2m} - i\hbar g \left(a e^{i\theta_j} - a^+ e^{-i\theta_j} \right) \right\}$$

R. Bonifacio, L. De Salvo, NIMA 341 (1994) 360

$$[z_j, p_{zj'}] = i\hbar\delta_{jj'} \qquad [a, a^+] = 1$$

$$\theta_j = 2kz_j \qquad g = g_1 \left(\frac{\Omega_p}{\Delta_0} \right) \qquad g_1 = \sqrt{\frac{\omega d^2}{2\hbar\varepsilon_0 V}}$$

Ω_p : pump Rabi frequency

$\Delta_0 = \omega - \omega_0$: pump detuning

CLASSICAL REGIME of CARL:

- Operators (z_j, p_{zj}) and a are considered as classical variables

$$\begin{cases} \dfrac{d\theta_j}{dt} = 2\omega_r p_j \\ \dfrac{dp_j}{dt} = -g(ae^{i\theta_j} + c.c.) \\ \dfrac{da}{dt} = g\sum_{j=1}^{N} e^{-i\theta_j} + i\Delta a \end{cases}$$

$$p_j = \frac{p_{zj}}{2\hbar k}$$ momentum in recoil units

$$\omega_r = \frac{(2\hbar k)^2}{2m}$$ recoil frequency

$$b = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$ bunching

$$\Delta = \omega - \omega_{pump}$$ pump – probe detuning

$$\tau = 2\omega_r \rho t$$

$$\bar{p}_j = \frac{p_j}{\rho}$$

$$A = \frac{a}{\sqrt{\rho N}}$$

$$\delta = \frac{\Delta}{2\omega_r \rho}$$

$$\begin{cases} \dfrac{d\theta_j}{d\tau} = \bar{p}_j \\ \dfrac{d\bar{p}_j}{d\tau} = -(Ae^{i\theta_j} + c.c.) \\ \dfrac{dA}{d\tau} = b + i\delta A \end{cases}$$

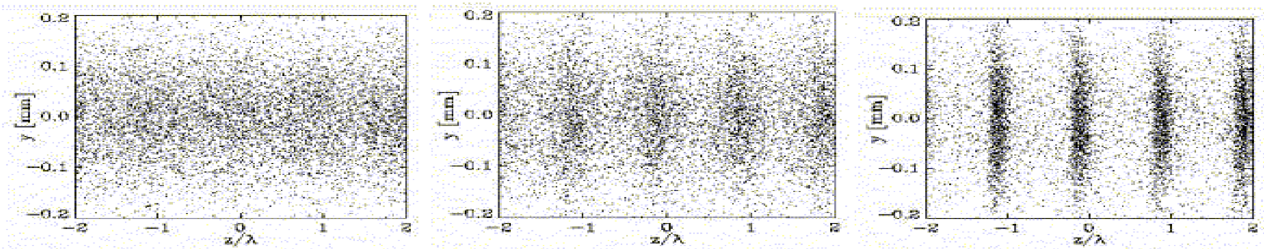
$$\rho = \left(\frac{g\sqrt{N}}{2\omega_r}\right)^{2/3}$$

CARL parameter

CARL instability

- exponential growth of scattered intensity and bunching
- saturation at A~1 (N_{photon} ~ρ N $\propto$ $N^{4/3}$)

b~0 → b~0.8



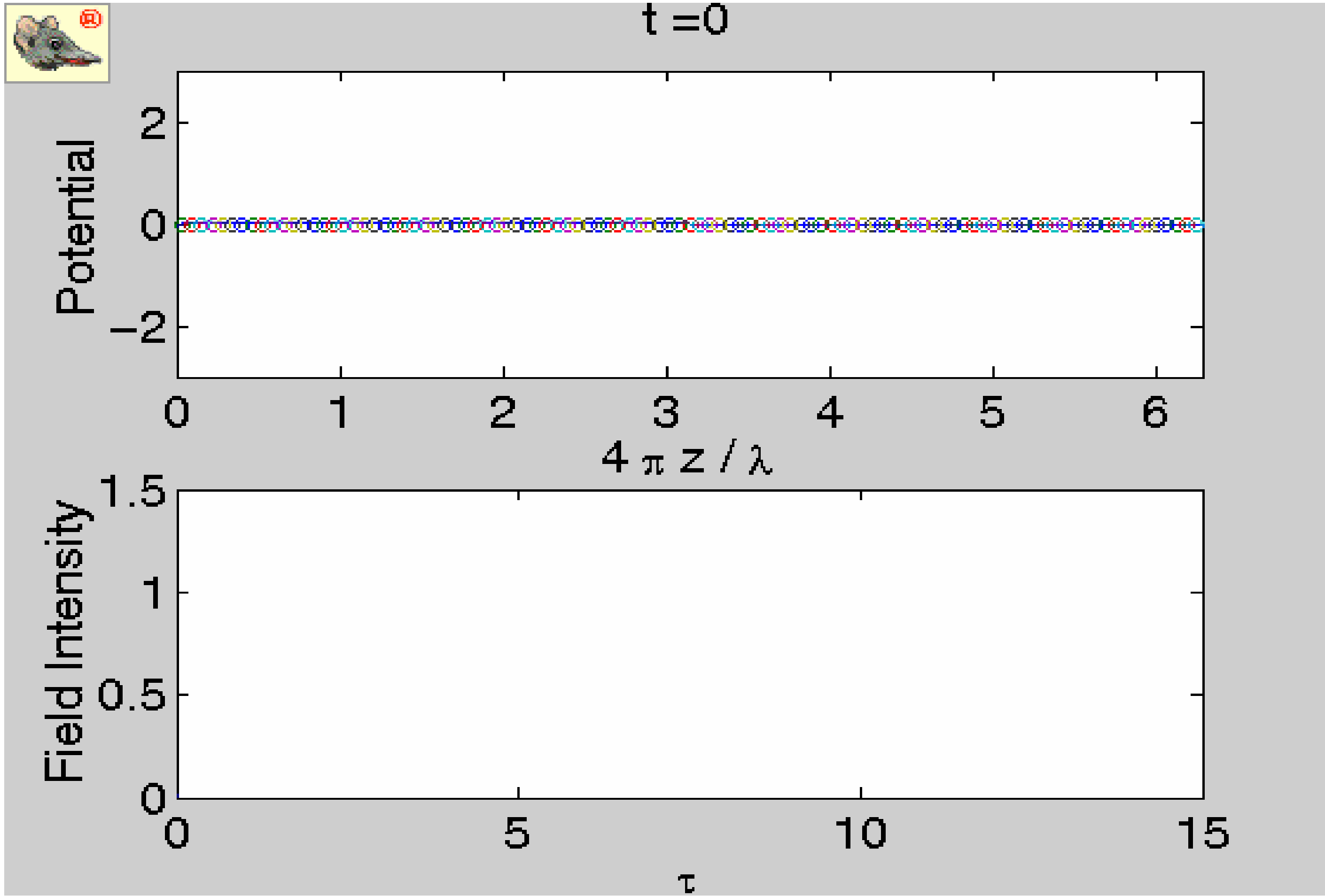
time

bunching:

$$b = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

Atoms behave as coupled pendula in a self-consistent potential:

$$V(\theta + \varphi) = 2\,|A|\cos(\theta + \varphi)$$



$$|A|^2 \propto e^{\sqrt{3}\tau}$$

SUPERRADIANT REGIME

In free space:

(EXACT PROPAGATION)

$$\frac{\partial^2\theta_j}{\partial\tau^2} = -(Ae^{i\theta_j} + c.c.)$$

$$\frac{\partial A}{\partial\tau} + \frac{\partial A}{\partial\zeta} = b$$

$$\left(\zeta = 2\omega_r\rho(z/c)\right)$$

self-similar solution:

$$A(\zeta,\tau) = \zeta A_1(\sqrt{\zeta}(\tau-\zeta))$$

for $t >> L_a/c$:

$$|A|^2 \propto \exp\left[\mathrm{cost}\cdot\left(\sqrt{L_a}\,\tau\right)^{2/3}\right]$$

In optical cavity:

(MEAN FIELD APPROXIMATION)

$$\frac{d^2\theta_j}{d\tau^2} = -(Ae^{i\theta_j} + c.c.)$$

$$\frac{dA}{d\tau} = b - KA$$

$$K = \frac{K_{cav}}{2\omega_r\rho}$$

$$K_{cav} = \frac{cT}{L_{cav}}$$

for $K > 1 \quad (K_{cav} > 2\omega_r\rho)$

$$A(\tau) \approx \frac{b}{K} \propto \rho \Rightarrow N_{photon} \propto N\rho^3 \propto N^2$$

$$|A|^2 \propto \exp[\sqrt{2/K}\,\tau]\ , \quad \sigma_t \propto \frac{1}{\sqrt{N}}$$

SUPERRADIANT REGIME : SELF-SIMILAR SOLUTION

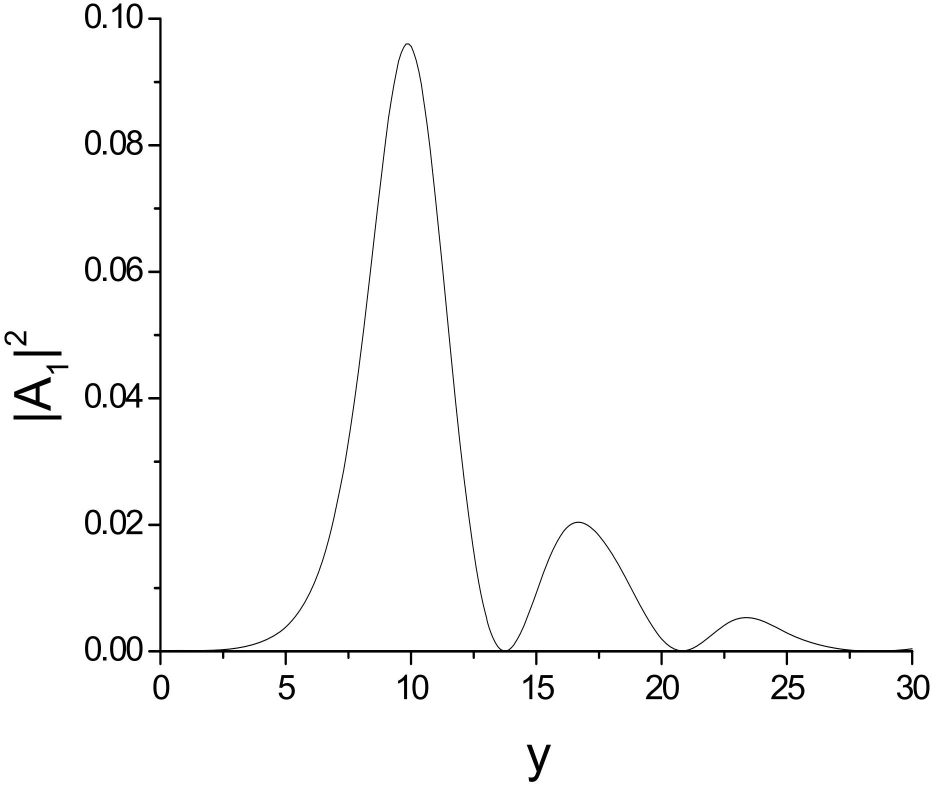
$$A(\tau,\zeta) = \zeta A_1(y)$$

$$\theta_j(\tau,\zeta) = \theta_{1j}(y)$$

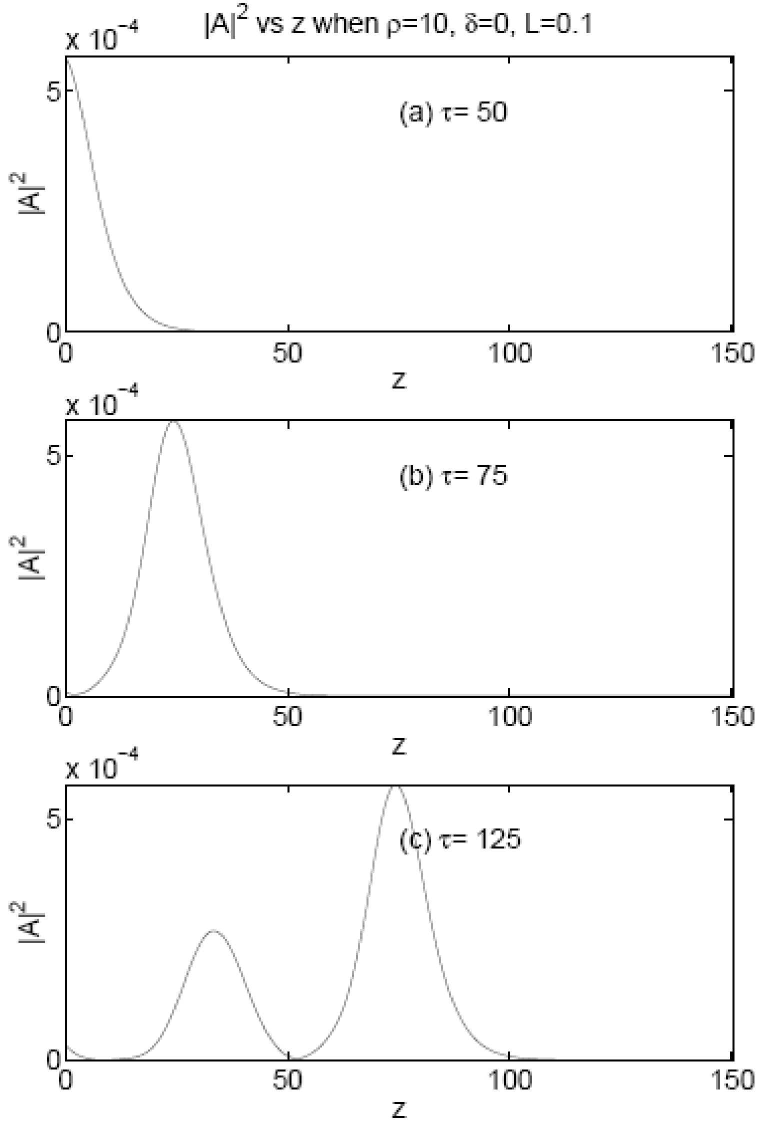
$$y = \sqrt{\zeta}(\tau - \zeta)$$

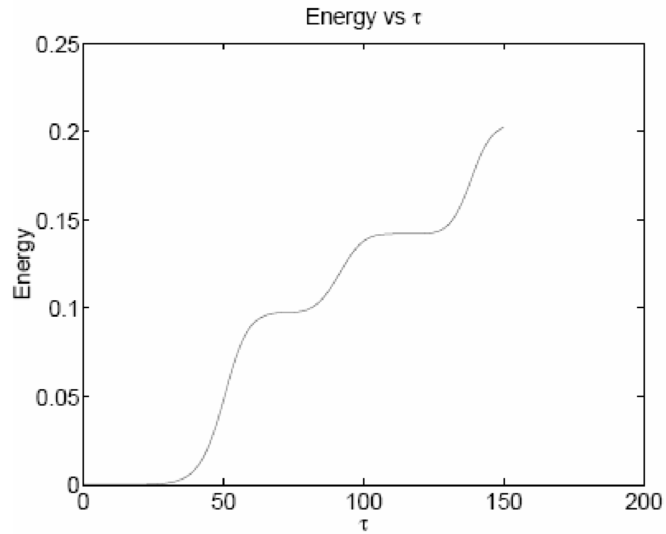
$$\frac{d^2\theta_{1j}}{dy^2} = -(A_1 e^{i\theta_{1j}} + c.c.)$$

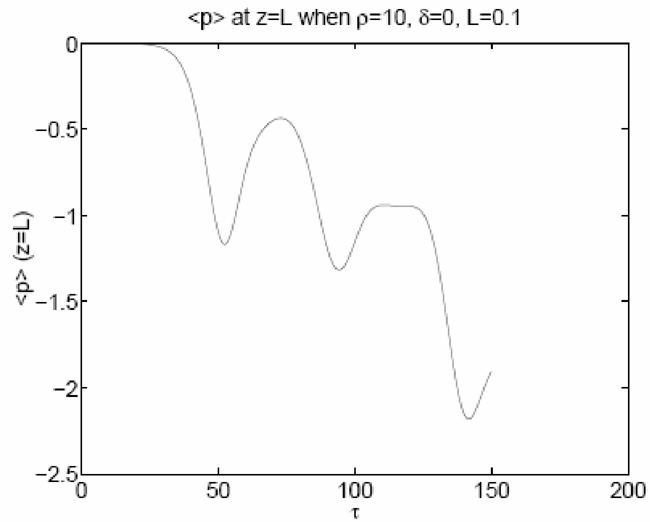
$$\frac{y}{2}\frac{dA_1}{dy} + A_1 = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_{1j}}$$



SUPERRADIANT EMISSION (EXACT PROPAGATION)



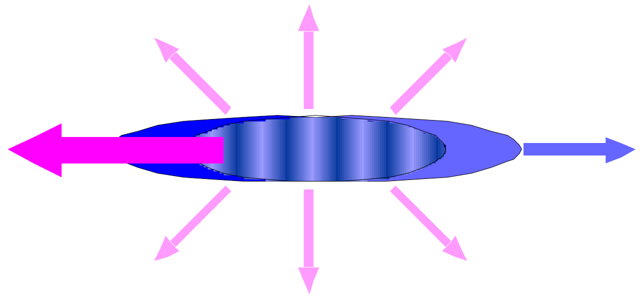




QUANTUM EFFECTS ARE EXPECTED WHEN THE ATOMS ARE INITIALLY COLDER THAN T_{recoil}

momentum spread $\sigma_p \leq \hbar k$ ➡ $T < T_{recoil}$ i.e. a **BEC** !

pump laser ➡



two possible regimes:

CLASSICAL REGIME:

$$\rho >> 1 \Rightarrow g\sqrt{N} >> \omega_r$$

$$\langle p_z \rangle_{max} \approx \rho(2\hbar k) >> 2\hbar k$$

$$N_{photon} \approx \rho N >> 1$$

QUANTUM REGIME:

$$\rho < 1 \Rightarrow g\sqrt{N} < \omega_r$$

$$N_{photon} = N \qquad \langle p_z \rangle = 2\hbar k$$

QUANTUM MODEL FOR CARL

$$H=\sum_{j=1}^{N}\left\{\frac{p_{zj}^2}{2m}-i\hbar g\left(ae^{i\theta_j}-a^+e^{-i\theta_j}\right)\right\}$$

$$i\hbar\frac{\partial\Psi}{\partial t}=\hat{H}\Psi=-\hbar\omega_r\frac{\partial^2\psi}{\partial\theta^2}-i\hbar g\left(ae^{i\theta}-h.c.\right)$$

$$\frac{da}{dt}=gN\int_0^{2\pi}d\theta\,|\Psi(\theta,t)|^2\,e^{-i\theta}+(i\Delta-K_{cav})a$$

$$i\frac{\partial\Psi}{\partial\tau}=-\frac{1}{2\rho}\frac{\partial^2\psi}{\partial\theta^2}-i\rho\left(Ae^{i\theta}-h.c.\right)\Psi$$

$$\frac{dA}{d\tau}=\int_0^{2\pi}d\theta\,|\Psi(\theta,\tau)|^2\,e^{-i\theta}+(i\delta-K)A$$

the quantum CARL model depends on ρ !

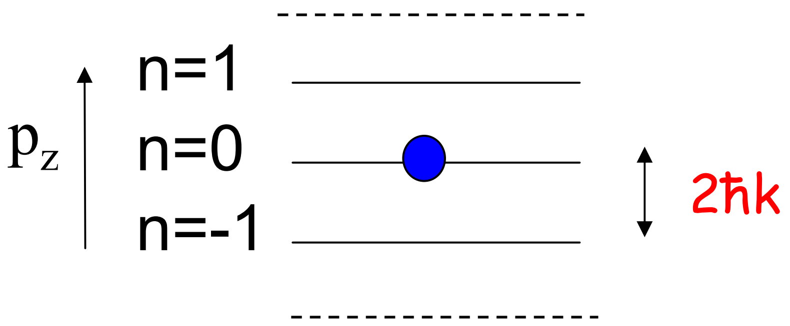
N. Piovella, M. Gatelli & R. Bonifacio, Opt. Comm. 194 (2001) 167
N. Piovella, M. Cola & R. Bonifacio, PRA 67 (2003) 01387

Momentum representation:

$$\Psi(\theta,\tau)=\sum_{n=-\infty}^{\infty}c_n(\tau)e^{in\theta}$$

$|c_n|^2$ = probability to find an atom with $p_z = n(2\hbar k)$

discrete values of momentum : $p_z = n\,(2\hbar k)$, $n=0,\pm1,..$



$$\frac{dc_n}{d\tau}=-\frac{in^2}{2\rho}c_n-\rho\left(Ac_{n-1}-A^*c_{n+1}\right)$$

$$\frac{dA}{d\tau}=\sum_{n=-\infty}^{\infty}c_n c_{n-1}^*+(i\delta-K)A$$

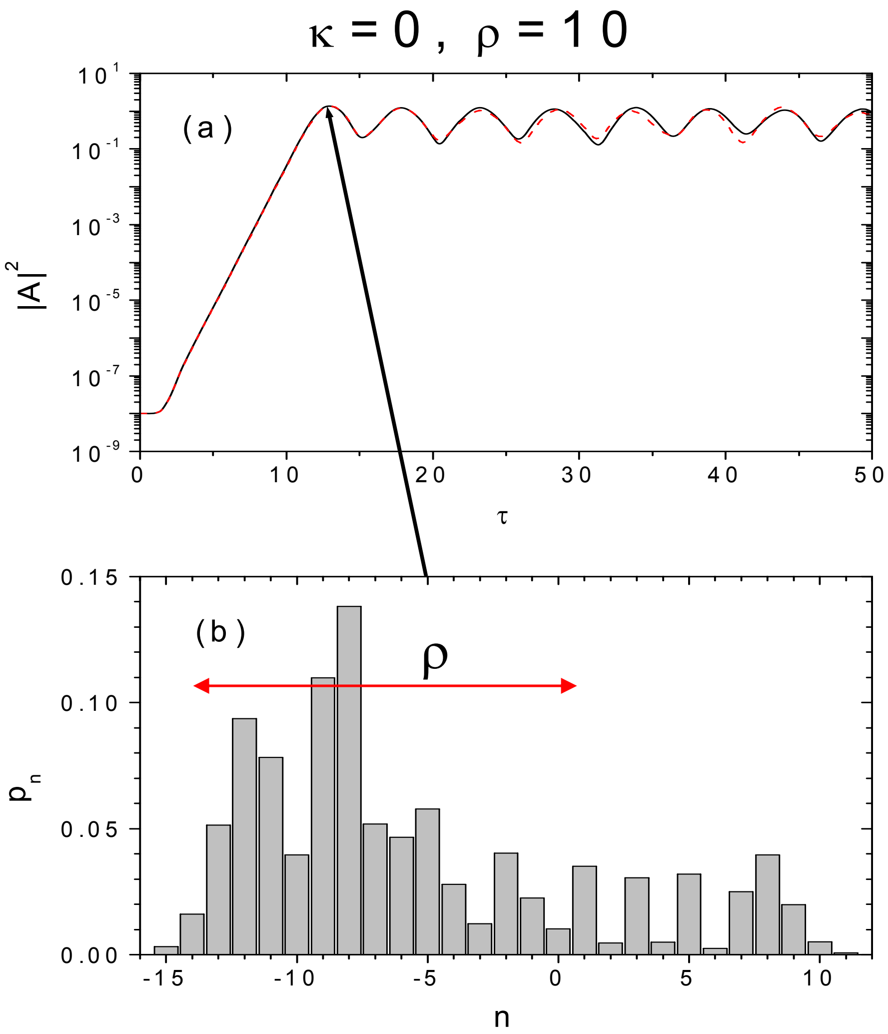
'Good-cavity limit' K=0
classical regime $\rho >> 1$

$g\sqrt{N} >> \omega_r$

$K_{cav} << 2\omega_r\rho$

initial state:
$c_0=1$ (all atoms with $p_z=0$)

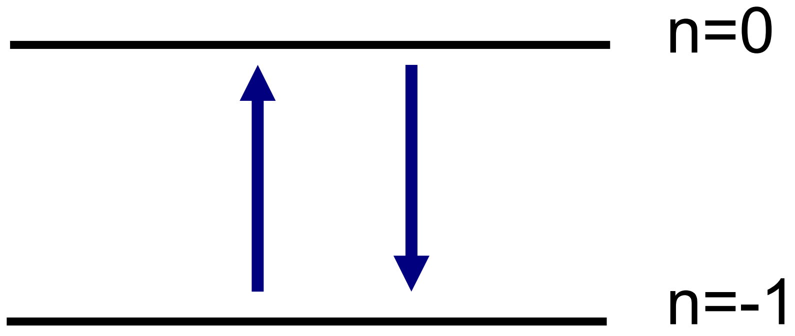
at the peak, the number of momentum states occupied is ~ ρ



'Good-cavity limit' K=0
quantum regime $\rho<1$

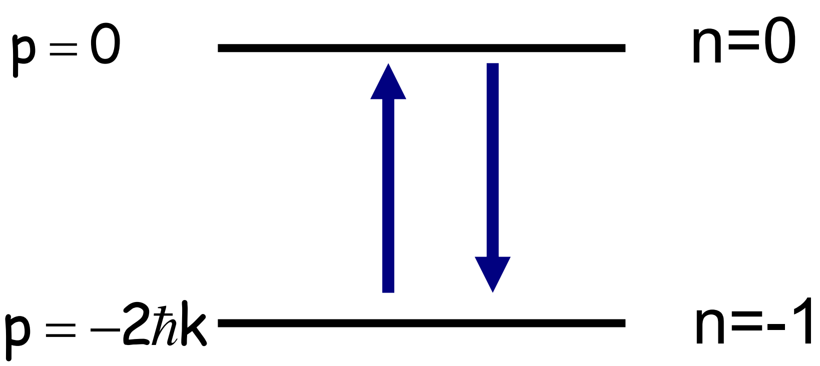
only TWO momentum states : $p_z=0$ and $p_z=-2\hbar k$

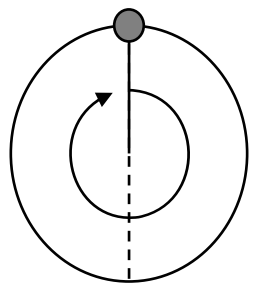
$$\Psi(\theta,\tau) \propto c_0(\tau) + c_{-1}(\tau)e^{-i\theta}$$



the condensate behaves as a two-level system (i.e. a laser !)

quantum CARL equations reduce to Maxwell-Bloch equations !





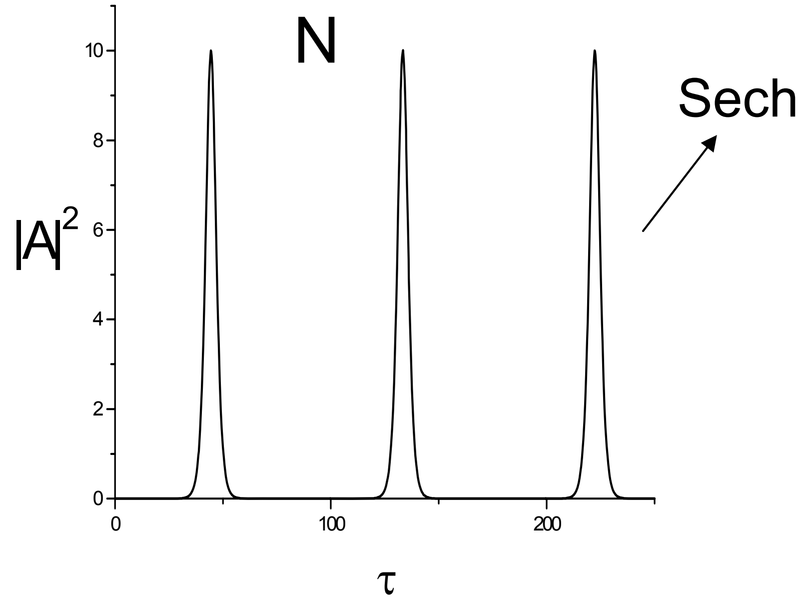
BEC behaves like an unstable pendulum

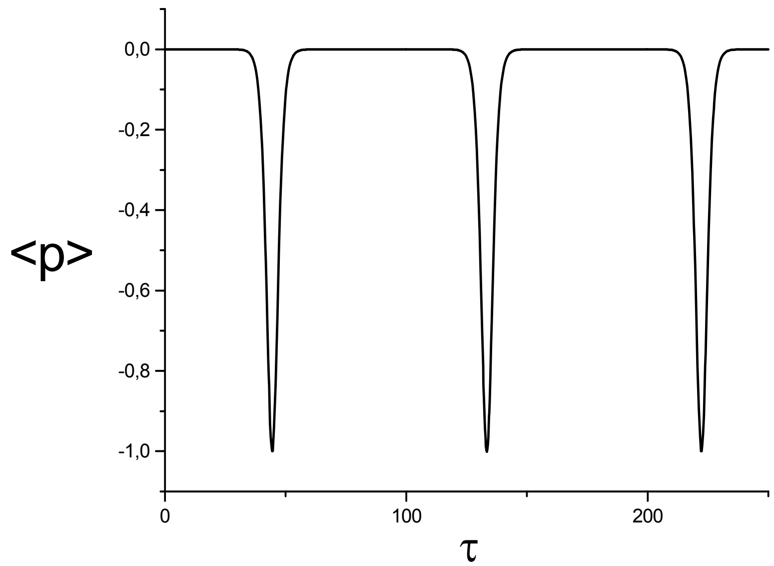
$$\ddot{\varphi}+K\dot{\varphi}=\rho\sin\varphi$$

$$\dot{\varphi}=\rho A$$

$$\langle p\rangle=-\sin^2(\varphi/2)$$

for K=0 train of sech^2 pulses (area=2π)





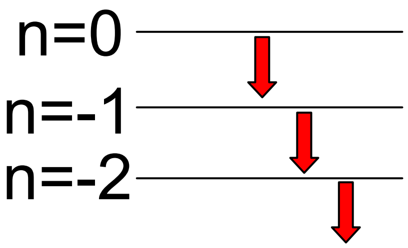
QUANTUM SUPERRADIANT REGIME:

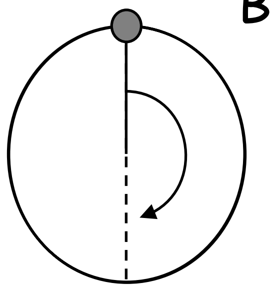
$$K_{cav} >> \omega_r > G_{sr} = \frac{g^2 N}{K_{cav}}$$

when photons escape fast from atoms (large K)..

SEQUENTIAL SUPERRADIANT SCATTERING:

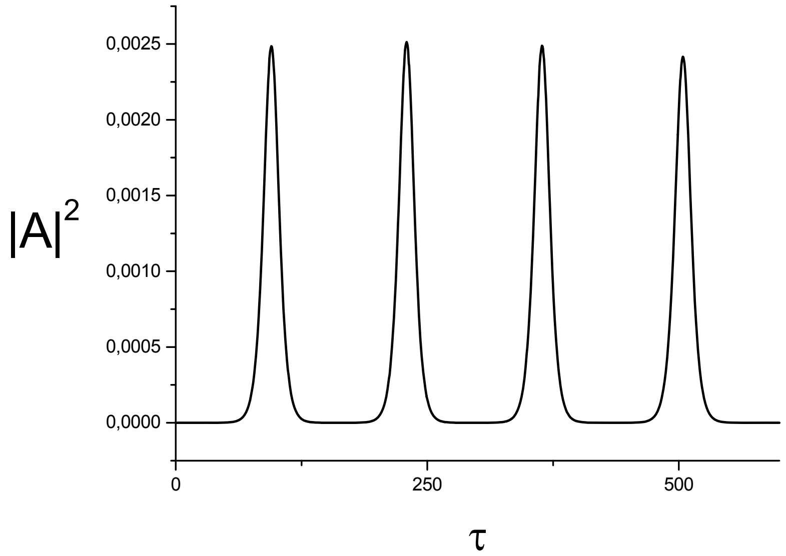
atoms recoil by 2ħk, emitting a SR pulse

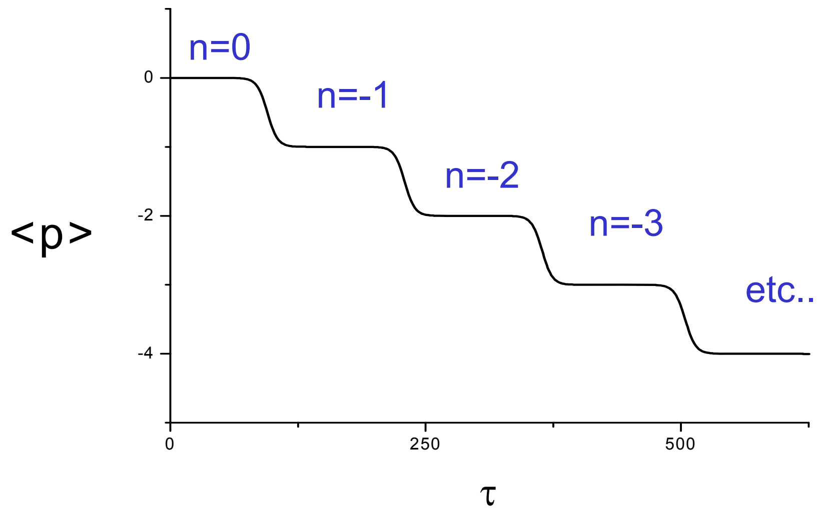




BEC behaves as an overdamped pendulum:

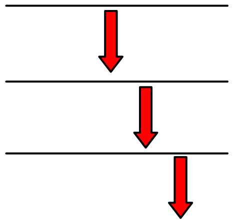
$$\dot{\varphi} \approx \frac{\rho}{K} \sin\varphi$$





CLASSICAL & QUANTUM SUPERRADIANT REGIME :

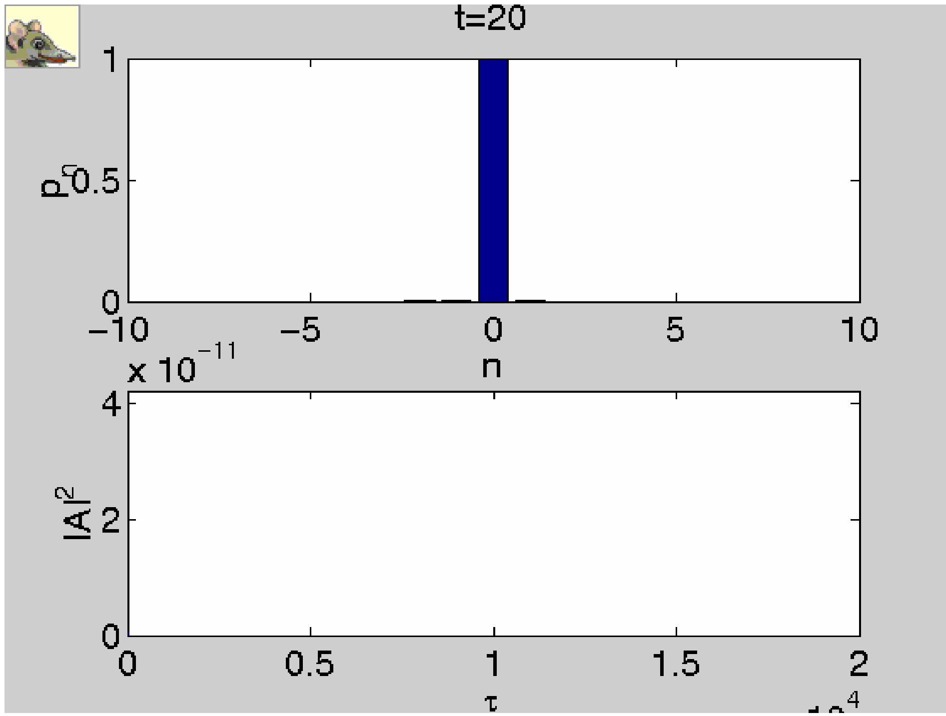
QUANTUM LIMIT
(sequential two-level SR)

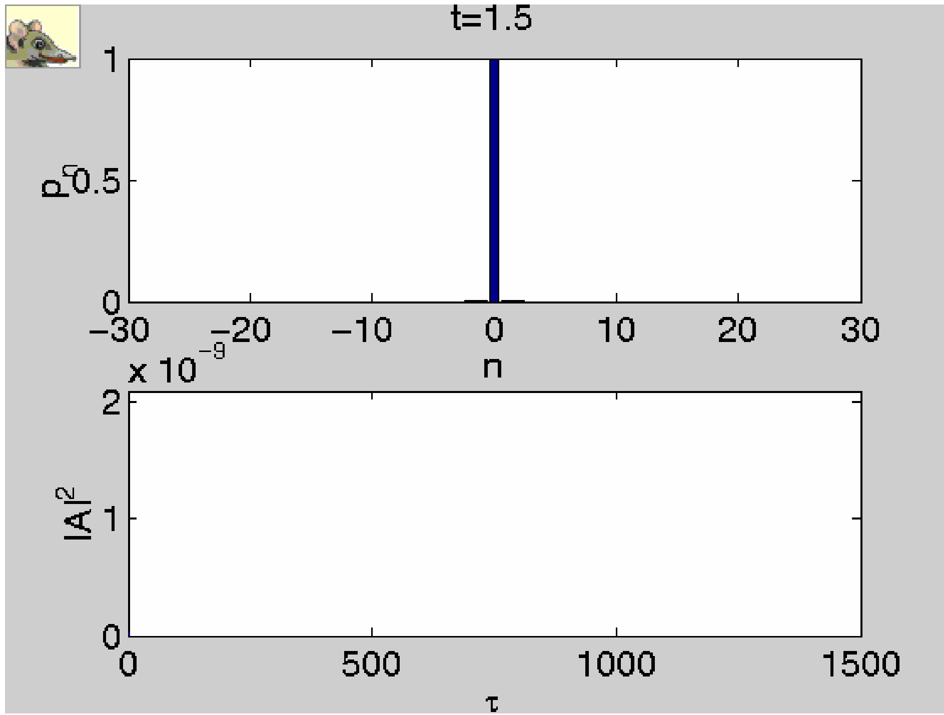


$$K_{cav} >> \omega_r > G_{sr} = \frac{g^2 N}{K_{cav}}$$

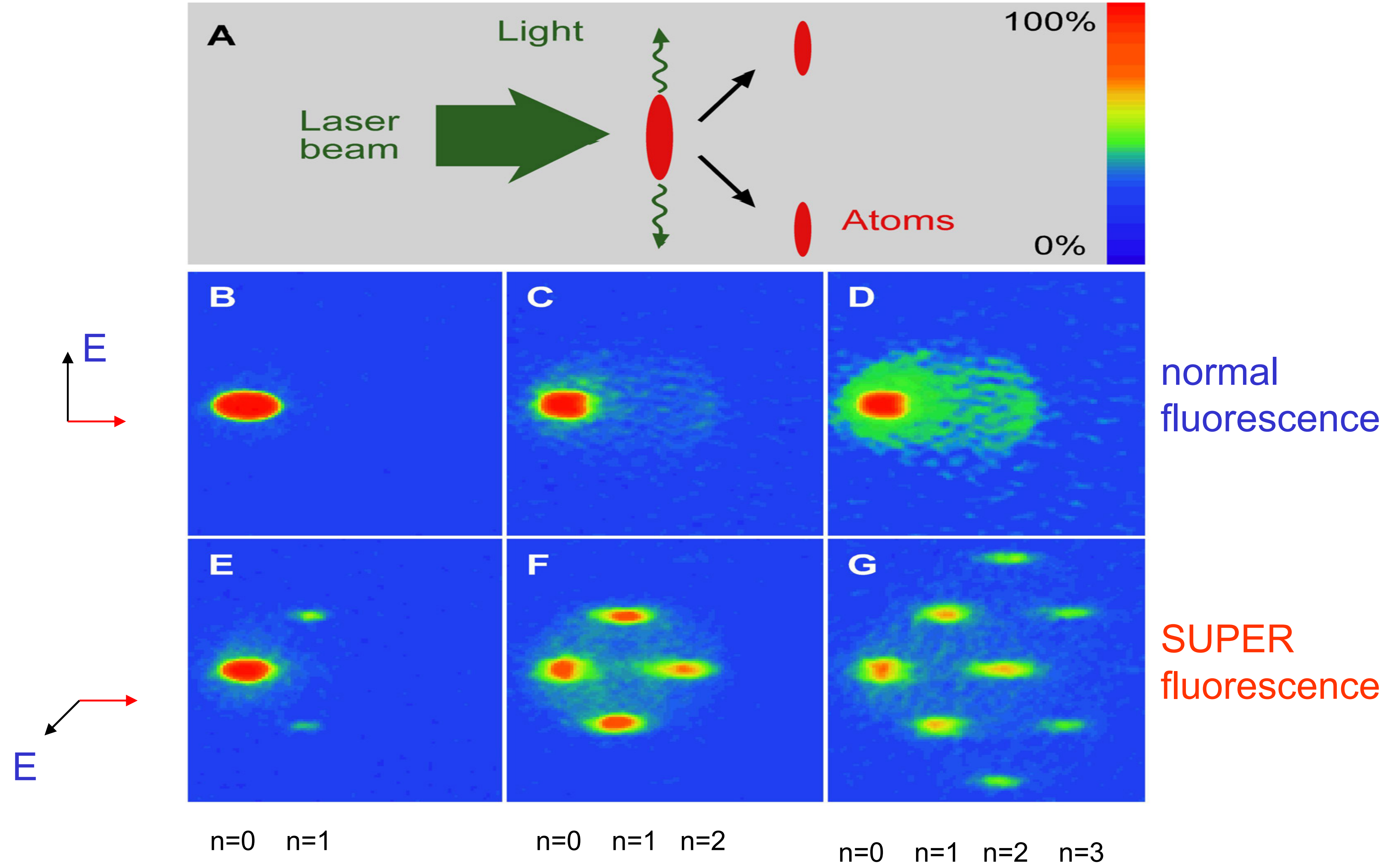
CLASSICAL LIMIT (CARL SR)

$$K_{cav} > G_{sr} = \frac{g^2 N}{K_{cav}} >> \omega_r$$



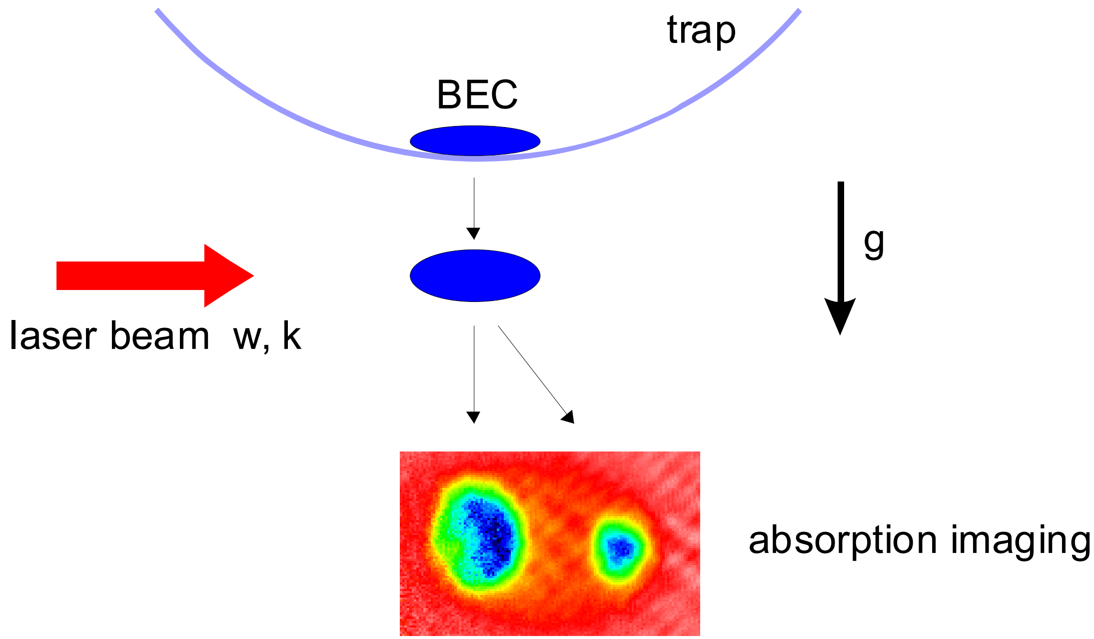


Quantum Superradiant CARL has been observed in 1999 by Ketterle at MIT and at LENS in 2002



The LENS experiment

- Production of an elongated ^{87}Rb BEC in a Ioffe-Pritchard magnetic trap
- Laser pulse during first expansion of the condensate
- Absorption imaging of the momentum components of the cloud



Experimental values:

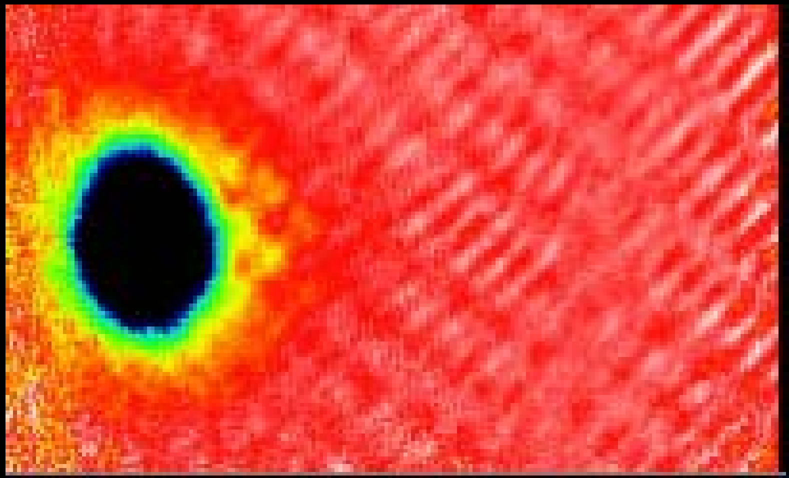
Δ = 13 GHz
w = 750 μm
P = 13 mW

L. Fallani et al., PRA (2006)

The LENS experiment

Temporal evolution of the population in the first three atomic momentum states during the application of the light pulse.

pump light



TRANSVERSE EFFECTS AND SELF-FOCUSING IN CARL

$$H = \frac{p_z^2 + p_\perp^2}{2m} - i\hbar g\left[a(\vec{x}_\perp, t)e^{i\theta} - c.c.\right]$$

$$\frac{\partial a}{\partial t} - \frac{ic}{Z_R}\nabla_\perp^2 a = gN\left\langle e^{-i\theta}\right\rangle + (i\Delta - K_{cav})a$$

$$\frac{d\left\langle \vec{p}_\perp \right\rangle}{dt} = \hbar\left(\dot{\phi} - \Delta\right)\left(\frac{\vec{\nabla}_\perp |a|^2}{N}\right) < 0$$

$\dot{\phi} > 0$, $\Delta = 0$

self-focusing force proportional to
the field intensity gradient and to the phase shift

SELF-FOCUSING IN QUANTUM REGIME

$$\Psi(\theta,\tau) \propto c_0(\tau) + c_{-1}(\tau)e^{-i\theta}$$

N. Piovella et al. , Laser Physics 1 (2007)

$$\frac{\partial c_0}{\partial \tau} = i\eta\nabla^2_{x'}c_0 - Ac_{-1}$$

$$\frac{\partial c_{-1}}{\partial \tau} = i\eta\nabla^2_{x'}c_{-1} + i\tilde{\Delta}c_{-1} - A^*c_0^*$$

$$\frac{\partial A}{\partial \tau} = iD\nabla^2_{x'}c_0 + c_{-1}^*c_0 - KA$$

$$\eta = \frac{\omega_\perp}{g\sqrt{N}} \qquad D = \frac{c}{Z_R g\sqrt{N}}$$

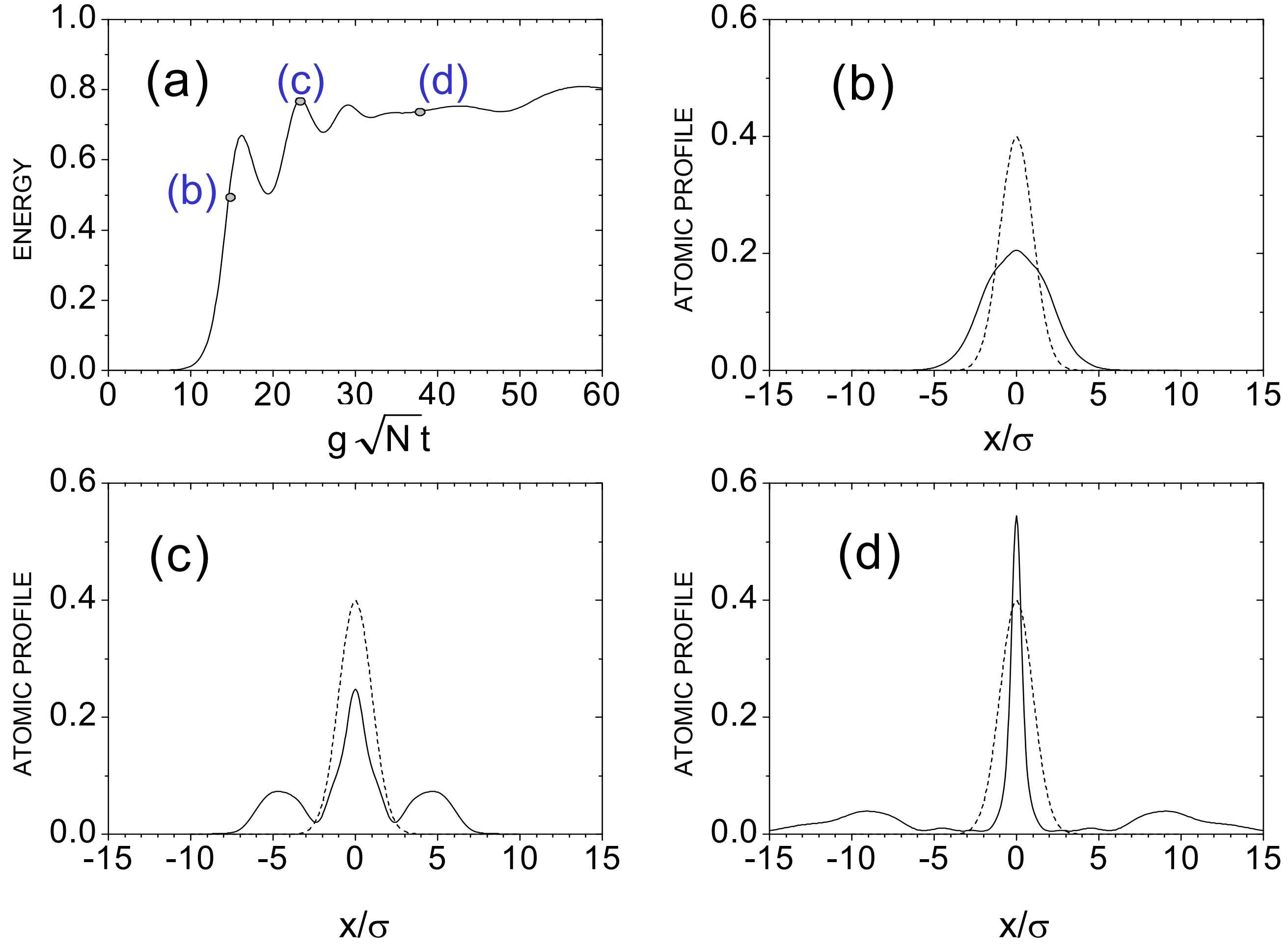
$$x' = \frac{x}{\sigma} \qquad \tau = g\sqrt{N}t$$

$$K = \frac{K_{cav}}{g\sqrt{N}} \qquad A = \frac{a}{\sqrt{N}}$$

$$v_{rec} = \frac{\hbar k}{m}, \quad D = \frac{c}{v_{rec}}\eta, \quad Z_R = \frac{4\pi\sigma^2}{\lambda}, \quad \tilde{\Delta} = \frac{\Delta - \omega_r}{g\sqrt{N}} \quad \omega_\perp = \frac{\hbar}{2m\sigma^2}$$

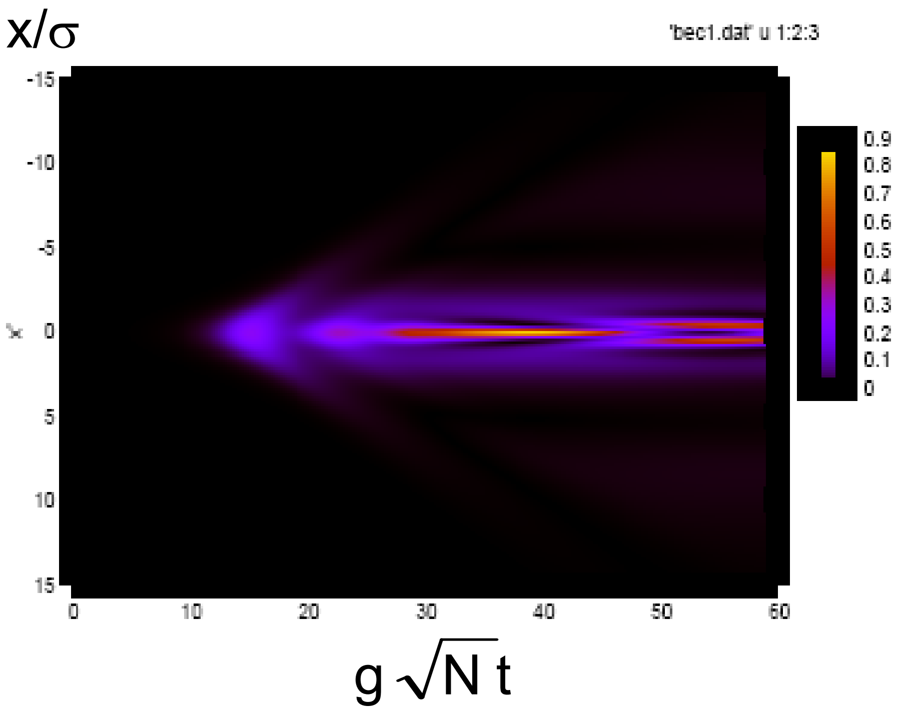
GOOD-CAVITY REGIME (K=0, D=0) and η=0.1

(neglecting radiation diffraction..)

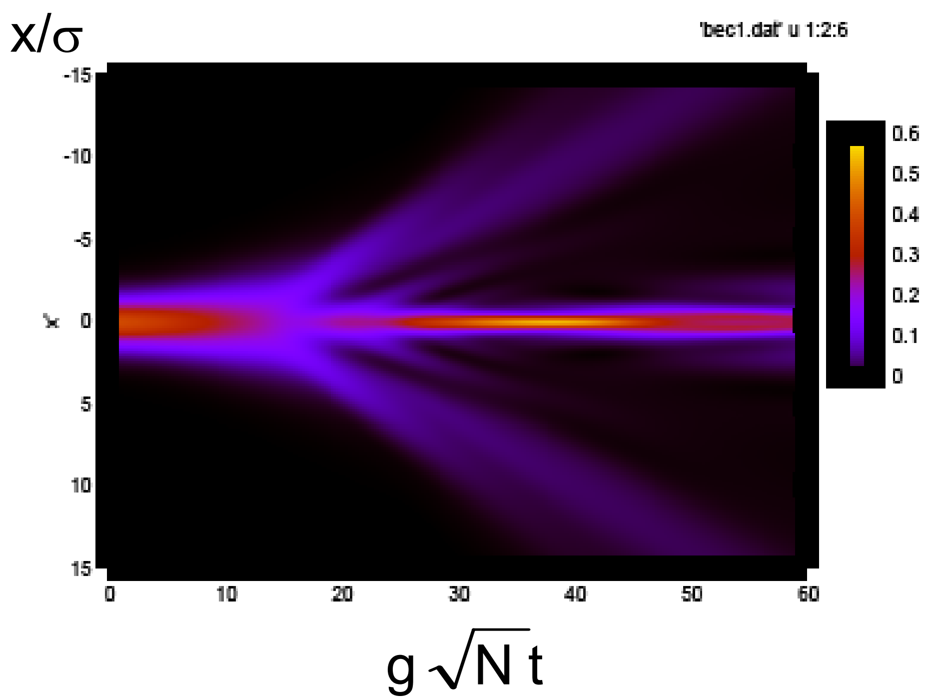


GOOD-CAVITY REGIME (K=0, D=0) and η=0.1

radiation intensity



atomic density



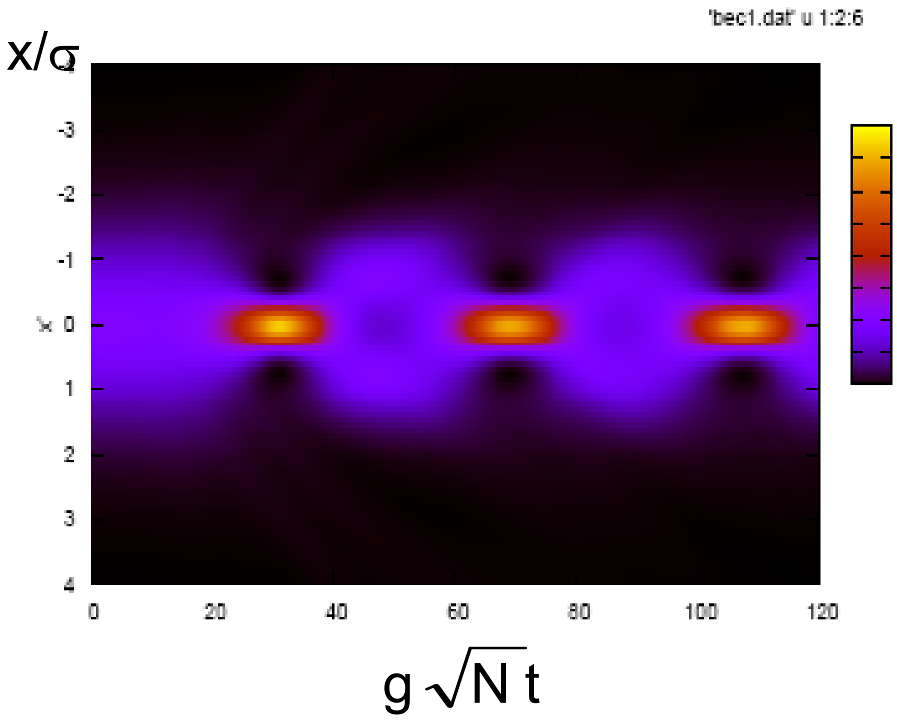
GOOD-CAVITY REGIME: effects of diffraction and cavity damping

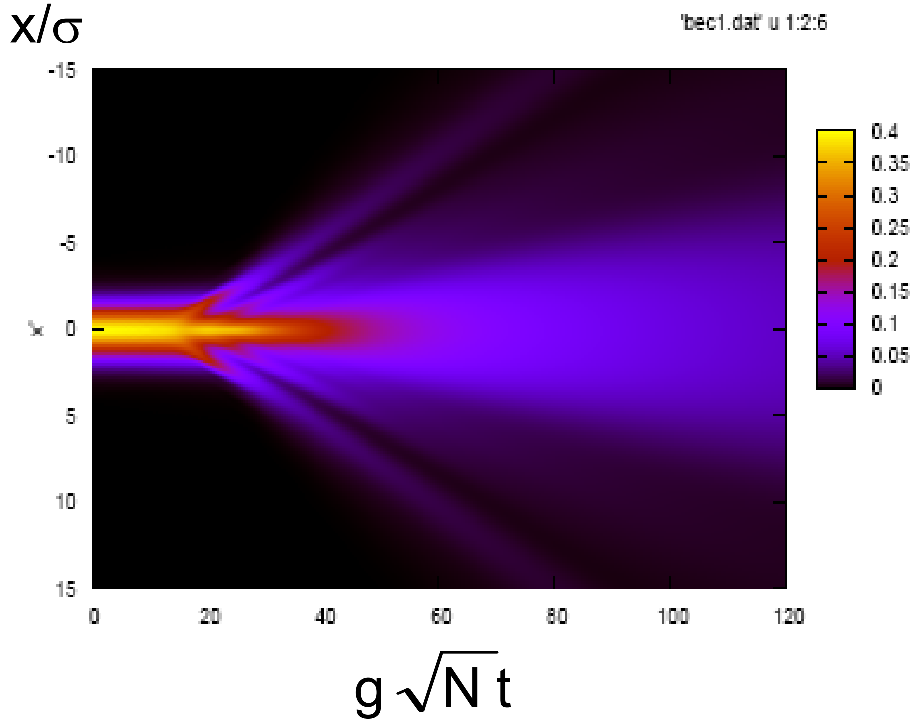
atomic density

$\eta=0.02$, $D=10\eta$, $K=0$

atomic density

$\eta=0.02$, $D=0$, $K=10\eta$





SUPERRADIANT REGIME (K>>D>1)

$$K_{cav} >> g\sqrt{N} \qquad (K >> 1)$$

$$Z_R >> L_{cav} \qquad (K >> D)$$

$$\Rightarrow \quad A \approx \frac{1}{K} c_{-1}^{*} c_0$$

$$\frac{\partial c_0}{\partial t'} = i\eta' \nabla_{x'}^2 c_0 - |c_{-1}|^2 c_0$$

$$\frac{\partial c_{-1}}{\partial t'} = i\eta' \nabla_{x'}^2 c_{-1} + |c_0|^2 c_{-1}$$

$$t' = Gt \qquad \eta' = \frac{\omega_\perp}{G}$$

$$\left(\omega_\perp = \frac{\hbar}{2m\sigma^2}\right)$$

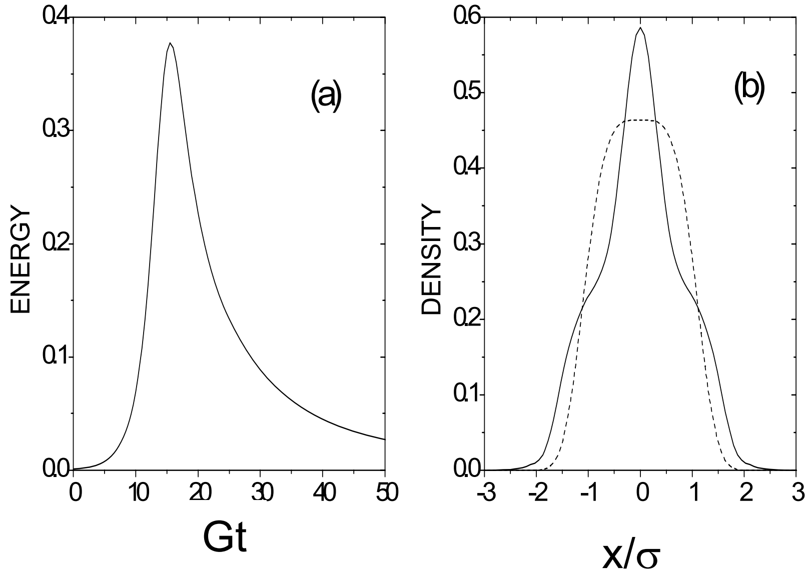
$$G = \frac{g^2 N}{K_{cav}}$$ (SR gain) (in free space $K_{cav} \sim c/L_a$)

SUPERRADIANT REGIME

ADIABATIC LIMIT

$K \gg D \quad (Z_R \gg L)$

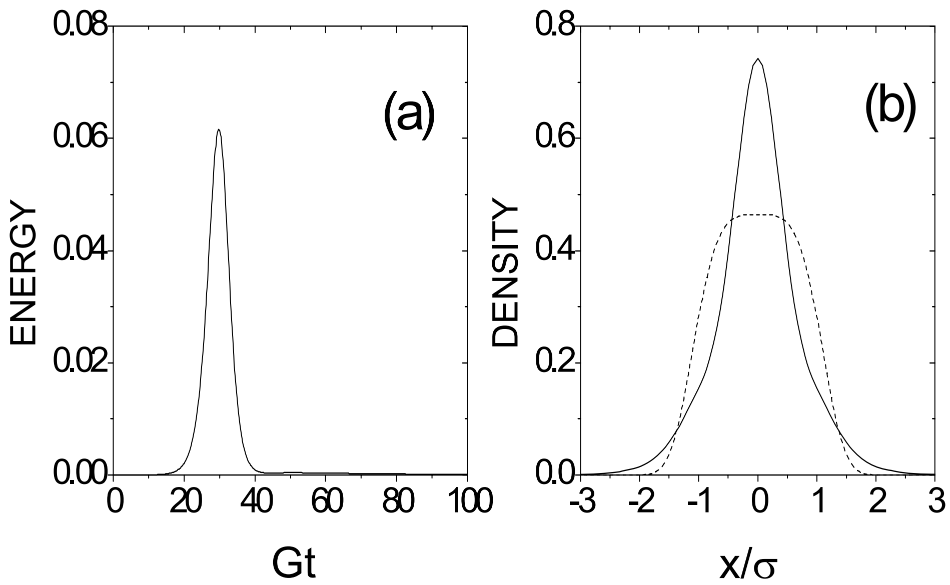
$\eta' = 0.005$



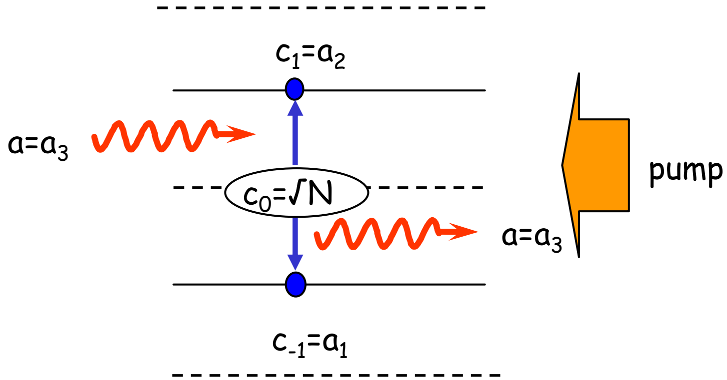
NON ADIABATIC LIMIT

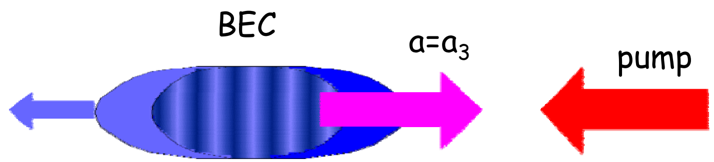
$K = D = 1 \quad (L = Z_R)$

$\eta' = 0.002$



Entanglement in CARL





$$H=\sum_n\left[n^2\omega_r c_n^+ c_n + ig\left(a^+ c_n^+ c_{n+1}^+ - h.c.\right)\right] - \Delta a^+ a$$ CARL Hamiltonian

linear approximation: $a_{1,2}=c_{\mp 1}e^{\pm i\Delta t}$, $a=a_3 e^{-i\Delta t}$

$$H=(\Delta+\omega_r)a_2^+ a_2-(\Delta-\omega_r)a_1^+ a_1+ig\left[\left(a_1^+ + a_2\right)a_3^+ - h.c.\right]$$

N. Piovella, M. Cola and R. Bonifacio, PRA 67 (2003) 013817

three-mode entanglement:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{1+N_1}} \sum_{m,n=0}^{\infty} \left(\frac{N_3}{1+N_1}\right)^{m/2} \left(\frac{N_2}{1+N_1}\right)^{n/2} e^{i(n\phi_2+m\phi_3)} \sqrt{\frac{(m+n)!}{m!n!}} |m+n,n,m\rangle \qquad (N_i = a_i^+ a_i)$$

$(N_3=N_1-N_2)$

in classical regime: $N_1 \sim N_2 \gg N_3$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{1+N_1}} \sum_{n=0}^{\infty} \left(\frac{N_2}{1+N_1}\right)^{n/2} e^{in\phi} |n,n,0\rangle$$

atom-atom entanglement

in quantum regime: $N_1 \sim N_3 \gg N_2$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{1+N_1}} \sum_{m=0}^{\infty} \left(\frac{N_3}{1+N_1}\right)^{m/2} e^{im\phi} |m,0,m\rangle$$

atom-photon entanglement

Entanglement is robust against decoherence (γ) and cavity damping (κ)

$$\frac{d\hat{\rho}}{d\tau} = -i\left[H,\hat{\rho}\right] + 2\gamma\left\{\hat{L}[a_1] + \hat{L}[a_2]\right\}\hat{\rho} + 2\kappa\hat{L}[a_3]\hat{\rho}$$

We have solved the MASTER EQUATION transforming it in an equation for

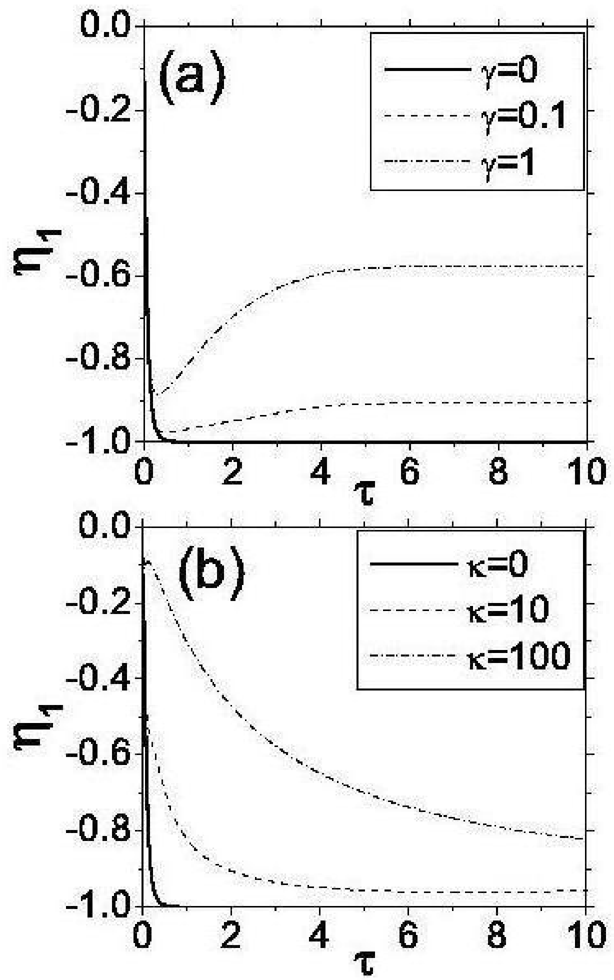
a three-mode WIGNER FUNCTION $W(\alpha_1,\alpha_2,\alpha_3)$

Then, we have calculated numerically the COVARIANCE MATRIX C (6x6)

$$\gamma = \frac{\Gamma_{decoherence}}{2\omega_r\rho} \quad , \quad \kappa = \frac{K_{cav}}{2\omega_r\rho}$$

M.M. Cola, M.G.A. Paris and N, Piovella PRA 70 (2004) 043809

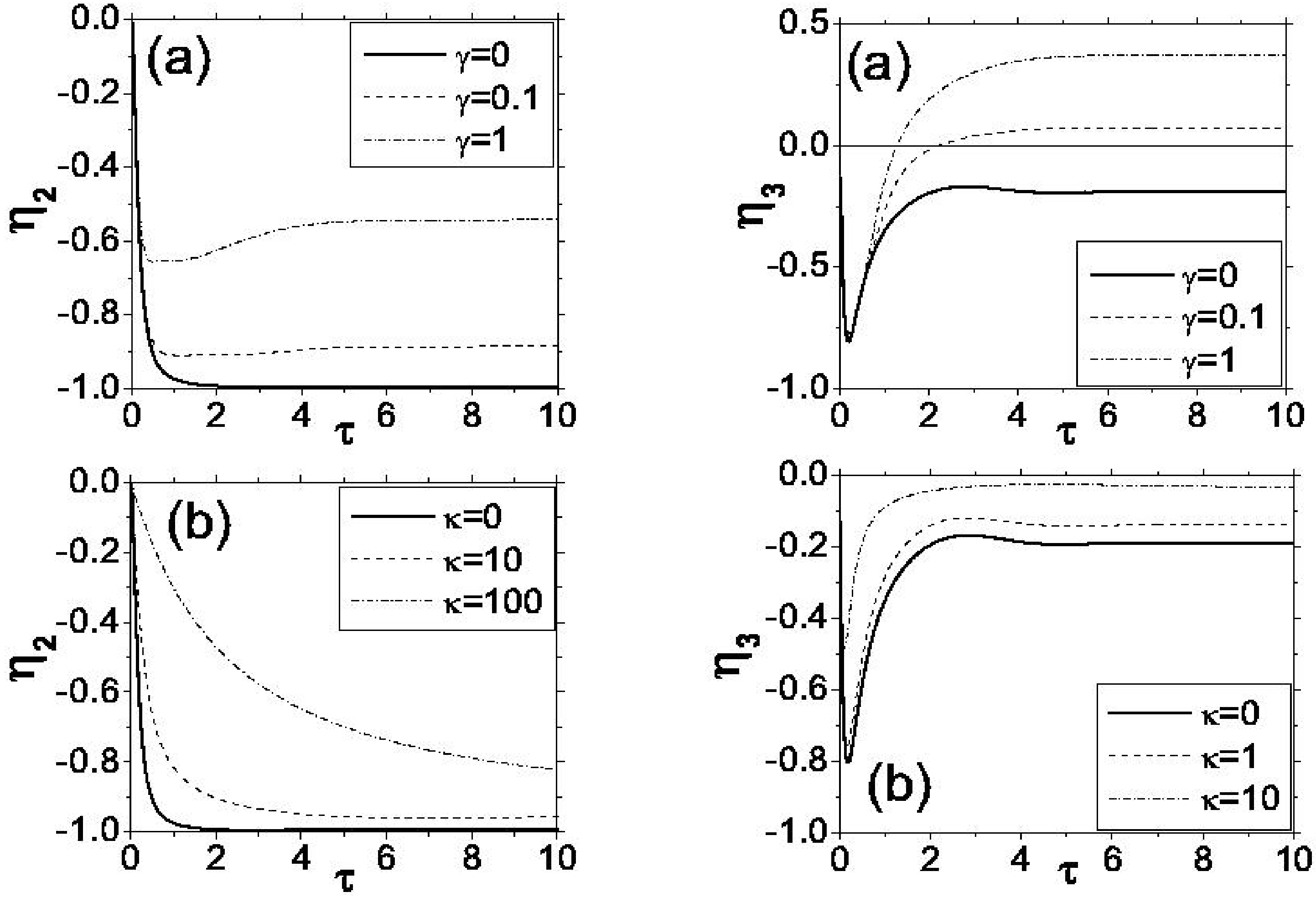
Classical regime ρ=100 and Δ=0



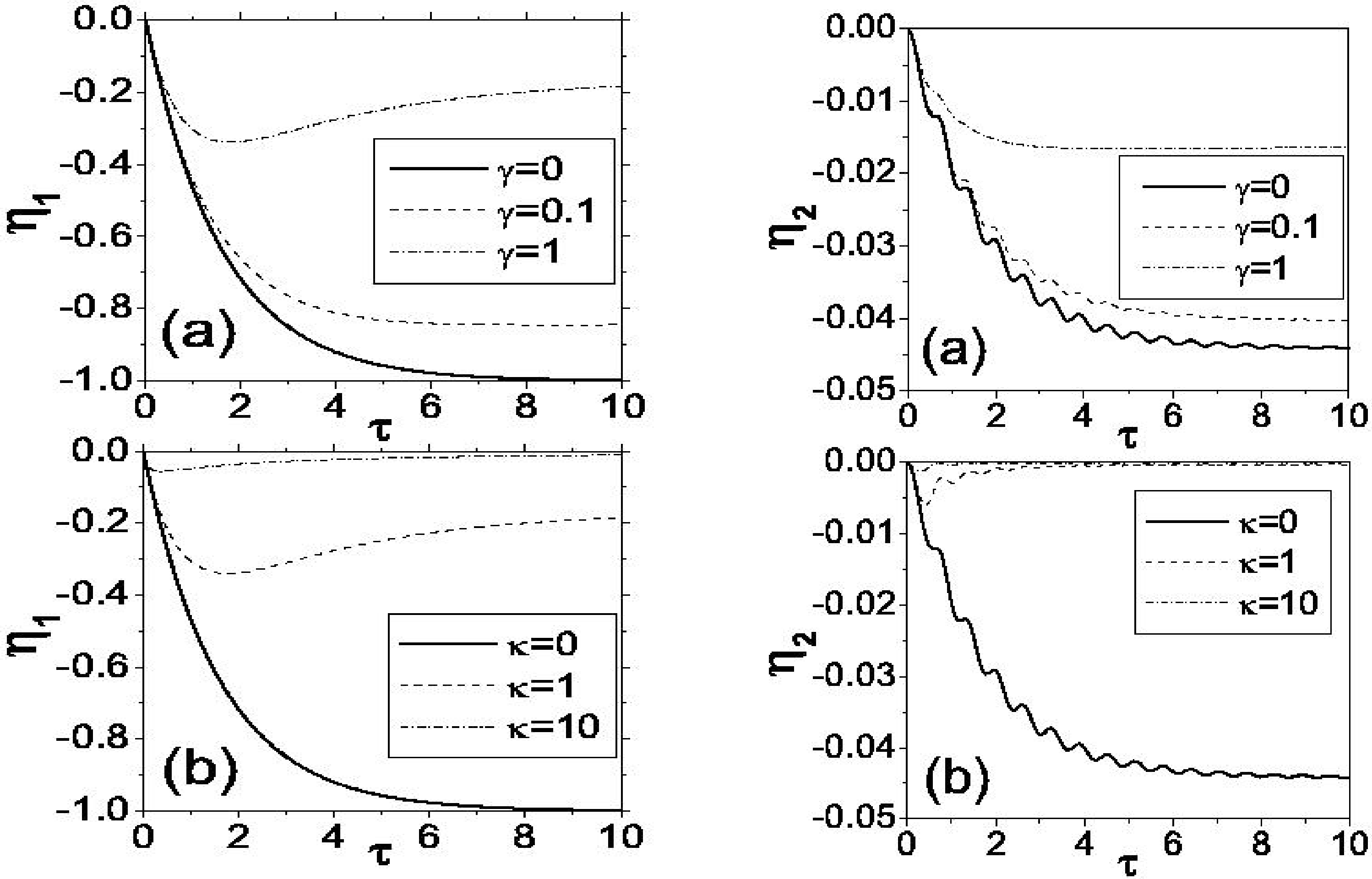
Entanglement and separability criteria have been applied to the covariance matrix $C(\tau)$

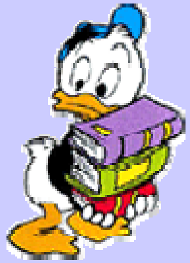
η_i>0 : separability of the i-th mode from the other two modes

Classical regime ρ=100 and Δ=0



Quantum regime $\rho=0.2$ and $\Delta=\omega_r$





Quantum Atoms in Milan

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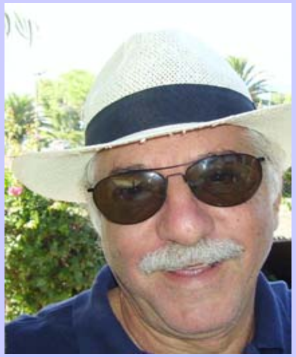

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