The energy expression is particularly simple in this limit: it is characterized by only four parameters but nonetheless has a very large region of applicability. The other two limits, the SU(3) and O(6), can be related to axial rotational motion and gamma-soft rotor spectra, respectively. We refer to the extensive literature on this subject for further details at the end of this chapter.

3.3.3 Conclusion

The field of symmetries, be it just the structural symmetries that can be associated with certain flow patterns, or the symmetries that are more directly connected to the dynamics of the nucleon–nucleon interactions reflected in the Hamiltonian of the nuclear system, presents a very rich source of information. It has been and still is used as a major guiding principle for comprehending and bringing order into the interacting nuclear many-body system.

3.4 The Nuclear Structure Phase Diagram

3.4.1 Introduction

The atomic nucleus with its \( A \) nucleons is governed by a large number of degrees of freedom. From this multitude one can single out a rather small number of variables that determine the major characteristics and can also be regarded as external 'parameters', which, when altered, cause the atomic nucleus to respond in a specific way. These variables can then be used to characterize some major 'axes' defining a space in which one can explore and map the nuclear behavior as a multidimensional system. Moreover, these major 'axes' can be directly connected to recent research efforts and to the actual experimental possibilities for probing and detecting nuclear phenomena:

(i) As a first axis we can take the nuclear angular momentum. This can be influenced in heavy-ion reactions in which an ion is accelerated and impinges at grazing conditions on a target nucleus, thereby transferring a large amount of angular momentum. This can modify the motion of individual nucleons in an important way since they will start moving in a rotating deformed field.

(ii) A second axis can be related to the internal temperature and to heating the nuclear many-body system. Heating may cause the internal occupation of the various orbitals to be strongly modified and may give rise to dramatic changes in the internal structural 'organization' of nucleons inside the nucleus.

(iii) For a third axis, one moves outside of the region of beta stability and explores the edges of stability in the nuclear landscape. Here, a typical variable might be the ratio \((N - Z)/A\) (relative neutron excess). This

Fig. 3.21. (a) Schematic representation of nuclear structure in the space of three major axes describing some of the most important variables characterizing nuclear properties. The axes are the nuclear temperature axis, \( T \), the nuclear rotational degree of freedom, \( J \), and the neutron-to-proton ratio described via the variable \((N - Z)/A\) (relative neutron excess). (b) A more detailed version of the \((T, J, (N - Z)/A)\) axes partitioning nuclear structure properties. A variety of nuclear structure effects and the methods used to explore these three main directions are indicated. (Adapted from A. Richter (1993) Nucl. Phys. A 533, 417c, Fig. 2. Elsevier Science, NL, with kind permission)
3.4 The Nuclear Structure Phase Diagram

3.4.2 Behavior of Rapidly Rotating Nuclei

Almost spherical nuclei in the vicinity of closed shells in which nucleons are preferentially found in $0^+$ coupled pairs show high rigidity against deformation of any type. Before studying the properties of the nucleus when rapid rotation is imposed on its constituent nucleons, we shall bring in a technical box (Box IV) that explains the basic issues that describe a nucleus moving in a deformed field that is subsequently set into rotation.

**Box IV**

**Nuclear Deformation and Rapid Rotation**

When a nucleus is moving in an axially deformed quadrupole average field (e.g., a three-dimensional harmonic oscillator potential with two equal frequencies in the $x$, $y$ plane but a different one in the $z$-direction), the otherwise $2j + 1$-fold degeneracy is split into $j + 1$ two-fold degenerate orbitals (Fig. IV.1). Here only the projection of the single-particle angular momentum remains a good quantum number. In evaluating now the total energy of the nucleus as a function of the shape of the deformed potential, one may think of adding up all individual single-particle energies corresponding to the occupied states. This is not fully correct and a much used method relies on renormalizing the bulk energy to the liquid drop model value for the same deformation but now adding the local fluctuating energy term that arises from the non-uniform level distribution near to the Fermi level (Fig. IV.2).

This latter term is also called the shell-correction energy and when added to the global liquid drop term gives a good prescription for determining the total nuclear energy

$$E(\varepsilon_2) = E_{\text{liquid drop}}(\varepsilon_2) + \delta E(\varepsilon_2).$$  \hspace{1cm} (IV.1)

It allows one to determine equilibrium deformed shapes for many nuclei in regions where large quadrupole deformations appear: rare-earth nuclei, actinides, etc.
One can now add the external rotational term which will break the time-reversal invariance that was present in the non-rotating field. Depending on the orientation of the angular momenta in the various substates relative to the external rotation field (which is put along the z-axis for convenience), this splitting will also affect the total nuclear energy, which now becomes not only a function of deformation but also of rotational frequency (Fig. IV.3). Using the same Strutinsky method, the resulting expression is

\[ E(\varepsilon_2, \omega) = E_{\text{rig. drop}}(\varepsilon_2, \omega) + \delta E(\varepsilon_2, \omega), \]

which can be minimized in deformation space. One can thus follow the trajectory of a given nucleus in its minimum energy state as a function of both the deforming and rotating external agents. A very interesting result is that in, e.g., the rare-earth nuclei, minima have been obtained at high spin and near to an axis ratio of 2:1 (major to minor axis), called superdeformed states. Ample experimental evidence for such excitations has accumulated during recent years and some excellent review articles have been written on the subject [3,115]–[3,123].

As was discussed in Box IV, nuclei can acquire a superdeformed energy minimum at high rotational frequencies. We first give some examples of how the total energy evolves as a function of quadrupole deformation and as a function of angular momentum of the nucleus for \(^{152}\text{Ce}\) and \(^{152}\text{Dy}\) (Fig. 3.22). A more elaborate theoretical study of \(^{152}\text{Dy}\) is shown in Fig. 3.23 as a three-dimensional energy surface where, besides quadrupole deformation, the hexadecapole deformation degree of freedom is also taken into account. Figure 3.23 is constructed at angular momentum spin 80\(\hbar\) and exhibits the state-of-the-art in producing such landscapes.
3. Introducing the Atomic Nucleus: Nuclear Structure

**Total energy:** $^{152}_{66}\text{Dy}_{86}$

$I^+ = 80^+$ \( \beta_4 = \text{minim.} \)

![Three-dimensional total energy surface for $^{152}$Dy](image)

**Fig. 3.23.** Three-dimensional total energy surface for $^{152}$Dy (see also Fig. 3.22) at spin 80$. Besides quadrupole deformation, hexadecapole deformation also is allowed to occur. The color code indicates the total energy scale. The specific deformed, rotating shapes at the three distinct minima are also shown. (By courtesy of J. Dudek)

One may wonder how such large amounts of angular momentum can be given to a nucleus. A typical method is 'fusion-evaporation' which consists in accelerating a medium-heavy fragment into a target nucleus (we illustrate this, in Fig. 3.24, for the case of $^{49}$Ar accelerated into $^{124}$Sn at grazing angles and with high velocity such that the resulting compound nucleus (in this case $^{173}$Er) is set into rapid rotation. This system is in general "hot" and will evaporate a number of nucleons (in our example four neutrons) before ending up as a bound nucleus $^{169}$Er, which then cools further through the emission of gamma radiation. In the course of this cooling process, the emitted gamma rays contain information about the cooling route and the physics of the region the nucleus is passing through. In order to find out about this decay route one should detect all the gamma rays as efficiently as possible using gamma-spectroscopic tools. A special 4\( \pi \) geometry has been set up

**Fig. 3.24.** The various steps in the reaction $^{124}$Sn($^{49}$Ar,3n)$^{169}$Er: (i) the $^{49}$Ar + $^{124}$Sn initial reaction, (ii) the fusion of the two nuclei into a compound system $^{164}$Er, (iii) the emission of four neutrons from the rapidly spinning compound system, and (iv) the final cooling of $^{169}$Er via gamma-ray emission. (Adapted from J. Goldhaber (1991) LBL Research Review, Vol. 16, No. 1, p. 24, with kind permission)

in order to reach a very high efficiency in detecting information that will allow a reconstruction of the decay path. A big "ball" of highly efficient Ge detectors with BGO shielding is set up around the reaction point. A number of such systems are now active, e.g., Gammasphere (Fig. 3.25), which has been operating at Lawrence Berkeley Laboratory for quite some time and is now moving to Argonne National Laboratory. These detectors (containing up to 110 elementary units) are very expensive and used mainly by relatively large
(still rather small compared to a typical-sized high-energy experiment at one of the CERN LEP sites) collaborations amongst a number of laboratories. Thus, the ball is set up in such a way that it can be transported from one accelerator to another with the aim of studying different mass regions.

The first discovery of superdeformed bands in the decay of very rapidly rotating nuclei was made at the Daresbury Nuclear Structure Facility (which is now closed!!!) by a team headed by P. Twin working at the TESSA-3 spectrometer. States of a very particular nature were discovered in $^{120}$Dy after the $^{109}$Pd ($^{48}$Ca, $^{20}$) reaction. They form a very regular deformed rotor spectrum that is very rigid and stays visible over an extended region of spin values (Fig. 3.26). European research efforts using such spherical detectors include the EUROGAM and EUROBALL projects.

![Diagram of gamma sphere]

**Fig. 3.25.** Gamma sphere, the powerful new nuclear gamma detector array constructed at Berkeley. The hollow spherical array of germanium and bismuth germanate crystals surrounds a heavy metal target that can be bombarded with ion beams from a cyclotron or van de Graaff accelerator. The array's inner and outer diameters are 50 and 90 cm. The liquid nitrogen dewars that cool the Ge detector elements extend the total diameter of the system to 2 m.

**Box V**

**EUROBALL: Probing the Rapidly Rotating Nucleus**

At present, nuclear physicists are enjoying a most interesting and exciting period with the advent of the most recent generation of powerful gamma-ray spectrometers. In particular set-ups like EUROGAM (UK/France), GASP (Italy), GAMMASPHERE (USA) and, in the near future, EUROBALL, will be able to observe even the tiniest rotational motion of the atomic nucleus.

The total photoneak efficiency has gone up to about 10% and the gamma-ray detection efficiency has increased to $10^{-3}$ or better of the production cross section.

At present, at the Vivitron in Strasbourg, the newest implementation of EUROGAM, a full 4$\pi$ gamma multidetector system contains up to 30 large-volume Compton-suppressed Ge detectors, up to 24 new polarization sensitive BGO shielded clovers to study how a highly excited nucleus in a state of very rapid rotation finds its way to the most stable configuration when cooled to a state having, in a quantum mechanical sense, the lowest possible angular momentum.

On July 1, 1994 an agreement was made between six European countries (Denmark, France, Germany, Italy, Sweden, and the UK) to construct...
EUROBALL, the new gamma-ray spectrometer. This project will be the culmination of many years of collaborative efforts in this field of nuclear physics.

EUROBALL should become the most sensitive and sophisticated array of detectors. Its basic aim, as indicated before, is to understand how nucleons behave under extreme conditions of high excitation energy and rapid rotation. The newest array should be able to follow with unprecedented precision the way the nucleus dissipates its energy and rotational energy via gamma-ray emission.

The overall cost (about 20 million ECU) will be distributed between the various partners. The first places the apparatus will go to, at the anticipated date of finishing the array, will be Legnaro (near to Padua in Italy) and the CRN at Strasbourg in France.

The construction of the EUROBALL array with its unique capabilities will involve over a hundred scientists, engineers, and technical staff and it will provide a research facility not only for the participating teams but also for many outside users. It is a good example of cooperative efforts amongst a large number of laboratories to create a powerful and breathtaking facility on the threshold of the next millennium. It also proves that innovative technologies and forefront nuclear physics research can go hand-in-hand and can be supported in a combined effort over the borders of the contributing European countries.

Recent experiments in this field have now studied examples of nuclei in:
(i) the rare-earth region,
(ii) the Hg, Pb mass region and,
(iii) the medium-heavy mass A=80 and A=100 regions. Also the gamma transitions that connect these superdeformed structures to the nuclear structure in the slow rotation cold regime have been found and allow a good estimate of the excitation energy at which the superdeformed bands are formed.

Finally, one can even think of more exotic objects like hyperdeformed states with axes ratios of 3 : 1 (major to minor axis) for the axially symmetric case. Indications that such forms exist have been very recently. Furthermore, the additions of such large amounts of angular momentum will inevitably push the nucleus to the limits beyond which it cannot sustain more angular momentum and cause fission or destabilization due to the very strong rotation.

3.4.3 Heating the Atomic Nucleus: Towards Chaotic Motion

Moving outside the region of cold nuclei by successively increasing the nuclear excitation energy, the regular nuclear structure (coherent types of nuclear collective motion like vibrational and rotational excitations) will, as a general rule, become "dissipated" through its coupling to the many intrinsic nucleonic excitations. The simple angular momentum $0^+$ coupled pair correlations will become destroyed by the heating process. Looking only at the temperature and rotational types of degrees of freedom in the nuclear diagram, one sees, rather similar to the regular superconducting region on the $(T, H)$ diagram (temperature, external magnetic field), that a superfluid region can be isolated at low excitation energies and small angular momenta (Fig. 3.27). By either making the nucleus rotate very rapidly and/or heating the nucleus intrinsically, one can leave this region of superfluidity where correlated $0^+$ nucleon pairs dominate the properties of the interacting $A$-body system.

The general properties of such complex $A$-body systems can then, under certain conditions, start to exhibit chaotic features which are reflections of the basic interactions amongst the many nucleons inside the nucleus.

Before discussing some of the main features of this transition from more ordered motion, where symmetries in the interacting $A$-nucleon system dominate the nuclear structure properties, to more random distributions of nuclear excitations, we present a short technical box on statistical level distributions.

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Box VI

Statistical Level Distributions

A property that is especially sensitive to the interacting $A$-nucleon system is the nuclear level spacing distribution or level densities.

We start from a simple $2 \times 2$ model Hamiltonian in which the elements of the matrix