Pairing Matrix Elements and Pairing Gaps with Bare, Effective and Induced Interactions

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Xavier Viñas celebration 2017 workshop at Milano "Nuclear Structure and Astrophysical Applications"
Milano (during the last 20 years)

L. Viverit, A. Pastore, S. Baroni, F. Raimondi
P. Avogadro, A. Idini, G. Potel

“External” collaborations with

H. Esbensen, P. Schuck, X. Viñas
- Introduction

- Putting together the Bare Pairing Interaction and The Interaction Induced from the Exchange of Surface Modes
  \[ V_{\text{bare}}, \ V_{\text{ind}} \]

- Including Exchange of (volume) Spin Modes, Application to the proton-neutron (T=0) pairing.

- Including Fragmentation

- Evidences from data
Commonly used mean-field approach: HFB theory with Gogny force

Overall agreement with ‘experimental’ $^1S_0$ pairing gaps, but many open questions remain. Among them: connection with the bare force many-body effects detailed isotopic dependence spectroscopic factors


\[ V(r-r') = V_c \left( 1 - \eta \frac{\rho(r)}{\rho_0} \right) \delta^3(r-r') \]

\[ \eta = 0; 0.5 \text{ and } 1.0 \text{ (vol.; mix.; surf.)} \]

\( (E_{\text{cut}} = 60 \text{MeV}) \)

TABLE IV: RMS residuals of \( \Delta^{(3)}_n \) obtained in various models. All energies are in MeV. The last column shows the ratio of proton and neutron effective pairing strengths obtained through the optimization procedure. The mass predictions of the HFB-14 model [16] were taken from [51].

<table>
<thead>
<tr>
<th>Theory</th>
<th>pairing</th>
<th>residual neutrons</th>
<th>residual protons</th>
<th>( V_0^{\text{eff}}(p)/V_0^{\text{eff}}(n) )</th>
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<tr>
<td>Constant</td>
<td></td>
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<tr>
<td>( c/A^\alpha )</td>
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<tr>
<td>HFB+LN</td>
<td>mixed</td>
<td>0.23</td>
<td>0.28</td>
<td>1.11</td>
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<tr>
<td>HFB-14</td>
<td></td>
<td>0.46</td>
<td>0.44</td>
<td>1.10</td>
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</table>
Various effective forces in the pairing channel have been proposed, with different features (finite/zero range, density dependence...)
Unfortunately it is difficult to discriminate among them comparing with available data.

We want to follow a different strategy:

Start from an Hartree-Fock calculation with a ‘reasonable’ interaction. Then solve the pairing problem with a bare interaction in the $^1S_0$ channel. And finally add correlations beyond mean field.
We know that these correlations strongly renormalize the density of single-particle levels (effective mass) and their occupation factors (fragmentation), and we expect that they can have a large effect on pairing properties.
Pairing matrix elements and pairing gaps with bare, effective, and induced interactions

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Bare Pairing interaction: V-14 Argonne 1S$_0$

\[
\left( \begin{array}{cc}
\varepsilon - \varepsilon_F & \Delta \\
-\Delta & -(\varepsilon - \varepsilon_F)
\end{array} \right)
\left( \begin{array}{c}
U^i \\
V^i
\end{array} \right) = E_i \left( \begin{array}{c}
U^i \\
V^i
\end{array} \right),
\]

(1)

\[
\Delta_{a_1a_2} = -\frac{1}{2} \sum_{b_1} \sum_{b_2} U^i_{b_1} V^i_{b_2} \langle a_1 \tilde{a}_2 | \nu(12) | b_1 \tilde{b}_2 \rangle.
\]

(2)

Fig. 1. (a) The pairing gaps obtained in a Wigner-Seitz cell of radius equal to 15 fm, and $\varepsilon_F = 20$ MeV, containing the nucleus Sn whose bound states are described in terms of a Woods-Saxon potential with a radius of 6.26 fm and a depth of 45.5 MeV, are shown by a solid and dash-dotted curve, respectively for a diffusivity $a = 0.67$ fm and $a = 0$ fm. The gaps obtained for the discrete states have been averaged over 4 MeV. The gaps obtained for uniform neutron matter are shown by the dashed line. (b) The pairing gaps obtained for the bound states in the potential used already in connection with Fig. (a) with the diffusivities $a = 0.67$ fm and 0 fm are shown by filled circles and by open triangles respectively. The Fermi energy has been set at the values $\varepsilon_F = -7.2$ MeV (for $a = 0.67$ fm) and $\varepsilon_F = -10.5$ MeV (for $a = 0$ fm), appropriate for the nucleus $^{120}$Sn. Also shown is the BCS result (small crosses).
\[
\Delta_k = -\frac{1}{2\pi^2} \int_0^\infty dk' k'^2 V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} = U_k V_k
\]
Fig. 1. The pairing gaps in the $^1S_0$ channel and the corresponding pairing potentials $V(k_F, k)$ as functions of the momentum $k$ for several fixed Fermi-momenta $k_F$. The black and grey lines refer to the $V_{\text{low}-k}$ and the Gogny interaction, respectively. The solid, dashed, dashed-dotted and dotted lines correspond to the values of $k_F$ equal 0.2, 0.6, 1.0 and 1.4 fm$^{-1}$.

Fig. 2. The pairing gaps in the $^1S_0$ channel as functions of Fermi-momentum (density) of the matter for two effective interactions. The heavy and light lines refer to the $V_{\text{low}-k}$ and the Gogny interaction, respectively. The solid lines correspond to the non-interacting single-particle spectrum, the dashed and dashed-dotted lines—to renormalized single particle spectrum in symmetric nuclear matter and neutron matter, respectively. The single particle spectra are computed in the Hartree–Fock theory for the Gogny interaction and the Brueckner–Hartree–Fock theory for the $V_{\text{low}-k}$ interaction.

FIG. 1. State dependent pairing gap $\Delta_\nu$ [cf. Eq. (3)] for the nucleus $^{120}_{50}$Sn, calculated making use of the induced interaction defined in Eq. (2) (cf. inset, where particles are represented by arrowed lines and phonons by a wavy line).
In the box: finite nucleus, neutron star, infinite matter

Fig. 30. Comparison between the gap obtained requiring that the radial single-particle wavefunctions $u_L(r)$ vanish at the edge of the spherical box for all orbital angular momenta $L$ (crosses), or that $u_L(R)$ (for $L$ even) and $du_L/dR$ (for $L$ odd) vanish at the edge of the box (stars), or that $u_L(R)$ (for $L$ odd) and $du_L/dR$ (for $L$ even) vanish at the edge of the box (circles). The solid curve shows the pairing gap calculated in infinite neutron matter.
FIG. 5. The nucleus $^{120}$Sn. The semiclassical matrix elements of the induced interaction, calculated according to Eq. (11) (dash-dotted curve), are compared with the matrix elements of the Gogny force (solid curve, cf. Fig. 3) and with those of the $u_{\text{low-}k}$ interaction (dashed curve). Calculations are performed with $m_k = m$ and with the same Woods-Saxon potential used in Figs. 1 and 3.

FIG. 7. Pairing gaps calculated in neutron matter as a function of the Fermi momentum, obtained with the Gogny interaction (solid line), the Argonne $u_{14}$ potential (dash-dotted line), and the $u_{\text{low-}k}$ potential (dashed line). The bare effective mass has been used in the calculation.
FIG. 3. The nucleus $^{120}$Sn. The same as Fig. 2 (i.e., also with $m_k = m$) but for the fact that the single-particle wave functions have been calculated making use of the Woods-Saxon potential including the spin-orbit term already used for Fig. 1, where the diagonal matrix elements have already been shown. The nondiagonal matrix elements are plotted here at an energy $e_\nu$, which is the average between the energies of the initial and final states.

FIG. 1. The nucleus $^{120}$Sn. Diagonal pairing matrix elements of the induced interaction (upper panel, solid diamonds) and of the Gogny force (lower panel, solid circles), displayed as a function of the single-particle energy, $e_\nu$, of the state $\nu$ calculated using the bare nucleon mass and the single-particle wave functions of a Woods-Saxon potential with standard parameters (depth $V_0 = -49$ MeV, diffusivity $a = 0.65$ fm, and radius $R_0 = 6.16$ fm), including the spin-orbit term, parametrized according to Ref. [21]. Also shown by means of vertical lines is the position of the Fermi energy, $e_F = -9.1$ MeV. Note the different scale in the two figures.
No fragmentation

Only surface (density) modes exchanged
Repulsive Vind by $S=1$ modes exchange
Well known from Neutron Matter studies.
Microscopic calculation of the matrix elements of the induced interaction in spherical open-shell nuclei including spin modes

Vibrations are calculated with QRPA and SKM* interaction

Particle-hole interaction:

\[ v_{ph}(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}')\{[F_0 + F'_0 \vec{\tau} \cdot \vec{\tau}'] + [(G_0 + G'_0 \vec{\tau} \cdot \vec{\tau}') \vec{\sigma} \cdot \vec{\sigma}']\} \]

\[ \nu_{ph}(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') \left\{ [F_0 + F_0' \vec{\tau} \cdot \vec{\tau}'] + \left[(G_0 + G_0' \vec{\tau} \cdot \vec{\tau}') \vec{\sigma} \cdot \vec{\sigma}' \right]\right\} \]

\[
\langle \nu'\nu'\bar{\nu}'|\nu_{ind}|\nu\nu\nu\bar{\nu}\rangle = \sum_{J^\pi Mi} \frac{2(f + g)^{\nu'm'}_{vm;JM}}{E_0 - E_{int}} \frac{(f - g)^{\nu'm'}_{vm;JM}}{E_0 - E_{int}}
\]

(a)

\[
f^{\nu'm'}_{vm;JM} = i^{l-l'} \langle j'm'||(i)^J Y_{JM} | jm \rangle \times \int dr \varphi_{\nu'} \left[ (F_0 + F_0')\delta \rho^i_{J\pi n} + (F_0 - F_0')\delta \rho^i_{J\pi p} \right] \varphi_{\nu}
\]

(b)

\[
g^{\nu'm'}_{vmJ^\pi Mi} = \sum_{L=J-1}^{J+1} i^{l-l'} \langle j'm'||(i)^L [Y_L \times \sigma]_{JM} | jm \rangle \times \int dr \varphi_{\nu'} \left[ (G_0 + G_0')\delta \rho^i_{J\pi Ln} + (G_0 - G_0')\delta \rho^i_{J\pi Lp} \right] \varphi_{\nu}
\]
Microscopic calculation of the matrix elements of the induced interaction

\[ S=0 \text{ (attractive)} \]

\[ f_{vm;J\pi Mi}^{\nu \nu'} = i^{l-l'} \langle j' m' | (i)^J Y_{JM} | j m \rangle \]

\[ \times \int dr \varphi_{\nu'} [(F_0 + F'_0) \delta \rho_{j\pi n}^i + (F_0 - F'_0) \delta \rho_{j\pi p}^i] \varphi_{\nu} \]

\[ S=1 \text{ (repulsive)} \]

\[ g_{vmJ\pi Mi}^{\nu \nu'} = \sum_{L=J-1}^{J+1} i^{l-l'} \langle j' m' | (i)^L [Y_L \times \sigma]_{JM} | j m \rangle \]

\[ \times \int dr \varphi_{\nu'} [(G_0 + G'_0) \delta \rho_{j\pi Ln}^i \]

\[ + (G_0 - G'_0) \delta \rho_{j\pi Lp}^i] \varphi_{\nu} \]
FINITE NUCLEI

\(^{120}\text{Sn}\):

The induced interaction arising from the coupling to surface and spin modes is attractive and leads to a pairing gap of about 0.7 MeV (50% of the experimental value). Excluding the coupling to spin modes, the gap increases to about 1.1 MeV.

One must then add the bare interaction.

FIG. 4. (a) The state-dependent pairing gap as a function of the single-particle energies obtained by solving the BCS equations associated with the total induced interaction matrix elements; (b) same as (a) but for the matrix elements associated with the spin-independent part of the particle-hole interaction.
$T=0$ pairing

$T=0$ (attractive)

$T_z = 0$

$T=1$ (repulsive)

$T_z = +1$
T=0 pairing problem

Vonderfecht et al. PLB253,1 (1991)
Baldo et al. PLB283, 8 (1992)
Proton-neutron pairing in symmetric nuclear matter 3S1-3D1

Using CDBonn

Dashed lines: quasiparticle poles

Solid lines: dressed nucleons

No pairing at saturation density!!!!

Dickhoff PRC72(2005)054313

Lombardo preliminary results: No SD pairing at about 0.14fm^-3
Strength of $T=0$ pairing interaction can have a large effect on beta-decay lifetimes

Y.F. Niu et al.,
PRC 94 (2016) 064328 and to be published
Microscopic calculation and local approximation of the spatial dependence of the pairing field with bare and induced interactions

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\[ v^\delta(\vec{r}_1, \vec{r}_2) = v_0 \left[ 1 - \eta \left( \frac{\rho \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right)}{\rho_0} \right)^\alpha \right] \delta(\vec{r}_1 - \vec{r}_2) \]

FIG. 11. (Color online) (a) Spatial dependence of the different local pairing interactions introduced in this work to simulate the local pairing gaps [cf. Eq. (19)] obtained with the corresponding microscopic, nonlocal interactions: bare+induced interaction \( v_{\text{Arg+ind}} \) (corresponding to the parameters \( \alpha = 2.0, \eta = 1.32 \); cf. Table II); bare+induced interaction neglecting the spin-dependent part (\( v_{\text{Arg+ind}, G_0 = 0, G_0' = 0} \), \( \alpha = 1.79, \eta = 1.0 \); cf. Table V in Appendix B); bare \( v_{14} \) interaction \( v_{\text{Arg}}(\alpha = 0.66, \eta = 0.84 \); cf. Table II); Gogny interaction \( v_{\text{Gogny}}(\alpha = 0.51, \eta = 0.63 \); cf. Table IV in Appendix A). (b) The spatial dependence of the bare+induced interaction with and without the spin-dependent part of the induced interaction, already shown in (a), compared with the volume, surface, and mixed interaction of Ref. [26] (see text).
\[ v_{\text{attr}}^G(R_{\text{c.m.}}, r_{12}) = -b_{\text{attr}} \cdot \exp\left\{-\left[(r_{12} - c)/a_{\text{attr}}\right]^2\right\} \]
\[ c = 2|R_{\text{nucl}} - R_{\text{c.m.}}| \]
\[ a_{\text{attr}} = 2 \text{ fm} \]

\[ v_{\text{rep}}^G(R_{\text{c.m.}}, r_{12}) = b_{\text{rep}} \cdot \exp\left[-\left(r_{12}/a_{\text{rep}}\right)^2\right] \Theta(R_{\text{c.m.}} - R_0) \]
\[ a_{\text{rep}} = 3.5 \text{ fm} \]

FIG. 12. (Color online) (a) The values of the parameter \(b_{\text{attr}}\), obtained by fitting the Gaussian interaction [Eq. (23)], shown as a function of the center of mass \(R_{\text{c.m.}}\) (filled dots) and compared with the function \(0.14R_{\text{nucl}} \frac{dU(r)}{dR_{\text{c.m.}}}\) (solid line). (b) The values of the parameter \(b_{\text{rep}}\), obtained by fitting the Gaussian interaction [Eq. (23)], also shown as a function of the center of mass \(R_{\text{c.m.}}\) (filled squares).
Medium polarization and finite size effects on the superfluidity of the inner crust of neutron stars

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\textsuperscript{b} INFN, Sezione di Milano, via Celoria 16, 20133 Milano, Italy.
\textsuperscript{c} TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C. V6T 2A3, Canada
\textsuperscript{d} Departamento de Fisica Aplicada III, Escuela Tecnica Superior de Ingenieros, Camino de los Descubrimientos s/n, 41029 Sevilla, Spain.
\textsuperscript{e} The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, 2100 Copenhagen \O, Denmark.
(Dated: July 6, 2017)

The $^1S_0$ pairing gap $\Delta$ associated with the inner crust of a neutron star is calculated, taking into account the coexistence of the nuclear lattice with the sea of free neutrons (finite size effects), as well as medium polarization effects associated with the exchange of density and spin fluctuations. Both effects are found to be important and to lead to an overall quenching of the pairing gap. This result, whose quantitative value is dependent on the effective interaction used to generate the single-particle levels, is a consequence of the balance between the attractive (repulsive) induced interaction arising from the exchange of density (spin) modes, balance which in turn is influenced by the presence of the protons and depends on the single-particle structure of the system.
Neutron matter without and with impurities (Neutron Star)

FIG. 8: Pairing gap calculated at the Fermi energy for the
INCLUDING FRAGMENTATION

Two step calculation:

i) HFB with Vbare in a large space ---> “unperturbed” qp-properties

ii) Using the above HFB/V_bare results, diagonalize the PVC effects using the One Shell Approximation.

Approximation justified by the large scale separation between induced and bare interaction.
\[
\left( \epsilon_a - \epsilon_F \right) + \Sigma_{a(n)}^{11} - \left( \epsilon_a - \epsilon_F \right) + \Sigma_{a(n)}^{22} = \tilde{E}_{a(n)} \begin{pmatrix} \tilde{u}_{a(n)} \\ \tilde{v}_{a(n)} \end{pmatrix}
\]

\[
\sum_{a(n)}^{11 \text{pho}} = \sum_{b,m,\lambda,\nu} \frac{(f+g)^2 \left[ ab(m) \lambda \nu \right] S^2}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar \omega_{\lambda \nu}} + \sum_{b,m,\lambda,\nu} \frac{(f-g)^2 \left[ ab(m) \lambda \nu \right] P^2}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar \omega_{\lambda \nu}}
\]

\[
S = u_a u'_b - v_a v'_b
\]

\[
P = u_a v'_b + v_a u'_b
\]

\[
G = S \times P
\]

\[
\tilde{\Sigma}_{a(n)}^{12,\text{pho}} = \Delta_{a}^{\text{BCS}} + \tilde{\Sigma}_{a(n)}^{12,\text{pho}}
\]

Terasaki et al.
(2j_a+1)\bar{\Delta}_{a(n)} = -Z_{a(n)} \sum_{b,m} V_{\text{eff}}[a(n)b(m)] \frac{\bar{\Delta}_{b(m)}}{2\tilde{E}_{b(m)}}

V_{\text{eff}}[a(n)b(m)] = V_{\text{bare}}[ab] + V_{\text{ind}}[a(n)b(m)]

V_{\text{ind}}[a(n)b(m)] = \sum_{j_z,j_z'} <a(n);j_z-j_z|V_{\text{ind}}|b(m);j_z'-j_z'>

\langle a(n);j_z-j_z|V_{\text{ind}}|b(m);j_z'-j_z'\rangle = \sum_{LJM} (f+g)^{LJM} (f-g)^{LJM} \times \left[ \frac{1}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar\omega_{\lambda\nu}} - \frac{1}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar\omega_{\lambda\nu}} \right].

Z_{a(n)} = \left(1 - \frac{\Sigma_{a(n)}^{11} - \Sigma_{a(-n)}^{11}}{2\tilde{E}_{a(n)}} \right)^{-1}

N_{a(n)} = x_{a(n)}^2 + y_{a(n)}^2

\tilde{E}_{a(n)} = \sqrt{\tilde{\epsilon}_{a(n)}^2 - \epsilon_F^2 + \bar{\Delta}_{a(n)}^2}

\tilde{\epsilon}_{a(n)} - \epsilon_F = Z_{a(n)} \left[(\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{a(n)}^{\text{even}} \right].

T=1\text{(time reversal)}\text{BCS with Self energy effects}

A.Idini, et al
PRC85,14331
COMPARISON WITH DATA

Open-shell nuclei around $^{50}\text{Sn}$

Which mean field?

Observables:

- Low-energy quasiparticle spectrum
- Multiplet splitting
- Electromagnetic transition strength
- One-neutron transfer reactions
- Two-nucleon transfer cross sections
- Pairing gaps
Pairing gaps
120Sn

excitation spectrum
odd neighbors

$E \text{ [MeV]}$

$\begin{align*}
&d_{5/2} \\
g_{7/2} \\
s^{1/2} \\
&h_{11/2} \\
d_{3/2}
\end{align*}$

HF  |  HFB  |  NFT  |  $^{119}\text{Sn}$  |  $^{121}\text{Sn}$
Transfer reactions: $^{120}\text{Sn}(d,p)^{121}\text{Sn}$
Transfer reactions: \(^{120}\text{Sn}(p,d)^{119}\text{Sn}\) (\(5/2^+\))

<table>
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<th>(\epsilon_i)</th>
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<th>SLy4 ((d_{5/2} \text{ shift}))</th>
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<td>(\sigma)</td>
<td>(d\sigma/d\Omega)</td>
<td>(\epsilon_i)</td>
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<td>1562</td>
<td>0.13</td>
<td>0.16</td>
<td>1910</td>
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<tr>
<td>1730</td>
<td>0.16</td>
<td>0.18</td>
<td>(7.93 \pm 2)</td>
</tr>
</tbody>
</table>

![Graphs](image)
Two particle transfer

\[ ^{120}\text{Sn}(p,t) \quad E_p=21 \text{ MeV} \]

\[ \frac{d\sigma}{d\Omega} \quad (\text{mb/sr}) \]

\[ \sigma_{exp}=2250\pm338 \pm 14 \text{ mb} \]

\[ [7.6^\circ < \theta < 69.7^\circ] \]

Potel et al. PRC96(2017)034606
The splitting would be zero without PVC, and is highly independent of the used mean field: ESSENTIALLY IT IS A MEASURE OF THE PVC VERTEX
Electromagnetic E2 Transitions

All of them would be zero without PVC!
(β2)o = 0.12    Go = 0.22 MeV mk = 0.7
About the necessity of $V_{\text{ind}}$ in Nuclear Structure Studies, note that it appears when BMF calculations are performed INEVITABLY.....

meaning that if you are not aware of $V_{\text{ind}}$, and you use empirical pairing interactions in your BMF calculations, as a matter of fact your results (whatever) are associated to a too large pairing gap.
Future developments

- $V_{\text{ind}}$ in larger spaces: coupling to continuum.

  Some experience in pair addition/removal cases
  In light halo nuclei $^{11}\text{Li}$.

- $T=0$ pairing in finite nuclei.
\[
\begin{align*}
\left( (\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{a(n)}^{11} \right) & \left( (\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{a(n)}^{12} \right) \\
- (\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{a(n)}^{22} & \right) \left( \tilde{u}_{a(n)} \right) = \tilde{E}_{a(n)} \left( \tilde{u}_{a(n)} \right)
\end{align*}
\]

\[
\Sigma_{a(n)}^{11\text{pho}} = \sum_{b,m,\lambda,\nu} \frac{(f+g)^2 \left[ ab(m)\lambda\nu \right] S^2}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar \omega_{\lambda\nu}} \\
+ \sum_{b,m,\lambda,\nu} \frac{(f-g)^2 \left[ ab(m)\lambda\nu \right] P^2}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar \omega_{\lambda\nu}}
\]

\begin{align*}
S &= u_{a'b} v_{a'b'} \\
P &= u_{a'b} v_{a'b'} + v_{a'b} u_{a'b'} \\
G &= S \ast P
\end{align*}

\[
\Sigma_{a(n)}^{12\text{pho}} = - \sum_{b,m,\lambda,\nu} \frac{(f^2-g^2)[ab(m)\lambda\nu] \ G}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar \omega_{\lambda\nu}} \\
\times \left[ \frac{1}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar \omega_{\lambda\nu}} \right] \\
- \frac{1}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar \omega_{\lambda\nu}} \\
\Delta_{a\text{BCS}} + \tilde{\Sigma}_{a(n)}^{12,\text{pho}}
\]

Terasaki et al.
\[(2j_a+1)\tilde{\Delta}_{a(n)} = -Z_{a(n)} \sum_{b,m} V_{\text{eff}}[a(n)b(m)] N_{b(m)} \frac{\tilde{\Delta}_{b(m)}}{2\tilde{E}_{b(m)}} \]

\[V_{\text{eff}}[a(n)b(m)] = V_{\text{bare}}[ab] + V_{\text{ind}}[a(n)b(m)]\]

\[V_{\text{ind}}[a(n)b(m)] = \sum_{j_z,j_z'} <a(n);j_z-j_z'|V_{\text{ind}}|b(m);j_z'-j_z'>\]

\[<a(n);j_z-j_z'|V_{\text{ind}}|b(m);j_z'-j_z'> = \sum_{LJM} (f+g)_{LJM} (f-g)_{LJM} x \left[ \frac{1}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar\omega_{\lambda\nu}} - \frac{1}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar\omega_{\lambda\nu}} \right] \]

\[Z_{a(n)} = \left( 1 - \frac{\Sigma_{a(n)}^{11} - \Sigma_{a(-n)}^{11}}{2\tilde{E}_{a(n)}} \right)^{-1}\]

\[\tilde{E}_{a(n)} = \sqrt{(\tilde{\epsilon}_{a(n)} - \epsilon_F)^2 + \tilde{\Delta}_{a(n)}^2}\]

\[\tilde{\epsilon}_{a(n)} - \epsilon_F = Z_{a(n)} \left[ (\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{a(n)}^{\text{even}} \right] \]

\[N_{a(n)} = x_{a(n)}^2 + y_{a(n)}^2\]

\[T=1(\text{time reversal})\text{BCS with Self energy effects}\]

A. Idini, et al
PRC85, 14331
Collegamento con neutron matter – role of spin fluctuations – Dickhoff+Polls - T=0 pairing

Le slides seguenti avevano materiale su questi argomenti, bisogna scegliere le migliori
T=0 pairing problem

\[ \text{Minimum gap (MeV)} \]

\[ k_F \text{ (fm}^{-1}) \]

\[ {^3}_S_1 - {^3}_D_1 \]

\[ {^1}_S_0 \]
Proton-neutron pairing in symmetric nuclear matter

\[ ^3S_1-^3D_1 \]

Using CDBonn

Dashed lines: quasiparticle poles

Solid lines: dressed nucleons

No pairing at saturation density!!!!

Dickhoff talk
A similar scheme could be adopted to calculate the $T=0$ pairing gap in finite nuclei.
Microscopic calculation of the matrix elements of the induced interaction in spherical open-shell nuclei including spin modes

Vibrations are calculated with QRPA and SKM* interaction

Particle-hole interaction:

Finirei qua..
PAIRING GAP IN FINITE NUCLEI

Medium effects **increase** the gap in $^{120}$Sn and bring it in agreement with experiment.

---

PAIRING GAP IN NEUTRON MATTER

Medium effects **decrease** the gap.

---


C. Shen et al., PRC 67(2003) 061302
Pairing interaction in neutron and nuclear matter and exchange of p.h. excitations

\[ V_{\text{pairing}} = \begin{align} & k\phi \quad - \quad \bar{k}\phi \\ & k \quad k \\ \end{align} + \begin{align} & k\phi \quad \bar{k}\phi \\ & k_1 \quad k_2 \\ \end{align} + \begin{align} & k\phi \quad \bar{k}\phi \\ & k_2 \quad k_3 \\ \end{align} \]
antiscreening
\[ V_{\text{ind}} > V_{\text{dir}} \]

screening
\[ V_{\text{ind}} < V_{\text{dir}} \]

Vibrations are calculated with QRPA and SKM\* interaction.

Particle-hole interaction:

Microscopic calculation of the matrix elements of the induced interaction in spherical open-shell nuclei including spin modes.

Microscopic calculation of the matrix elements of the induced interaction

\[ S=0 \text{ (attractive)} \]

\[ f_{vm;J^\pi_M i}^{j'm'} = i^{l-l'} \langle j'm'|(i)^J Y_{JM}|jm \rangle \]
\[ \times \int dr \varphi_{v'}[(F_0 + F_0')\delta \rho^{i}_{J^\pi n} + (F_0 - F_0')\delta \rho^{i}_{J^\pi p}]\varphi_v \]

\[ S=1 \text{ (repulsive)} \]

\[ g_{vmJ^\pi_M i}^{j'm'} = \sum_{L=J-1}^{J+1} i^{l-l'} \langle j'm'|(i)^L Y_L \times \sigma_{JM}|jm \rangle \]
\[ \times \int dr \varphi_{v'}[(G_0 + G_0')\delta \rho^{i}_{J^\pi Ln} \]
\[ + (G_0 - G_0')\delta \rho^{i}_{J^\pi Lp}]\varphi_v \]
FINITE NUCLEI

$^{\text{120}}$Sn:

The induced interaction arising from the coupling to surface and spin modes is attractive and leads to a pairing gap of about 0.7 MeV (50\% of the experimental value). Excluding the coupling to spin modes, the gap increases to about 1.1 MeV.

One must then add the bare interaction.

FIG. 4. (a) The state-dependent pairing gap as a function of the single-particle energies obtained by solving the BCS equations associated with the total induced interaction matrix elements; (b) same as (a) but for the matrix elements associated with the spin-independent part of the particle-hole interaction.
Crucial: the surface nature of density modes. This assures an important overlap between the transition density and the single-particle wave-function at the Fermi energy.

Why such a difference with neutron matter?

\[ (-)^{M+J} \sum_{L=J-1}^{J+1} (j^m') (j^j Y_{j-M} | jm) \int dr \varphi_j(r) \varphi_j(r) \xi_{JL}(r), \]

\[ \xi_{JL}(r) = (-)^{J+M+1} \sum_{L=J-1}^{J+1} (j^m') (j^j Y_{J-M} | jm) \int dr \varphi_j(r) \varphi_j(r) \xi_{JL}(r). \]
G-matrix

Gogny force

SkM* force

Density dependence of Landau parameters (at k=0)
The proton-neutron interaction in the particle vibration coupling plays an essential role. If we cancel it, a net repulsive effect is obtained for the induced interaction.

Landau parameters of SkM* force in $^{120}\text{Sn}$

Strong difference between induced interaction in neutron and nuclear matter

Why such a difference with neutron matter?
Attractive and repulsive contributions of medium fluctuations to nuclear superfluidity

G. Gori, F. Ramponi, F. Barranco, P. F. Bortignon, R. A. Broglia, G. Colò and E. Vigezzi

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(Received 10 December 2003; published 18 July 2005)

Oscillations of mainly surface character ($S = 0$ modes) give rise, in atomic nuclei, to an attractive (induced) pairing interaction, while spin ($S = 1$) modes of mainly volume character generate a repulsive interaction, the net effect being an attraction which accounts for a sizeable fraction of the experimental pairing gap. Suppressing the particle-vibration coupling mediated by the proton degrees of freedom, i.e., mimicking neutron matter, the total surface plus spin-induced pairing interaction becomes repulsive.

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PACS number(s): 21.30.Fe, 21.60.Jz, 21.65.+f, 27.60.+j
\[ v_{ph}(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') \left[ [F_0 + F'_0 \bar{\tau} \cdot \bar{\tau}'] + [(G_0 + G'_0 \bar{\sigma} \cdot \bar{\sigma}')] \right] \]

\[ \langle \nu' \nu' m' m'|v_{ind}|\nu m \nu m \rangle = \sum_{J^\pi M_i} \frac{2(f + g)_{\nu m; J^\pi M_i}^\nu m' (f - g)_{\nu m; J^\pi M_i}^\nu m'}{E_0 - E_{\text{int}}} \]

\[ f_{\nu m; J^\pi M_i}^{\nu' m'} = i^{l - l'} \langle j' m'|i)^{J} Y_{JM} |j m \rangle \]

\[ \times \int dr \varphi_{\nu'} \left[ (F_0 + F_0') \delta \rho_{J^\pi_n}^i + (F_0 - F_0') \delta \rho_{J^\pi p}^i \right] \varphi_{\nu} \]

\[ g_{\nu m J^\pi M_i}^{\nu' m'} = \sum_{L = J - 1}^{J + 1} i^{l - l'} \langle j' m'|i)^{L} [Y_L \times \sigma]_{JM} |j m \rangle \]

\[ \times \int dr \varphi_{\nu'} \left[ (G_0 + G'_0) \delta \rho_{J^\pi L_n}^i + (G_0 - G'_0) \delta \rho_{J^\pi L_p}^i \right] \varphi_{\nu} \]
FIG. 1. Generalized Landau parameters associated with the interaction SkM* defining the strength of the particle-hole interaction in the isoscalar ($F_0$), isovector ($F'_0$), spin isoscalar ($G_0$), and spin isovector ($G'_0$) channels. In the inset, the functions $F_0 + F'_0$ ($n$-$n$ interaction), $F_0 - F'_0$ ($n$-$p$), $G_0 + G'_0$ ($n$-$n$), and $G_0 - G'_0$ ($n$-$p$) are also shown.
FIG. 3. Diagonal induced pairing matrix elements resulting from the exchange of phonons with natural parity (filled circles) and those resulting from the exchange of phonons with non-natural parity vibrations (empty circles), displayed as a function of the energy $\varepsilon_v$ of the single-particle state $\varepsilon_v$.

FIG. 6. The diagonal matrix elements associated with the exchange of phonons of natural (filled circles) and non-natural (open circles) parity as a function of the energy of the single-particle states. The proton part of the phonon wave functions was not included in the calculation.
FIG. 4. (a) The state-dependent pairing gap as a function of the single-particle energies obtained by solving the BCS equations associated with the total induced interaction matrix elements; (b) same as (a) but for the matrix elements associated with the spin-independent part of the particle-hole interaction.
T=0 pairing problem

Vonderfecht et al. PLB253, 1 (1991)
Baldo et al. PLB283, 8 (1992)
Proton-neutron pairing in symmetric nuclear matter
\(3S_1-3D_1\)

Using CDBonn

Dashed lines: quasiparticle poles

Solid lines: dressed nucleons

No pairing at saturation density!!!!

Dickhoff et al.
Slides che non mostrerei
Quasiparticle renormalization and pairing correlations in spherical superfluid nuclei

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(Received 25 May 2011; revised manuscript received 15 November 2011; published 31 January 2012)

Within the framework of nuclear field theory (NFT), the spectrum of atomic nuclei is described in terms of collective and quasiparticle degrees of freedom, that is, of elementary modes of excitation that are directly related to experiment and of their coupling, whose strength and form factors are the basic ingredients entering in the calculations of absolute cross sections of inelastic and of one- and two-particle transfer reactions. We present a detailed discussion of the solution of the Dyson equation, also known as the Nambu-Gor’kov equations in the case of a superfluid system, which propagates medium polarization processes calculated making use of NFT to all orders of perturbation, resulting in the dressing of quasiparticles and in the induced pairing interaction. The formalism is applied to the superfluid nucleus 120Sn. Results concerning the low-energy spectrum, that is, the quasiparticle strength distribution of the neighboring odd-A nuclei 119Sn and 121Sn, are presented and compared with the experimental findings.
General (HFB like) beyond time-reversal BCS is needed in coupling to continuum and in non-homogeneous systems.
Pairing interaction: V-14 Argonne 1S\textsubscript{0}

\[
\begin{pmatrix}
\varepsilon - \varepsilon_F & \Delta \\
-\Delta & -(\varepsilon - \varepsilon_F)
\end{pmatrix}
\begin{pmatrix}
U_i^i \\
V_i^i
\end{pmatrix}
= E_i
\begin{pmatrix}
U_i^i \\
V_i^i
\end{pmatrix},
\]

(1)

\[
\Delta_{a_1a_2} = -\frac{1}{2} \sum_{b_1} \sum_{b_2} U_{b_1}^i V_{b_2}^i \langle a_1 \tilde{a}_2 | \nu(12) | b_1 \tilde{b}_2 \rangle.
\]

(2)

Fig. 1. (a) The pairing gaps obtained in a Wigner-Seitz cell of radius equal to 15 fm, and \( \varepsilon_F = 20 \text{ MeV} \), containing the nucleus Sn whose bound states are described in terms of a Woods-Saxon potential with a radius of 6.26 fm and a depth of 45.5 MeV, are shown by a solid and dash-dotted curve, respectively for a diffusivity \( a = 0.67 \text{ fm} \) and \( a = 0 \text{ fm} \). The gaps obtained for the discrete states have been averaged over 4 MeV. The gaps obtained for uniform neutron matter are shown by the dashed line. (b) The pairing gaps obtained for the bound states in the potential used already in connection with Fig. (a) with the diffusivities \( a = 0.67 \text{ fm} \) and 0 fm are shown by filled circles and by open triangles respectively. The Fermi energy has been set at the values \( \varepsilon_F = -7.2 \text{ MeV} \) (for \( a = 0.67 \text{ fm} \)) and \( \varepsilon_F = -10.5 \text{ MeV} \) (for \( a = 0 \text{ fm} \)), appropriate for the nucleus \(^{120}\text{Sn}\). Also shown is the BCS result (small crosses).
\[ \Delta_k = -\frac{1}{2\pi^2} \int_0^\infty dk' k'^2 V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} = U_k V_k \]
**Effective forces: $V_{\text{low-k}}$ vs. Gogny**

Fig. 1. The pairing gaps in the $^1S_0$ channel and the corresponding pairing potentials $V(k_F, k)$ as functions of the momentum $k$ for several fixed Fermi-momenta $k_F$. The black and grey lines refer to the $V_{\text{low-k}}$ and the Gogny interaction, respectively. The solid, dashed, dashed-dotted and dotted lines correspond to the values of $k_F$ equal 0.2, 0.6, 1.0 and 1.4 fm$^{-1}$.

Fig. 2. The pairing gaps in the $^1S_0$ channel as functions of Fermi-momentum (density) of the matter for two effective interactions. The heavy and light lines refer to the $V_{\text{low-k}}$ and the Gogny interaction, respectively. The solid lines correspond to the non-interacting single-particle spectrum, the dashed and dashed-dotted lines—to renormalized single particle spectrum in symmetric nuclear matter and neutron matter, respectively. The single particle spectra are computed in the Brueckner–Hartree–Fock theory for the Gogny interaction and the Brueckner–Hartree–Fock theory for the $V_{\text{low-k}}$ interaction.
Semiclassical approximation to neutron star superfluidity corrected for proximity effects

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4Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

(Received 23 January 1998)

The inner crust of a neutron star is a superfluid and inhomogeneous system, consisting of a lattice of nuclei immersed in a sea of neutrons. We perform a quantum calculation of the associated pairing gap and compare it to the results one obtains in the local density approximation (LDA). It is found that the LDA overestimates the spatial dependence of the gap, and leads to a specific heat of the system which is too large at low temperatures, as compared with the quantal result. This is caused by the neglect of proximity effects and the delocalized character of the single-particle wave functions close to the Fermi energy. It is possible to introduce an alternative, simple semiclassical approximation of the pairing gap which leads to a specific heat that is in good agreement with the quantum calculation. [S0556-2813(98)07308-7]
FIG. 1. The Woods-Saxon potential $U(R)$ in the Wigner-Seitz cell of radius $R_s = 29$ fm is shown. We also indicate the Fermi energy $\epsilon_F$ and the outer neutron density $\rho_{\text{ext}}$.

$$\Phi_{S=0}(\vec{r}_1, \vec{r}_2) = \sum_{nn'lj} \left( \sum_q U_{nlj}^q V_{n'lj}^q \right) \frac{2j+1}{8\pi} \times \phi_{nlj}(r_1) \phi_{n'lj}(r_2) P_l(\cos(\theta_{21})).$$

$$\Delta(\vec{r}_1, \vec{r}_2) = -v(|\vec{r}_1 - \vec{r}_2|) \Phi_{S=0}(\vec{r}_1, \vec{r}_2)$$

$$\Delta(\vec{k}, \vec{R}) = \int d^3r_{12} \Delta(\vec{r}_1, \vec{r}_2) e^{-i\vec{k}\cdot\vec{r}_{12}},$$
Quantum calculation of vortices in the inner crust of neutron stars

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(Received 26 January 2006; revised manuscript received 26 June 2006; published 12 January 2007)

The self-consistent mean-field quantum mechanical solution of a vortex and a nucleus immersed in a sea of free neutrons, a scenario representative of the inner crust of neutron stars, is presented for the first time. Because of quantal size effects the phase space for vortices inside the nucleus is essentially zero, so that the vortex core opens up and surrounds the nucleus. As a consequence, pinned configurations (in which a vortex becomes anchored to the nucleus) are favored at low and high densities in the inner crust. This result is qualitatively different from that obtained in all previous models, which predict pinning at intermediate densities.
\[ [K + V(\rho, z) - E_F]U_\alpha + \Delta V_\alpha = E_\alpha U_\alpha, \]
\[-[K + V(\rho, z) - E_F]V_\alpha + \Delta^*U_\alpha = E_\alpha V_\alpha \]

\[ U_\alpha(\rho, z, \phi) = e^{i l_\alpha \phi} \sum_{n,m} U_{\alpha}^{n,m} \psi_{n,l_\alpha}(\rho) \chi_m(z), \]

\[ V_\alpha(\rho, z, \phi) = e^{i [l_\alpha - v] \phi} \sum_{n,m} V_{\alpha}^{n,m} \psi_{n,l_\alpha - v}(\rho) \chi_m(z), \]
Putting together bare and Induced Interaction
Surface effects in nuclear Cooper pair formation

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(Received 21 February 2001; published 25 March 2002)

The formation of Cooper pairs resulting from the exchange of vibrations between pairs of fermions moving in time reversal states close to the Fermi surface in a semi-infinite system of nuclear matter (slab model) with parameters adjusted so as to mimic real nuclei, leads to pairing gaps which account for a substantial fraction of the experimental value throughout the mass table. The predictions are in qualitative agreement with detailed calculations carried out for finite nuclei and testify to the central role the surface of nuclei plays in the phenomenon of nuclear superfluidity.
FIG. 1. (a) Pairing gap of particles as a function of the particle energy $\varepsilon = (\hbar k_p)^2/(2m)$, for $R = 6$ fm. For each energy value, the pairing gap $\Delta$ has been calculated as an average over the gaps of particles having the same $k_p^2$. Detailed results for the nucleus $^{120}$Sn are also displayed (open dots, cf. Fig. 1, Ref. [4]). (b) The pairing gap of a particle at the Fermi energy as a function of the momentum component parallel to the surface of the slab. The gap goes to zero when $k_{vp} = k_F = 1.337$ fm$^{-1}$, corresponding to the case of particles moving parallel to the surface of the slab ($k_z = 0$).
T=0 pairing problem

Vonderfecht et al. PLB253,1 (1991)
Baldo et al. PLB283, 8 (1992)
Proton-neutron pairing in symmetric nuclear matter $^3S_1-^3D_1$

Using CDBonn

Dashed lines: quasiparticle poles

Solid lines: dressed nucleons

No pairing at saturation density!!!!

Dickhoff et al.
LOMBARDO'S
COMPARISON WITH DATA:

Mamotretus and mk problem

PRC 120Sn con tanti osservabili

PRC 120Sn TPT transfer
THIS IS THE END, MY ONLY FRIEND,
THE END.
e-'s Cooper Pair Model; BCS (S=0;L=0); (S=0;L=2, HiTc?)

Superconductivity, Superfluidity

V. mutans mutandisi

n-n pairing in Uniform Neutron Matter (S=0;L=0),(S=1;L=1),(S=0;L=2)

NeutronStars: Cv, Vortices-Glitches

V. add protons (Z=N)

n-n in Unif. Symm. Nucl. Matt. (S=0;L=0),(S=1;L=1),(S=0;L=2) T=1;Tz=-1
p-p in Unif. Symm. Nucl. Matt. (S=0;L=0),(S=1;L=1),(S=0;L=2) T=1;Tz=+1

AND
n-p in Unif. Symm. Nucl. Matt. (S=0;L=0),(S=1;L=1),(S=0;L=2) T=1;Tz=0
n-p in Unif. Symm. Nucl. Matt. (S=1;L=0),(S=0;L=1),(S=1;L=2) T=0;Tz=0

..... OR
Quaternions??

V. add less protons (Z<N)

n-n in U. Asymm. Nucl. M. (S=0;L=0),(S=1;L=1),(S=0;L=2) T=1;Tz=-1
p-p in U. Asymm. Nucl. M. (S=0;L=0),(S=1;L=1),(S=0;L=2) T=1;Tz=+1
Typical forces employed in finite nuclei pairing HFB-like calculations:

I) Monopole: $H_p = -G P^+P$ with $G = -25/A$ MeV

being $P^+ = \sum_{m>0} a^+_m a^+_{-m}$; restricted to one shell

ii) Contact interactions $V_c(1-\eta^*\rho(r)/\rho_0) (E_{cut} = 60\text{MeV})$

Commonly used with Skyrme forces

iii) Gogny force

Continuum is relevant

iv) Argonne int.

Continuum is crucial

Up to 800MeV

v) Nuclear Matter Calculation Based Pairing Interaction

Show some Illustrative results
Contact interaction

\[ V(r-r') = V_c (1 - \eta^* \rho(r)/\rho_0) \delta^3(r-r') \]

\( \eta = 0; 0.5 \text{ and } 1.0 \) (vol.; mix.; surf.)

\( (E_{\text{cut}} = 60\text{MeV}); \)

---

**Table IV:** RMS residuals of \( \Delta_0^{(3)} \) obtained in various models. All energies are in MeV. The last column shows the ratio of proton and neutron effective pairing strengths obtained through the optimization procedure. The mass predictions of the HFB-14 model [16] were taken from [51].

<table>
<thead>
<tr>
<th>Theory</th>
<th>pairings</th>
<th>residual neutrons</th>
<th>residual protons</th>
<th>( V_0^{\text{eff}}(p)/V_0^{\text{eff}}(n) )</th>
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<tr>
<td>Constant</td>
<td></td>
<td>0.31</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>( c/A^\alpha )</td>
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<td>HFB+LN mixed</td>
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PHYSICAL REVIEW C 86, 064313 (2012)

Pairing gaps in the Hartree-Fock-Bogoliubov theory with the Gogny D1S interaction

FIG. 1. (Color online) Neutron pairing gaps $\Delta^{(3)}$ in the oxygen isotope chain. Energies were computed in the $N_{sh} = 10$ harmonic oscillator space.

FIG. 2. (Color online) Neutron pairing gaps $\Delta^{(3)}$ in the Sn isotope chain. Energies were computed in the $N_{sh} = 12$ harmonic oscillator space.
Another correlation is associated with the polarization of the nucleus by the valence nucleons. The resulting induced pairing interaction has been calculated to give as strong a contribution as the nucleon-nucleon interaction in the pairing channel [32]. Such induced interactions are long-ranged and energy-dependent, and vary from nucleus to nucleus depending on its structure. It would not be surprising that the effects were beyond the scope of the simple energy functionals in current use.
We have repeated the calculations, but this time setting the Fermi energy at $-7.2$ MeV, that is, for the isolated atomic nucleus $^{120}\text{Sn}$. The value of $R_{\text{box}}$ has been varied within the range 8–15 fm to check the stability of the results. The values of the pairing gaps become stable once $R_{\text{box}} \geq 12$ fm. The results of the calculation with $R_{\text{box}} = 15$ fm are displayed in Fig. 1(b). The value of the pairing gap at the Fermi energy is $2.2^{+0.4}_{-0.8}$ MeV, the “errors” reflecting the conspicuous state dependence of $\Delta$.\(^1\) This value is about 50% higher than the experimental value of 1.47 MeV deduced from the odd-even mass differences [22]. This also in keeping with the fact that the free nucleon-nucleon interaction seems to provide too strong pairing correlations in finite nuclei (cf. e.g. [23]). In any case, theory provides a sound framework for an eventual quantitative description of pairing in nuclei.

$m=1.0 \, m_0 \Rightarrow$ Too large gap
Finite size effects on the pairing field (BCS with the bare force)

Potential in the Wigner cell

Pairing gap in uniform neutron matter

\[ \varepsilon_F = 13.5 \text{MeV} \]

P.M. Pizzochero, F. Barranco, E. Vigezzi, R.A. Broglia, APJ 569 (2003) 381
HFB Equations are expanded on a basis

\[
\left( \begin{array}{cc}
    h_{nn'lj} - \lambda & \Delta_{nn'lj} \\
    \Delta_{nn'lj} & -h_{nn'lj} + \lambda
\end{array} \right) \left( \begin{array}{c}
    U_{nlj}^\alpha \\
    V_{nlj}^\alpha
\end{array} \right) = E_{lj}^\alpha \left( \begin{array}{c}
    U_{nlj}^\alpha \\
    V_{nlj}^\alpha
\end{array} \right)
\]

\[
\kappa(R_{cm}, r_{12}) = \frac{1}{8\pi} \sum_{nn'lj\alpha} (2j + 1) U_{nlj}^{*\alpha} V_{n'lj}^{\alpha} \varphi_{nlj}^{*} (r_1) \varphi_{n'lj} (r_2) P_l (\cos(\theta_{12}))
\]

\[
\Delta(R_{cm}, r_{12}) = -V_{eff} (r_{12}) \kappa(R_{cm}, r_{12})
\]

\[
\Delta(R_{cm}, k) = \int dr_{12} \Delta(R_{cm}, r_{12}) \exp(ikr_{12})
\]
Spatial description of (non-local) pairing gap
Essential for a consistent description of vortex pinning!

The range of the force is small compared to the coherence length, but not compared to the diffusivity of the nuclear potential.

The local-density approximation overestimates the decrease of the pairing gap in the interior of the nucleus. (PROXIMITY EFFECTS)
Mean field calculation with $V_{\text{low-k}}$ pairing force: 3-body force reduces the pairing gaps.

120Sn with $V_{18}$ and Gogny HF-field: 1.1MeV

Dependence on the effective mass at high momenta? ($V_{\text{low-k}}$ vs $V_{18}$)

Results depend on the Central Potential parameters, including nucleon effective mass.

Any way some solid conclusions can be extracted

1. Bare Argonne is tractable at hfB level

2. With “reasonable” potentials as Sly4 a close to experiment pairing gap is obtained: 1.1MeV vs 1.3MeV.

3. Using $V_{\text{low-K}}$ (and NNN forces) similar results are obtained

Resonable potential are related to a $m^*=0.7m_0$

which also leave room to self-energy corrections ($m_{\text{exp}}=1.0m_0$)
Beyond mean field needed?

a). Neutron matter shows important screening

b). Single-particles are importantly affected, why not quasi-particle properties like the pairing gap?

c). In B&M-II book an estimate was made...

d). If simple DF are used:

(\textit{Robledo,Bernard,Bertsch,PRC86(2012)})

Another correlation is associated with the polarization of the nucleus by the valence nucleons. The resulting induced pairing interaction has been calculated to give as strong a contribution as the nucleon-nucleon interaction in the pairing channel \cite{32}, Such induced interactions are long-ranged and energy-dependent, and vary from nucleus to nucleus depending on its structure. It would not be surprising that the effects were beyond the scope of the simple energy functionals in current use.\footnote{Alternative sophisticated DF: I. Stetcu (LANL, 2013 talk)}
In second order, the particle-vibration coupling gives rise to an interaction between two particles, which can be evaluated in a manner similar to the particle-phonon interaction considered in Sec. 6-5d. To illustrate the magnitude of the polarization force, we consider the limiting case in which the frequency of the exchanged phonon is large compared to the energy differences between the particle states. In this case, one can view the interaction as resulting from the static deformation (6-217) produced by the first particle acting on the second. Thus, for a mode of multipolarity $\lambda$, one obtains (see Eq. (6-68))

$$V_\lambda(1,2) = -\frac{2\lambda + 1}{4\pi C_\lambda} k_\lambda(r_1) k_\lambda(r_2) P_\lambda(\cos \theta_{12})$$

(6-228)

The order of magnitude of the polarization interaction (6-228) is given by $f_\lambda^2 \hbar \omega_\lambda$ (as for the particle self-energy). For the high-frequency modes, we have $f_\lambda^2 \hbar \omega_\lambda \sim \epsilon_F A^{-1}$, which is comparable to the average interaction between nucleons in the nucleus ($\sim V_0 A^{-1}$). The magnitude of the polarization interaction can be seen directly from the fact that the deformation of the closed shells produced by a single particle implies polarization moments comparable with the bare moments of the particles (see p. 510); hence, the corresponding contribution to the polarization interaction (6-228) is similar in magnitude to the direct force.

The polarization interaction resulting from the coupling to the low-frequency modes may be considerably larger than the bare force; since the frequencies of these modes may be comparable with the particle frequencies, it may be necessary to go beyond the static approximation (6-228), as in the evaluation of the particle-phonon interaction.
B. Particle Vibration Coupling

and

Single-particle Self-energy
Effective charges

One-particle transfer leading to Vibrational states

\[ h_{j,j'}\lambda = \beta\lambda(2\lambda+1)^{-1/2} \langle j \mid \mid \mathbf{Ro} \frac{dU}{dr} Y\lambda \mid j' \rangle (2j+1)^{-1/2} \]

PVC directly related observables

The vertex: Basic Ingredient
The paradigmatic case; 208Pb(3-) + 1p(h9/2)

Fig. 11. Observed low-lying energy levels of $^{209}$Bi. The coupling of different excitation modes are discussed in the text.

Table 6
Energy shifts of the septuplet members ($h_{9/2}$ 3-) $f^\pi$ in $^{209}$Bi from the unperturbed vibrational frequency 2.614 MeV. Experimental values are taken from ref. [29]

<table>
<thead>
<tr>
<th>I</th>
<th>$\delta E_{\text{exp}}$ (keV)</th>
<th>$\delta E_{\text{calc}}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2</td>
<td>-120</td>
<td>+36 $\rightarrow$ -190</td>
</tr>
<tr>
<td>5/2</td>
<td>+4</td>
<td>+7</td>
</tr>
<tr>
<td>7/2</td>
<td>-29</td>
<td>-6</td>
</tr>
<tr>
<td>9/2</td>
<td>-49</td>
<td>-89</td>
</tr>
<tr>
<td>11/2</td>
<td>-14</td>
<td>-31</td>
</tr>
<tr>
<td>13/2</td>
<td>-14</td>
<td>-63</td>
</tr>
<tr>
<td>15/2</td>
<td>+130</td>
<td>+156</td>
</tr>
</tbody>
</table>

I. Hamamoto, 1973
SELF ENERGY RENORMALIZATION OF SINGLE-PARTICLE STATES: CLOSED SHELL

\[ {}^{208}\text{Pb} \]

Relativistic Mean Field Calculations
Litvinova et al, PRC84, 014305 (2011)
Full Skyrme (Sly5), including momentum dependent terms

Colo' et al, PRC82, 064307

The vertex is deduced from the Transition Densities obtained in RPA calculation.
Similar difficulties in:

Propagation of uncertainties in the Skyrme energy-density-functional model

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2Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, ul. Hoża 69, PL-00-681 Warsaw, Poland
3Institute for Nuclear Research and Nuclear Energy, 1784 Sofia, Bulgaria

(Dated: January 28, 2013)

TABLE I: The $^{208}\text{Pb}$ proton (top) and neutron (bottom) s.p. energies $e_{\text{HF}}$ (b) and their standard errors (c), as compared to the empirical values (d) [25], residuals, $\Delta(e_{\text{HF}}) = e_{\text{HF}} - e_{\exp}$ (e), PVCs $\delta_{\text{PVC}}$ (f), and residuals of the PVC-corrected s.p. energies $\Delta(e_{\text{PVC}}) = e_{\text{HF}} + \delta_{\text{PVC}} - e_{\exp}$ (g). Where applicable, we also give the root-mean-square (rms) values of entries shown in a given column. All energies are in MeV.

<table>
<thead>
<tr>
<th>orbital</th>
<th>$e_{\text{HF}}$</th>
<th>$\sigma(e_{\text{HF}})$</th>
<th>$e_{\exp}$</th>
<th>$\Delta(e_{\text{HF}})$</th>
<th>$\delta_{\text{PVC}}$</th>
<th>$\Delta(e_{\text{PVC}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi 3p_{1/2}$</td>
<td>0.219</td>
<td>0.181</td>
<td>-0.16</td>
<td>0.38</td>
<td>-0.440</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\pi 3p_{3/2}$</td>
<td>-1.045</td>
<td>0.139</td>
<td>-0.68</td>
<td>-0.37</td>
<td>-0.662</td>
<td>-1.03</td>
</tr>
<tr>
<td>$\pi 2f_{5/2}$</td>
<td>-1.284</td>
<td>0.229</td>
<td>-0.97</td>
<td>-0.31</td>
<td>-0.480</td>
<td>-0.79</td>
</tr>
<tr>
<td>$\pi 1i_{13/2}$</td>
<td>-2.794</td>
<td>0.238</td>
<td>-2.10</td>
<td>-0.60</td>
<td>-0.221</td>
<td>-0.92</td>
</tr>
<tr>
<td>$\pi 1h_{9/2}$</td>
<td>-3.501</td>
<td>0.282</td>
<td>-3.80</td>
<td>0.30</td>
<td>-0.280</td>
<td>0.02</td>
</tr>
<tr>
<td>$\pi 2f_{7/2}$</td>
<td>-3.725</td>
<td>0.115</td>
<td>-2.90</td>
<td>-0.83</td>
<td>-0.284</td>
<td>-1.11</td>
</tr>
<tr>
<td>$\pi 3s_{1/2}$</td>
<td>-8.036</td>
<td>0.140</td>
<td>-8.01</td>
<td>-0.03</td>
<td>-0.108</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\pi 2d_{3/2}$</td>
<td>-8.378</td>
<td>0.199</td>
<td>-8.36</td>
<td>-0.02</td>
<td>0.220</td>
<td>0.20</td>
</tr>
<tr>
<td>$\pi 1h_{11/2}$</td>
<td>-9.153</td>
<td>0.207</td>
<td>-9.36</td>
<td>0.21</td>
<td>-0.141</td>
<td>0.07</td>
</tr>
<tr>
<td>$\pi 2d_{5/2}$</td>
<td>-10.117</td>
<td>0.117</td>
<td>-9.82</td>
<td>-0.30</td>
<td>0.116</td>
<td>-0.18</td>
</tr>
<tr>
<td>$\pi 1g_{7/2}$</td>
<td>-10.908</td>
<td>0.229</td>
<td>-12.00</td>
<td>1.09</td>
<td>0.131</td>
<td>1.22</td>
</tr>
</tbody>
</table>

rms | n.a. | 0.196 | n.a. | 0.52 | 0.328 | 0.70 |

<table>
<thead>
<tr>
<th>orbital</th>
<th>$e_{\text{HF}}$</th>
<th>$\sigma(e_{\text{HF}})$</th>
<th>$e_{\exp}$</th>
<th>$\Delta(e_{\text{HF}})$</th>
<th>$\delta_{\text{PVC}}$</th>
<th>$\Delta(e_{\text{PVC}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu 3d_{3/2}$</td>
<td>-1.856</td>
<td>0.250</td>
<td>-1.40</td>
<td>-0.46</td>
<td>-0.104</td>
<td>-0.56</td>
</tr>
<tr>
<td>$\nu 4s_{1/2}$</td>
<td>-2.051</td>
<td>0.235</td>
<td>-1.90</td>
<td>-0.15</td>
<td>-0.668</td>
<td>-0.82</td>
</tr>
<tr>
<td>$\nu 2g_{7/2}$</td>
<td>-2.141</td>
<td>0.258</td>
<td>-1.44</td>
<td>-0.70</td>
<td>-0.280</td>
<td>-0.98</td>
</tr>
<tr>
<td>$\nu 1f_{5/2}$</td>
<td>-2.231</td>
<td>0.167</td>
<td>-2.51</td>
<td>0.28</td>
<td>-0.226</td>
<td>0.05</td>
</tr>
<tr>
<td>$\nu 3d_{5/2}$</td>
<td>-2.549</td>
<td>0.166</td>
<td>-2.37</td>
<td>-0.18</td>
<td>-0.384</td>
<td>-0.56</td>
</tr>
<tr>
<td>$\nu 1i_{11/2}$</td>
<td>-2.680</td>
<td>0.223</td>
<td>-3.16</td>
<td>0.48</td>
<td>-0.271</td>
<td>0.21</td>
</tr>
<tr>
<td>$\nu 2g_{9/2}$</td>
<td>-4.336</td>
<td>0.071</td>
<td>-3.94</td>
<td>-0.40</td>
<td>-0.183</td>
<td>-0.58</td>
</tr>
<tr>
<td>$\nu 3p_{1/2}$</td>
<td>-7.855</td>
<td>0.109</td>
<td>-7.37</td>
<td>-0.49</td>
<td>0.152</td>
<td>-0.33</td>
</tr>
<tr>
<td>$\nu 3p_{3/2}$</td>
<td>-8.503</td>
<td>0.065</td>
<td>-8.26</td>
<td>-0.24</td>
<td>-0.119</td>
<td>-0.36</td>
</tr>
<tr>
<td>$\nu 2f_{5/2}$</td>
<td>-8.519</td>
<td>0.143</td>
<td>-7.94</td>
<td>-0.58</td>
<td>0.156</td>
<td>-0.42</td>
</tr>
<tr>
<td>$\nu 1i_{13/2}$</td>
<td>-8.603</td>
<td>0.162</td>
<td>-9.24</td>
<td>0.64</td>
<td>-0.130</td>
<td>0.51</td>
</tr>
<tr>
<td>$\nu 1h_{9/2}$</td>
<td>-9.922</td>
<td>0.190</td>
<td>-11.40</td>
<td>1.48</td>
<td>0.120</td>
<td>1.60</td>
</tr>
<tr>
<td>$\nu 2f_{7/2}$</td>
<td>-10.407</td>
<td>0.103</td>
<td>-9.81</td>
<td>-0.60</td>
<td>0.179</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

rms | n.a. | 0.176 | n.a. | 0.61 | 0.273 | 0.68 |
We will show mainly results obtained using the simple B&MII vertex, based on Experimental beta's
C. Phonon exchange Induced Pairing Interaction (old theory)


\[
M_{\nu,\lambda \nu'}^n = \frac{\langle \nu' \| R_o \frac{\partial U}{\partial \tau} Y_{\lambda} \| \nu \rangle}{\sqrt{2j_\nu + 1}} \frac{\beta_\lambda(n)}{\sqrt{(2\lambda + 1)}}
\]  

(1)

can be calculated. The quantities entering in the reduced matrix element appearing in Eq. (1) are the nuclear radius \(R_o\), the derivative of the Saxon-Woods potential, and a spherical harmonic of multipolarity \(\lambda\). Once these matrix elements are known, one can calculate the induced pairing interaction matrix elements (cf. inset of Fig. 1)

\[
\nu_{\nu' \nu'} = \langle (j_\nu m_{\nu'}) (j_\nu m_{\nu'}) | \nu \rangle (j_\nu m_{\nu'}) (j_\nu m_{\nu'}) \rangle_{a.s.} = \sum_{\lambda n} \frac{2}{(2j_\nu' + 1)} \frac{2(M_{\nu,\lambda \nu'}^n)^2}{E_o - [e_\nu + e_\nu' + \hbar \omega_\lambda(n)]},
\]  

(2)

and thus determine the state dependent BCS pairing gap [2]

\[
\Delta_\nu = - \sum_{\nu'} \frac{(2j_\nu' + 1)}{2} \frac{\Delta_\nu'}{2E_{\nu'}} \nu_{\nu' \nu'}.
\]  

(3)

\[\Delta \nu\]  

IG. 1. State dependent pairing gap \(\Delta_\nu\) [cf. Eq. (3)] for the nucleus \(^{120}\text{Sn}\) calculated making use of the induced interaction defined in Eq. (2) (cf. inset, where particles are represented by arrowed lines and phonons by a wavy line).
Comparison between Gogny and Induced Interaction matrix elements

FIG. 1. The nucleus $^{120}$Sn. Diagonal pairing matrix elements of the induced interaction (upper panel, solid diamonds) and of the Gogny force (lower panel, solid circles), displayed as a function of the single-particle energy, $e_{\nu}$, of the state $\nu$ calculated using the bare nucleon mass and the single-particle wave functions of a Woods-Saxon potential with standard parameters (depth $V_0 = -49$ MeV, diffusivity $a = 0.65$ fm, and radius $R_0 = 6.16$ fm), including the spin-orbit term, parametrized according to Ref. [21]. Also shown by means of vertical lines is the position of the Fermi energy, $e_F = -9.1$ MeV. Note the different scale in the two figures.
Semiclassical (TF averaged) estimate of diagonal pairing matrix elements in $^{120}$Sn.

The coupling with the phonons induces a surface-peaked interaction and pairing gap.

\[ \Delta \approx Z(v_{14} + v_{\text{ind}}) \kappa \]

But the pairing density \( k(r) \) is much less affected.

Density modes (surface): Attractive
Spin modes (volume); Repulsive !!

Equivalent DDDI parameters **Volume Repulsive** Interaction!!
Induced Interaction in the Slab Model (Giovanardi, PRC65,041304(2002)):

Slab Model: Semi-infinite Nuclear Matter with realistic surface statical and dynamical properties (Esbensen&Bertsch Ann.Phys.(1984)).
D. Microscopic description of superfluid nuclei beyond mean field:

**Quasiparticle Vibration Coupling**

(used formalism)  
(cf. Van der Sluys et al., NPA551(1993)210)

For each single-particle state $a = (\epsilon_a, n_a, l_a, j_a)$ the equation-of-motion method leads to a system of linear equations of dimension $2(N+1)$ with $N$ the dimension of the $(1qp \times \text{phonon})$ space:

$$
\begin{pmatrix}
E_a & V(abJ\nu) & 0 & W(abJ\nu) \\
V(abJ\nu) & (E_{J\nu} + E_b) & W(abJ\nu) & 0 \\
0 & W(abJ\nu) & -E_a & -V(abJ\nu) \\
W(abJ\nu) & 0 & -V(abJ\nu) & -(E_{J\nu} + E_b)
\end{pmatrix}
\begin{pmatrix}
x_0 \\
C_{bJ\nu} \\
y_0 \\
-D_{bJ\nu}
\end{pmatrix}
= \tilde{E}_a
\begin{pmatrix}
x_0 \\
C_{bJ\nu} \\
y_0 \\
-D_{bJ\nu}
\end{pmatrix}.
$$

Neglecting the mutual interaction between the $(1qp \times \text{phonon})$ states, it becomes easy to rewrite the secular equation (23) as a two-dimensional, hermitian and non-linear eigenvalue problem, by projecting onto the $1qp$ space:

$$
\begin{pmatrix}
E_a & 0 \\
0 & -E_a
\end{pmatrix} + \begin{pmatrix}
\Sigma_{11}(E) & \Sigma_{12}(E) \\
\Sigma_{12}(E) & \Sigma_{22}(E)
\end{pmatrix}
\begin{pmatrix}
x_0 \\
y_0
\end{pmatrix}
= \tilde{E}
\begin{pmatrix}
x_0 \\
y_0
\end{pmatrix}.
$$

The energy-dependent self-energy matrix elements $\Sigma_{ij}(E)$ stand for:

$$
\Sigma_{11}(E) = \sum_{bJ\nu} \left( \frac{|V(abJ\nu)|^2}{E - (E_b + E_{J\nu})} + \frac{|W(abJ\nu)|^2}{E + (E_b + E_{J\nu})} \right),
$$

$$
\Sigma_{22}(E) = \sum_{bJ\nu} \left( \frac{|W(abJ\nu)|^2}{E - (E_b + E_{J\nu})} + \frac{|V(abJ\nu)|^2}{E + (E_b + E_{J\nu})} \right),
$$

$$
\Sigma_{12}(E) = -\sum_{bJ\nu} V(abJ\nu) W(abJ\nu) \left( \frac{1}{E - (E_b + E_{J\nu})} - \frac{1}{E + (E_b + E_{J\nu})} \right).
$$

---

**Correcting HFB/BCS**

Quasiparticles: 2 steps in calculating pairing properties
USED FORMALISM
(Idini, Barranco, Vigezzi, PRC85, 014331(2012))

\[
\begin{pmatrix}
E_a + \Sigma_{11}(\tilde{E}_{a(n)}) \\
\Sigma_{12}(\tilde{E}_{a(n)})
\end{pmatrix}
\begin{pmatrix}
\Sigma_{11}(\tilde{E}_{a(n)}) \\
- E_a + \Sigma_{22}(\tilde{E}_{a(n)})
\end{pmatrix}
\begin{pmatrix}
x_{a(n)} \\
y_{a(n)}
\end{pmatrix}
= \tilde{E}_{a(n)}
\begin{pmatrix}
x_{a(n)} \\
y_{a(n)}
\end{pmatrix}
\]

where one has introduced the normal and abnormal self-energies \( \Sigma_{11}(E) \) (being \( \Sigma_{22}(E) = -\Sigma_{11}(-E) \)) and \( \Sigma_{12}(E) \), given by

\[
\Sigma_{11} = \sum_{b,m,J,\nu} \frac{V^2(a(n)b(m)J\nu)}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar \omega_{J,\nu}} + \sum_{b,m,J,\nu} \frac{W^2(a(n)b(m)J\nu)}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar \omega_{J,\nu}}
\]

and

\[
\Sigma_{12} = \Sigma_{12}^{\text{pho}} + \Sigma_{12}^{\text{bare}}
\]

\[
\Sigma_{12}^{\text{pho}} = - \sum_{b,m,J,\nu} V(a(n), b(m), J, \nu) W(a(n), b(m), J, \nu) \left[ \frac{1}{\tilde{E}_{a(n)} - \tilde{E}_{b(m)} - \hbar \omega_{J,\nu}} - \frac{1}{\tilde{E}_{a(n)} + \tilde{E}_{b(m)} + \hbar \omega_{J,\nu}} \right].
\]

\[
\Sigma_{12}^{\text{bare}} = \pm \sum_{b,n} V_{b,n}\left( 2j_b + 1 \right) \tilde{u}_{b(n)} \tilde{v}_{b(n)}
\]

\[
V(jj'\lambda) = \beta \lambda (2\lambda + 1)^{-1/2} \langle j | R dU/dr Y\lambda | j' \rangle (uj' - vj') (2j + 1)^{-1/2}
\]

\[
W(jj'\lambda) = \beta \lambda (2\lambda + 1)^{-1/2} \langle j | R dU/dr Y\lambda | j' \rangle (uj' + vj'') (2j + 1)^{-1/2}
\]

Optional: 1 or 2 steps?
Since self-energies are energy dependent many solutions are obtained: \( n=1,2,.. \)  
Each carrying a quasi-particle strength \( u(a,n)^2 + v(a,n)^2 < 1 \)  
Closure requires \( \Sigma_n u(a,n)^2 + v(a,n)^2 = 1 \)
A generalized gap equation. Different versions

Expressing $u^*v$ as a function of sigma12, and reintroducing them in the sigma12 expression, a close expression for sigma is obtained:

$$\Delta_{a(n)} = - Z_{a(n)} \sum_{b(m)} V_{eff}(a(n), b(m)) N_{b(m)} \frac{\Sigma_{12}(b(m), E_{b(m)})}{2 \sqrt{(\epsilon_b - e_F + \Sigma_{even}(b(m), E_{b(m)})^2 + \Sigma_{12}^2(b(m), E_{b(m)})^2)}}$$

where $N$ is the proper quasi-particle normalization:

$$N_{b(m)} = \tilde{u}_{b(m)}^2 + \tilde{v}_{b(m)}^2 = \left(1 - \frac{\partial \Sigma_{11}(a, E_{a(n)})}{\partial E_{a(n)}} \tilde{u}_{b(m)}^2 - \frac{\partial \Sigma_{22}(a, E_{a(n)})}{\partial E_{a(n)}} \tilde{v}_{b(m)}^2 - 2 \frac{\partial \Sigma_{12}(a, E_{a(n)})}{\partial E_{a(n)}} u_{b(m)} v_{b(m)}\right)^{-1} < 1$$

which gives the properly normalized $u,v$'s starting from the normalized to $1$ $u,v$'s,

and where the effective interaction is

$$V_{eff}(a(n), b(m)) = V_{bare}(a, b) + \sum_{J,v} \hbar^2 (a, b, J, v) \left(\frac{1}{E_{an} - E_{bm} - \hbar \omega_{J,v}} - \frac{1}{E_{an} + E_{bm} + \hbar \omega_{J,v}}\right)$$

Reintroducing the $Z_b$-factor

$$\Delta_{a(n)} = - Z_{a(n)} \sum_{b(m)} V_{eff}(a(n), b(m)) N_{b(m)} \frac{Z_{b(m)} \Sigma_{12}(b(m), E_{b(m)})}{2 \sqrt{Z_{b(m)}^2 (\epsilon_b - e_F + \Sigma_{even}(b(m), E_{b(m)})^2 + Z_{b(m)}^2 \Sigma_{12}^2(b(m), E_{b(m)})^2)}}$$

$$\Delta_{a(n)} = - \sum_{b(m)} V_{eff}(a(n), b(m)) \frac{Z_{a(n)} \Delta_{b(m)} N_{b(m)}}{2 \sqrt{(\epsilon_b - e_F)^2 + \Delta_{b(m)}^2}}$$

$$\Delta_{a(n)} = - \sum_{b(m)} V_{eff}(a(n), b(m)) \frac{Z_{a(n)} \Delta_{b(m)} N_{b(m)}}{2 E_{b(m)}}$$

c.f.Baldov
Z=1 free Fermi gas
Z<1 correlated Fermi system

\[ \Delta_p = -\frac{1}{2} \int d^3 p' \left( \frac{Z V_{pp'} Z_{p'}}{\sqrt{\left( \tilde{\varepsilon}_{p'} - \varepsilon_F \right)^2 + \Delta_{p'}^2}} \right) \]

Quasiparticle strength <1
Bare+Induced interaction
Renormalized s.p. energy
Experimental phonons were used for the density modes $L=2,3,4,5$. Spin modes not included in this results.

1st iteration

RENORMALIZING Argonne (on Sly4) pairing gaps (2 steps calculation)
FIG. 17. (Color online) The state-dependent pairing gap $\Delta^{\text{BCS}}$ calculated in BCS with the bare $v_{14}$ interaction is compared to the renormalized gap $\tilde{\Delta}$ [cf. Eq. (38)] obtained solving the Nambu-Gor'kov equations. We compare results obtained with a mean field produced with the SGII interaction (a) and with the SKa interaction (b) (cf. Fig. 8 for the corresponding calculation with the SLy4 mean field).

FIG. 18. The theoretical quasiparticle spectra obtained at the various steps of the calculation are compared to the experimental data. We compare results obtained with a mean field produced with the SGII interaction (a) and with the SKa interaction (b) (cf. Fig. 8 for the corresponding calculation with the SLy4 mean field).
FIG. 26. (Color online) Renormalized gaps $\tilde{\Delta}$ obtained solving the Nambu Gor’kov equations in the one-step and the two-steps diagonalization schemes with the monopole pairing force as a function of the pairing constant $G_0$, averaged over the five valence orbitals. Also shown is the gap $\Delta^{\text{BCS}}$ obtained solving the BCS equation.

Application to $^{10}$Li and $^{11}$Be: Talk by E. Vigezzi

Figure A.1: Feynman representation of the components of self energy matrix $\{A.2.5\}$. The Green's functions lines with the empty arrow represent respectively the positive and negative quasiparticle eigenvalue $^{(\pm)}$ of the basis $^{\{A.1.14\}}^{A.1.15}$.

<table>
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<tr>
<th>$-(\hat{T} + \hat{V} - \lambda)_{hh}$</th>
<th>$\hat{\Delta}_{ph}$</th>
<th>$\hat{\Delta}_{ph}$</th>
<th>$-(\hat{T} + \hat{V} - \lambda)_{hp}$</th>
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<td>$\hat{\Delta}_{pp}$</td>
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</tbody>
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+ 

Barranco, Broglia, PRL29 (1987)

Esbensen, Bertsch, ZPF's: PRC28

Allows correcting $r$-dependence

HF

HF + (a) + (b) + (c) + (d)
Fig. 1. Mean square radius of the Ca isotopes. In (a) the charge mean square radius is displayed as a function of the mass number [25–27]. The quantity \( \langle r^2 \rangle_{\text{ch}}^{1/2} \) was assumed to have a linear dependence with \( A \). Its slope was fitted by requiring that the quantity (1) coincides with the experimental values for \(^{40}\text{Ca}\) and \(^{48}\text{Ca}\) making use of the values of \( \Delta r^2 \) calculated through eqs. (2) – (4). These values are collected in table 1. The resulting values of \( \langle r^2 \rangle_{\text{ch}} \) are displayed as a full drawn curve. In (b) the mass mean square radius is displayed [27]. The quantity \( \langle r^2 \rangle_{\text{matt}}^{1/2} \) was given an \( A^{1/3} \) dependence, and its coefficient adjusted to get the best fit when use is made of eq. (1) and the quantities \( \Delta r^2 \) (cf. table 1). The corresponding results are shown with a solid curve. The resulting bare mean square radius is \( \langle r^2 \rangle_{\text{matt}}^{1/2} = A^{1/3} \times 0.96 \text{ fm} \).
Figure A.2: Feynman representation of $V$ and $W$ vertices (A.2.8)-(A.2.11) for the case of particle $a$ represented in the quasiparticle basis (A.1.14)-(A.1.15) $(\pm)$ scattering into quasiparticle $b_{\mu}^{(\pm)}$ and a phonon $\lambda_{\nu}^{\pi}$.

\[ V(p_k^{(+)} , b_{\mu}^{(+)} , \lambda_{\nu}^{\pi}) \equiv \langle p_k^{(+)} | V_{res} | b_{\mu}^{(+)} \lambda_{\nu}^{\pi} \rangle = - \langle p_k^{(-)} | V_{res} | b_{\mu}^{(-)} \lambda_{\nu}^{\pi} \rangle \\
= \langle 0 | a_{p_k} V_{res} \alpha_{b_{\mu},\lambda_{\nu}}^{\dagger} \Gamma_{\lambda_{\nu}^{\pi}} | 0 \rangle = \langle 0 | a_{p_k} \hat{V} \left( \sum_l \tilde{u}_{b_{\mu},l} a_{b_{\mu}}^{\dagger} + \tilde{\nu}_{b_{\mu},l} a_{b_{\mu}} \right) \Gamma_{\lambda_{\nu}^{\pi}} | 0 \rangle \]

\[ = \sum_l (f+g)(p_k, b_{\mu}, \lambda_{\nu}^{\pi}) \tilde{\nu}_{b_{\mu},l} \quad \text{cf. Fig. A.2(a)} \quad (A.2.8) \]

\[ V(h_k^{(+)}, b_{\mu}^{(+)} , \lambda_{\nu}^{\pi}) \equiv \langle h_k^{(+)}, V_{res} | b_{\mu}^{(+)} \lambda_{\nu}^{\pi} \rangle = - \langle h_k^{(-)} | V_{res} | b_{\mu}^{(-)} \lambda_{\nu}^{\pi} \rangle \\
= \sum_l -\tilde{\nu}_{b_{\mu},l}(f-g)(h_k, b_{\mu}, \lambda_{\nu}^{\pi}) \quad \text{cf. Fig. A.2(b)} \quad (A.2.9) \]

\[ W(p_k^{(+)}, b_{\mu}^{(-)} , \lambda_{\nu}^{\pi}) \equiv \langle p_k^{(+)}, V_{res} | b_{\mu}^{(-)} \lambda_{\nu}^{\pi} \rangle = \langle p_k^{(-)} | V_{res} | b_{\mu}^{(+)} \lambda_{\nu}^{\pi} \rangle \\
= \sum_l \tilde{\nu}_{b_{\mu},l}(f-g)(p_k, b_{\mu}, \lambda_{\nu}^{\pi}) \quad \text{cf. Fig. A.2(c)} \quad (A.2.10) \]

\[ W(h_k^{(+)}, b_{\mu}^{(-)} , \lambda_{\nu}^{\pi}) \equiv \langle h_k^{(+)}, V_{res} | b_{\mu}^{(-)} \lambda_{\nu}^{\pi} \rangle = \langle h_k^{(-)} | V_{res} | b_{\mu}^{(+)} \lambda_{\nu}^{\pi} \rangle \\
= \sum_l \tilde{u}_{b_{\mu},l}(f+g)(h_k, b_{\mu}, \lambda_{\nu}^{\pi}) \quad \text{cf. Fig. A.2(d)} \quad (A.2.11) \]
Similar developments (Soma, et. al, PRC87, 011303(13))

Self-energy expansion

- Gorkov equations → energy dependent eigenvalue problem

\[
\sum_b \left( \frac{t_{ab} - \mu_{ab} + \Sigma_{ab}^{11}(\omega)}{\Sigma_{ab}^{21}(\omega)} - \frac{\Sigma_{ab}^{12}(\omega)}{-t_{ab} + \mu_{ab} + \Sigma_{ab}^{22}(\omega)} \right) \left|_{\omega_k} \begin{pmatrix} \mathcal{U}_b^k \\ \mathcal{V}_b^k \end{pmatrix} \right. = \left. \omega_k \begin{pmatrix} \mathcal{U}_a^k \\ \mathcal{V}_a^k \end{pmatrix} \right.
\]

Ab Initio, and HF side, 2\textsuperscript{nd} order

- 1\textsuperscript{st} order → energy-independent self-energy

\[\Sigma_{ab}^{11}(1) = \quad \Sigma_{ab}^{12}(1) = \]

- 2\textsuperscript{nd} order → energy-dependent self-energy

\[\Sigma_{ab}^{11}(2)(\omega) = \quad \Sigma_{ab}^{12}(2)(\omega) = \]

- 2\textsuperscript{nd} order → energy-dependent self-energy

\[\Sigma_{ab}^{22}(2)(\omega) = \quad \Sigma_{ab}^{22}(2)(\omega) = \]
**Scaling of Gorkov’s problem**

★ Transformed into an energy *independent* eigenvalue problem

\[
\begin{pmatrix}
  T - \mu + \Lambda & \tilde{h} & C & -D^\dagger \\
  \tilde{h}^\dagger & -T + \mu - \Lambda & -D & C \\
  C^\dagger & -D & E & 0 \\
  -D & C^\dagger & 0 & -E
\end{pmatrix}
\begin{pmatrix}
  U^k \\
  V^k \\
  W_k \\
  Z_k
\end{pmatrix}
= \omega_k
\begin{pmatrix}
  U^k \\
  V^k \\
  W_k \\
  Z_k
\end{pmatrix}
\]

★ Numerical scaling

\[
m_{p,1} \approx \left( \frac{N_b}{3} \right) \propto \frac{N_b^3}{6}
\]

\[
2N_b \begin{pmatrix}
  N_b \\
  \tilde{h} \\
  \tilde{h}^\dagger & C & -D^\dagger \\
  -h & -D^\dagger & C \\
  C^\dagger & -D & E & 0 \\
  -D & C^\dagger & 0 & -E
\end{pmatrix}
\]

- $N_b \rightarrow$ dimension of the s.p. basis
- $n \rightarrow$ number of iterations

\[
N_{tot,1} = 2N_b + M_{p,1} \approx N_b^3 \\
\ldots \\
N_{tot,n} = 2N_b + M_{p,n} \approx N_b^{3n}
\]
Pairing gaps

Three-point mass differences

\[ \Delta_{n}^{(3)}(A) = \frac{(-1)^{A}}{2} \left[ E_{0}^{A+1} - 2E_{0}^{A} + E_{0}^{A-1} \right] \]
FIG. 3. (Color online) Three-point mass differences $\Delta_n^{(3)}$ in the calcium isotopes calculated to third order in MBPT with and without the leading chiral 3N forces, and in comparison with experiment [24, 67]. The legend is as in Fig. 1. Panel (a) shows the results of the third-order ladder contributions. Panels (b) and (c) include all MBPT diagrams to third order in the $pf$-shell and the extended $pg_{9/2}$ valence space, respectively. The results in the $pf$-shell are with empirical SPEs. For the $pg_{9/2}$ space, we show pairing gaps for both the MBPT and empirical SPEs.

When particle-hole contributions are included in a full third-order calculation, we find in Fig. 3 a clear improvement compared to including only ladder diagrams. In the $pf$-shell, the three-point mass differences are increased, leading to reasonable agreement with experimental data. This clearly demonstrates the importance of particle-hole many-body processes, such as core-polarization, on pairing in nuclei. Our results show that they can provide the missing pairing strength required to reproduce experiment on top of the direct NN+3N interactions. Analogously, the systematic differences between theoretical and experimental pairing gaps found in the EDF approach of Ref. [15] may be attributed to these effects.
Combines
i) particle,
ii) ph-RPA and
iii) pp(hh)-RPA
consistently

FIG. 1. (Color online) Feynman diagrams representation of the self energy. The first diagram on the right-hand side represents the Hartree-Fock-like contribution to the mean field. The remaining ones describe core polarization effects in the particle (2p1h) and hole (2h1p) part of the spectrum.

FIG. 2. (Color online) Example of a diagrammatic contribution included in the Faddeev expansion for $R^{(2p1h)}$ (see Fig. 1). A quasiparticle is coupled to the response function $\Pi^{(ph)}$ that describes the target nucleus. It can also participate in pairing processes, which are accounted for by the two-body propagator $g^{II,(pp)}$. 
CONCLUSIONS

a. At mean field level surface pairing interaction preferred. Bare Argonne tractable as pairing interaction.

b. Phonon Exchange Pairing Induced Interaction very surface peaked. In volume could be repulsive due to spin modes. Strong A-dependence from the Slab Model.

c. Close connection to self-energy effects from Dyson Gorkov's eqs.: Quasi-particle Phonon Coupling: Fragmentation and Dense Espectra $\Rightarrow$ Induced Pairing Int. Bare Argonne plus Induced Pairing good description of data.

d. Shell Model confirms relevance of surface phonons mediated pairing interaction.

e. Multishell Self-energy: Full HFB renormalization. Ab Initio calculations in progress ($2^{nd}$ order).