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Classical and quantum regimes in the collective atomic recoil laser from a Bose–Einstein condensate

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Abstract. A collective atomic recoil laser (CARL) realized with a Bose–Einstein condensate offers the possibility to investigate new effects in the coherent interaction between optical and matter waves. This paper discusses some aspects of the nonlinear evolution of scattered radiation and the matter-wave field in the high-Q cavity and superradiant CARL regimes.

1. Introduction

So far, experiments attempting to observe the collective atomic recoil laser (CARL), first proposed by Bonifacio and coworkers in 1994 [1], have only been undertaken in hot atomic vapours [2, 3], in which the thermal broadening of the atomic velocity distribution makes the CARL gain very small and comparable to that of the related phenomenon of recoil-induced resonances (RIR) [4]. Whereas in RIR the gain is proportional to the derivative of the velocity distribution, in CARL in principle one should be able to observe the exponential growth of a probe field oriented reversely to a strong pump interacting with the atoms. Only recently, CARL action has been observed injecting a cold atomic gas in a high-Q optical ring cavity driven by an external laser and applying an additional friction force to the atoms via an optical molasses [5].

In general, for CARL the best active medium is a Bose–Einstein condensate (BEC), now routinely produced in many laboratories, due to the complete absence of Doppler broadening. Furthermore, the extremely long coherence length and the wave-like nature of a BEC allows for new and unexpected quantum regimes of CARL [6,7]. Recent experiments on superradiant Rayleigh scattering [8–11], realized sending a single far-off resonance laser on an elongated BEC, have been proved to be related to the CARL mechanism. In the following we will introduce a model able to describe the CARL action in a one-dimensional BEC, and we will derive the main features of the different regimes of operation.

2. Basic model

Consider a gas of two-level atoms exposed to a laser incident along the negative direction of the symmetry axis z of the atomic sample. The laser is far-detuned from the atomic resonance, so that radiation pressure due to absorption and

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subsequent random incoherent, isotropic emission of a photon, can be neglected. In this regime, the atoms backscatter photons at frequency ω_s and wave vector $k_s = \omega_s/c \approx k$, recoiling with a momentum $2\hbar k$ along the same direction of the incident laser beam. The semiclassical regime of CARL is described by the following model of equations [1]:

$$\frac{d\theta_j}{d\tau} = \bar{p}_j \tag{1}$$

$$\frac{d\bar{p}_j}{d\tau} = -\left[Ae^{i\theta_j} + \text{c.c.}\right] \tag{2}$$

$$\frac{dA}{d\tau} = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j} + (i\delta - \kappa)A \tag{3}$$

where $\tau = \rho\omega_r t$ is the interaction time in units of the collective recoil bandwidth, $\rho\omega_r$, $\omega_r = 2\hbar k^2/m$ is the recoil frequency, m is the atomic mass. In equations (1)–(3), $\theta_j = 2\,kz_j$ and $\overline{p}_j = 2kv_{zj}/\rho\omega_r$ and $|A| = (\epsilon_0/n_a\hbar\omega\rho)^{1/2}|E_s|$, where z_j and v_{zj} are position and velocity of the jth atom and E_s is the electric field of the scattered radiation, $\delta = (\omega - \omega_s)/\rho\omega_r$, $\rho = (\Omega_0/2\Delta_0)^{2/3}(\omega d^2 n_a/\hbar\epsilon_0\omega_r^2)^{1/3}$ is the collective CARL parameter, $\Omega_0 = dE_0/\hbar$ is the Rabi frequency of the pump, E_0 is the electric field of the incident laser, d is the electric dipole moment, n_a is the atomic density and ϵ_0 is the permittivity of free space. Equation (3) has been written in the 'mean-field' limit, which models the propagation effects of the light by replacing the nonuniform electric field by an average value and by adding to the equation a damping term with decay constant $\kappa = \kappa_c/\rho\omega_r$, where $\kappa_c = cT/2L$, T is the mirror transmission of the optical ring cavity enclosing the backscattered radiation, L is the cavity length and c is the speed of light in vacuum. In the free-space case, $T = 1$ and L is the condensate length.

The CARL in an elongated Bose–Einstein condensate can be described by the following CARL–BEC model, consisting of a Gross–Pitaevskii equation for the matter-wave field $\Psi(\theta, \tau)$, coupled self-consistently with the equation for the radiation field A:

$$\frac{\partial\Psi}{\partial\tau} = \frac{i}{\rho}\frac{\partial^2\Psi}{\partial\theta^2} - \frac{\rho}{2}\left[Ae^{i\theta} - \text{c.c.}\right]\Psi - 2i\pi\beta\Psi|\Psi|^2 \tag{4}$$

$$\frac{dA}{d\tau} = \int d\theta|\Psi|^2 e^{-i\theta} + (i\delta - \kappa)A. \tag{5}$$

The matter-wave field Ψ is normalized to one, i.e. $\int d\theta|\Psi(\theta, t)|^2 = 1$. A detailed derivation of equations (4) and (5) can be found in [12] or, in a slightly different form, in the works by Moore and co-workers [6]. The last nonlinear term of equation (4) takes into account for a 'mean-field' atomic interaction due to binary collisions, where $\beta = 8\pi\hbar k a_s N/m\Sigma\omega_r\rho$, a_s is the scattering length, d Σ is the condensate cross-section and N is the atom number.

If the condensate is much longer than the radiation wavelength and the density is uniform, then periodic boundary conditions can be assumed on θ and the wavefunction can be written as a Fourier series

$$\Psi(\theta, t) = \sum_n c_n(t)u_n(\theta)e^{-in\delta t}, \tag{6}$$

where $u_n(\theta) = (1/\sqrt{2\pi})\exp(in\theta)$ are momentum eigenfunctions with eigenvalues $p_z = n(2\hbar k)$. Using equation (6), equations (4) and (5) reduce to an infinite set of ordinary differential equations,

$$\frac{dc_n}{d\tau} = -i\delta_n c_n + \frac{\rho}{2}(A^* c_{n+1} - Ac_{n-1}) - i\beta \sum_{m,l} c_m c_l c^*_{m+l-n} \tag{7}$$

$$\frac{dA}{d\tau} = \sum_n c^*_n c_{n+1} - \kappa A, \tag{8}$$

where $\delta_n = n^2/\rho - n\delta$. Assuming that the only two momentum levels involved in the process are the initial level n and the final level $n-1$, equation (7) and (8) reduce, defining $S = c_n c^*_{n-1}$ and $W = |c_n|^2 - |c_{n-1}|^2$, to:

$$\frac{dS}{d\tau} = -i(\Delta - \beta W)S + \frac{\rho}{2}AW \tag{9}$$

$$\frac{dW}{d\tau} = -\rho(AS^* + \text{c.c.}) \tag{10}$$

$$\frac{dA}{d\tau} = S - \kappa A, \tag{11}$$

where $\Delta = \delta + (2n-1)/\rho$. We note that when the atomic interaction is neglected ($\beta = 0$), $\Delta = 0$ is the Bragg condition of the scattering process, arising from momentum and energy conservation [13]. We observe from equation (9) that the atomic interaction term has a dynamical dispersive effect on the Bragg resonance, proportional to the population difference W.

3. High-Q cavity limit

Let us now assume that the scattered radiation is circulating in a high-Q optical ring cavity, with $\kappa \approx 0$. We assume that the condensate is sufficiently dilute that the nonlinear term in equation (4) may be neglected ($\beta = 0$) [14]. In this case the total momentum is conserved, i.e. $\langle p_z \rangle + 2\hbar k(\rho/2)|A|^2$ is constant, where $\langle p_z \rangle = (2\hbar k)\sum_n n|c_n|^2$. This means that in the classical regime, described by equations (1)–(3), the maximum average momentum transferred to the atoms is $\langle p_z \rangle \approx -(2\hbar k)\rho$, since at saturation $|A|^2$ is of the order of one [1]. Hence, the collective parameter ρ can be interpreted as the maximum average number of photons scattered by each atom in the classical regime. Then, it is expected that for $\rho \gg 1$ the CARL from a BEC behaves classically. This is shown by figure 1(a), in which $\langle p_z \rangle$ versus τ, obtained from the classical model (1)–(3) (dashed line), is compared with the one obtained from the quantum model (7)–(8) (continuous line), for $\rho = 10$, $\delta = 0$, an initial seed $A_0 = 10^{-4}$ and $\bar{p}_j(0) = 0$ ($c_n(0) = \delta_{n,0}$). In the classical limit $\rho \gg 1$ the small frequency shift $\omega - \omega_s = \omega_r$, due to the recoil energy transferred to the atom after scattering a photon, is negligible. On the contrary, for $\rho < 1$ each atom may scatter no less than one photon, and the recoil shift becomes important. Figure 1(b) shows the average momentum $\langle p_z \rangle$ versus τ, obtained from the quantum model (7)–(8), for $\rho = 0.2$ and $\delta = 1/\rho$: the average momentum oscillates between zero and $-(2\hbar k)$ and the BEC behaves as a system of only two momentum levels, described by equations (9)–(11). This regime of

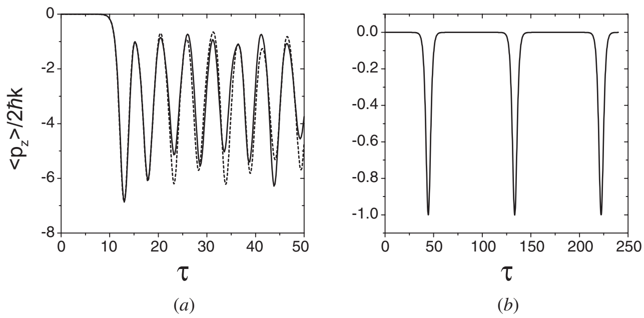

Figure 1. Average momentum $\langle p_z \rangle / 2\hbar k$ in the high-Q cavity limit. (*a*) Classical regime for $\rho = 10$, $\delta = 0$, from equations (1)–(3) (dashed line) and from equations (7), (8) (continuous line). (*b*) Quantum regime for $\rho = 0.2$, $\delta = 1/\rho$, from equations (7), (8).

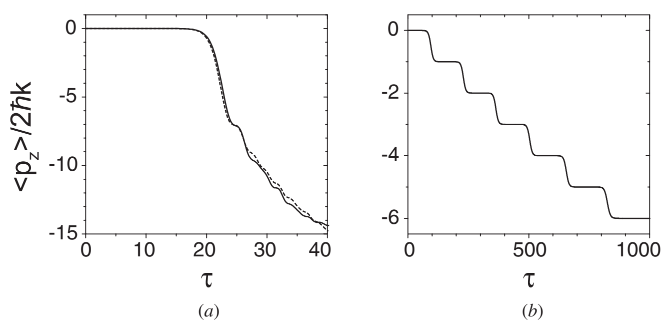

Figure 2. Average momentum $\langle p_z \rangle / 2\hbar k$ for superradiance. (*a*) Classical regime for $\kappa = 2$, $\rho = 10$, $\delta = 0$, from equations (1)–(3) (dashed line) and from equations (7), (8) (continuous line). (*b*) Quantum regime for $\kappa = 10$, $\rho = 2$, $\delta = 1/\rho$, from equations (7), (8).

CARL is the analogue of the coherent spontaneous emission regime predicted by Bonifacio and Preparata for an atomic two-level system [15]. The solution of equations (9)–(11) for $\kappa = \beta = \Delta = 0$ is $\langle p_z \rangle = (W - 1)/2 = -(2\hbar k)\{1 - \tanh^2[\sqrt{\rho/2}(\tau - \tau_0)]\}$, where $\tau_0 = \sqrt{2/\rho}\ln(\rho/2)$.

4. Bad-cavity limit and superradiance

In the 'bad-cavity' limit $\kappa \gg 1$, the emission is superradiant, i.e. $|A|^2 \approx \kappa^{-2} \propto \rho^2$ and the scattered power P_s is proportional to N^2 [16]. We define a classical superradiant regime for $\rho \gg \sqrt{2\kappa}$ and a quantum superradiant regime for $\rho < \sqrt{2\kappa}$. Figure 2(a) compares the solution of the classical (dashed line) and quantum (continuous line) models for $\kappa = 2$, $\rho = 10$ and $\delta = 0$. The average momentum is described by the approximated solution [15] $\langle p_z \rangle \approx -(2\hbar k)(\rho/\sqrt{2\kappa})\{1 + \tanh[(\tau - \tau_D)\sqrt{2\kappa}]\}$, where τ_D is a delay time depending on the initial conditions. On the contrary, in the quantum superradiant regime the average momentum decreases by discrete steps of recoil momentum $2\hbar k$, as can be seen in Figure 2(b), showing the solution of the quantum model for $\rho = 2$, $\kappa = 10$ and $\delta = 1/\rho$. The atoms, initially at $p_z = 0$, 'decay' to the lower momentum state $p_z = -(2\hbar k)$ emitting a superradiant hyperbolic-secant pulse and with average momentum $\langle p_z \rangle \approx -(2\hbar k)\{1 + \tanh[(\rho/2\kappa)(\tau - \tau'_D)]\}/2$, where τ'_D is a delay time.

Whereas in the classical regime the momentum changes of many quantum recoil units $2\hbar k$ during the superradiant process, in the quantum regime the scattering is sequential, with p_z changing each time by a single quantum step $2\hbar k$.

This regime has been observed in several experiments of superradiant Rayleigh scattering [8, 9], in good agreement with the CARL–BEC model where, however, a decoherence term must be added to describe the loss of coherence between the two motional states n and $n-1$ [11]. A strong interest for the quantum regime of CARL arises from the fact that, in the absence of decoherence, all the atoms can be transferred from a momentum state with p_z to a state with $p_z - 2\hbar k$, in a similar way to that usually done by the Bragg scattering technique. What is more appealing in CARL is that it is a spontaneous phenomenon starting from noise and, at least in the high-Q cavity limit, it can generate atom–atom or atom–photon entangled states [12, 18]. The persistence of entanglement also in the superradiant regime and in the presence of atomic decoherence is still an unresolved problem, which we are currently investigating.

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