Parity violating asymmetry, giant resonances, and the neutron skin thickness

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INTRODUCTION
The Nuclear Many-Body Problem:

- **Nucleus**: from few to more than 200 strongly interacting and self-bound fermions.
- **Underlying interaction** is not perturbative at the (low)energies of interest for the study of masses, radii, deformation, giant resonances,...
- **Complex systems**: spin, isospin, pairing, deformation, ...
- **Many-body** calculations based on NN scattering data in the vacuum are not conclusive yet:
  - different nuclear interactions in the medium are found depending on the approach
  - EoS and (only very recently) few groups in the world are able to perform extensive calculations for light and medium mass nuclei
- Based on effective interactions, **Nuclear Energy Density Functionals** are successful in the description of masses, nuclear sizes, deformations, Giant Resonances,...
Nuclear Energy Density Functionals:
Main types of successful EDFs for the description of masses, deformations, nuclear distributions, Giant Resonances, ...

Relativistic mean-field models, based on Lagrangians where effective mesons carry the interaction:

$$L_{\text{int}} = \bar{\Psi} \Gamma_\sigma (\bar{\Psi}, \Psi) \Psi \Phi_\sigma + \bar{\Psi} \Gamma_\delta (\bar{\Psi}, \Psi) \tau \Psi \Phi_\delta$$

$$- \bar{\Psi} \Gamma_\omega (\bar{\Psi}, \Psi) \gamma_\mu \Psi A^{(\omega)} \mu$$

$$- \bar{\Psi} \hat{Q} \gamma_\mu \Psi A^{(\gamma)} \mu$$

Non-relativistic mean-field models, based on Hamiltonians where effective interactions are proposed and tested:

$$V_{\text{eff}}^{\text{Nucl}} = V^{\text{long-range attractive}} + V^{\text{short-range repulsive}} + V_{\text{SO}} + V_{\text{pair}}$$

- Fitted parameters contain (important) correlations beyond the mean-field
- Nuclear energy functionals are phenomenological → not directly connected to any NN (or NNN) interaction
The Nuclear Equation of State: Infinite System

\[ \frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + \mathcal{O}(\beta^4) \]

- **Nuclear Matter**

\[
\beta = \frac{\rho_n - \rho_p}{\rho}
\]
The Nuclear Equation of State: Infinite System

\[ E_A(\rho, \beta) = E_A(\rho, \beta = 0) + S(\rho)\beta^2 + O(\beta^4) \]

- Nuclear Matter
- Symmetric Matter

\[ \beta = \frac{\rho_n - \rho_p}{\rho} \]
The Nuclear Equation of State: Infinite System

\[ E_A(\rho, \beta) = E_A(\rho, \beta = 0) + S(\rho)\beta^2 + O(\beta^4) \]

◮ Nuclear Matter
◮ Symmetric Matter
◮ Symmetry energy

\[ \beta = \frac{\rho_n - \rho_p}{\rho} \]
The Nuclear Equation of State: Infinite System

![Graph showing the relationship between density and energy]

\[
\frac{E}{A}(\rho, \beta) = \frac{E}{A}(\rho, \beta = 0) + S(\rho)\beta^2 + O(\beta^4)
\]

\[
= \frac{E}{A}(\rho, \beta = 0) + \beta^2 \left( J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + O(x^3) \right)
\]

\[
\begin{bmatrix}
\beta = \frac{\rho_n - \rho_p}{\rho}; \\
\chi = \frac{\rho - \rho_0}{3\rho_0}
\end{bmatrix}
\]
The Nuclear Equation of State: Infinite System

\[
\begin{align*}
\frac{E}{A}(\rho, \beta) &= \frac{E}{A}(\rho, \beta = 0) + \beta^2 \left( \frac{J}{L} + \frac{1}{2} \frac{K_{\text{sym}}}{9 \rho_0^2} \chi^2 + O(\chi^3) \right) \\
S(\rho_0) &= J \\
\frac{d}{d\rho} S(\rho) \bigg|_{\rho_0} &= \frac{L}{3\rho_0} = \frac{P_0}{\rho_0^2} \\
\frac{d^2}{d\rho^2} S(\rho) \bigg|_{\rho_0} &= \frac{K_{\text{sym}}}{9 \rho_0^2} \\
\beta &= \frac{\rho_n - \rho_p}{\rho} \quad ; \quad \chi = \frac{\rho - \rho_0}{3\rho_0}
\end{align*}
\]
In the past (and also in the present), neutron properties in stable medium and heavy nuclei have been mainly measured by using strongly interacting probes.

Limited knowledge of isovector properties

At present,

- the use of rare ion beams has opened the possibility of measuring properties of exotic nuclei ⇒ more info
- parity violating elastic electron scattering (PVES), a model independent technique, has allowed to estimate the neutron radius of a stable heavy nucleus like $^{208}\text{Pb}$

Promising perspectives for the near future
PARITY VIOLATING ASYMMETRY
Some basics ...

- **Electrons** interact by exchanging a $\gamma$ or a $Z_0$ boson.
- While **protons** couple basically to $\gamma$, **neutrons** do it to $Z_0$.
- Electron motion governed by the Dirac equation:
  \[
  [\vec{\alpha} \cdot \vec{p} + \beta m_e + V(r)]\psi = E\psi
  \]
  where $V(r) = V_C(r) + \gamma^5 V_W(r)$
- Dirac equation for helicity states ($m_e \approx 0$)
  \[
  [\vec{\alpha} \cdot \vec{p} + (V_C(r) \pm V_W(r))]\psi_\pm = E\psi_\pm
  \]
- **Ultra-relativistic electrons**, depending on their helicity, will interact with the nucleus seeing a slightly different potential "$\alpha Z$" $\pm$ "$G_F$".

Some basics ...

- **The interference** between the DCS of electrons with $+$ and $-$ helicity states,

$$\Lambda_{p\nu} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega}$$

- **Ultra-relativistic electrons** moving under the effect of $V_\pm$ where **Coulomb distortions** are important $\Rightarrow$ solution of the Dirac equation via the Distorted Wave Born Approximation (**DWBA**).

  - Input for the calculation of $V_\pm$ are the $\rho_n$ and $\rho_p$ (main uncertainty in $\rho_n$) and **nucleon form factors** for the e-m and the weak neutral current.
Qualitative considerations ...

Within the Plane Wave Born Approximation,

\[ A_{pv} = \frac{G_F q^2}{4\pi\alpha \sqrt{2}} \left[ 4\sin^2 \theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)} \right] \]

... which depends on \( F_n(q) - F_p(q) \). For \( q \to 0 \), it is approximately,

\[ -\frac{q^2}{6} (\langle r_n^2 \rangle - \langle r_p^2 \rangle) = -\frac{q^2}{6} \left[ \Delta r_{np}(\langle r_n^2 \rangle^{1/2} + \langle r_p^2 \rangle^{1/2}) \right] \]

\[ = -\frac{q^2}{6} \left( 2\langle r_p^2 \rangle^{1/2} \Delta r_{np} + \Delta r_{np}^2 \right) \]

variation of \( A_{pv} \) at a fixed \( q \) dominated by the variation of \( \Delta r_{np} \). \( F_p(q) \) well fixed by experiment
Qualitative considerations ...

Within the Plane Wave Born Approximation...

... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately, within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G}{4\pi a} \frac{1}{\sqrt{2}} \left[ 4\sin^2 \theta_W + F_n(q) - F_p(q) \right].$$

Variation of $A_{pv}$ at a fixed $q$ dominated by the variation of $\Delta r_{np}$. $F_p(q)$ well fixed by experiment.
Within the Plane Wave Born Approximation,

\[ A_{pv} = \frac{G_F q^2}{4\pi\alpha \sqrt{2}} \left[ 4\sin^2 \theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)} \right] \]

... which depends on \( F_n(q) - F_p(q) \). For \( q \to 0 \), it is approximately,

\[ -\frac{q^2}{6} \left( \langle r_n^2 \rangle - \langle r_p^2 \rangle \right) = -\frac{q^2}{6} \left[ \Delta r_{np} (\langle r_n^2 \rangle^{1/2} + \langle r_p^2 \rangle^{1/2}) \right] \]

\[ = -\frac{q^2}{6} \left( 2\langle r_p^2 \rangle^{1/2} \Delta r_{np} + \Delta r_{np}^2 \right) \]

variation of \( A_{pv} \) at a fixed \( q \) dominated by the variation of \( \Delta r_{np} \). \( F_p(q) \) well fixed by experiment
So, let us check DWBA results...
208 Pb: direct correlations

\[ \delta A_{pv} \sim 1\%; \]

\[ \delta \Delta r_{np} \sim 0.02 \text{ fm}; \]

\[ \delta L \sim 10 \text{ MeV} \]

X. Roca-Maza, et al., PRL 106 252501 (2011)

EDF correlations allows to determine \( \Delta r_{np} \) and \( L \) without direct assumpt. on \( \rho \), JLab and Mainz forthcoming experiments

Different experiments on proton elastic scattering, antirpotonic atoms and pion-photoproduction agrees with the correlation
Isovector Giant Resonances

- In isovector giant resonances neutrons and protons “oscillate” out of phase
e.g. within a classical picture: “e-m interacting probes basically excite protons, protons drag neutrons thanks to the nuclear strong interaction, when neutrons approach too much to protons, they are pushed out”

- Isovector resonances will depend on oscillations of the density $\rho_{iv} \equiv \rho_n - \rho_p \Rightarrow S(\rho)$ will drive such “oscillations”

- The excitation energy ($E_x$) within a Harmonic Oscillator approach is expected to depend on the symmetry energy:

$$\omega = \sqrt{\frac{1}{m} \frac{d^2U}{dx^2}} \propto \sqrt{k} \rightarrow E_x \sim \sqrt{\frac{\delta^2 e}{\delta \beta^2}} \propto \sqrt{S(\rho)}$$

where $\beta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$
Polarizability, Strength distribution and its moments

- The **linear response** or dynamic polarizability of a *nuclear system excited* from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the action of an external isovector oscillating field (dipolar/quadrupolar in our case) of the form $(Fe^{iwt} + F^\dagger e^{-iwt})$:

$$F_{JM} = \sum_i A r^J Y_{JM}(\hat{r})\tau_z(i) \quad (\Delta L = 1, 2 \rightarrow \text{Dipole, Quadrupole})$$

- is proportional to the **static polarizability** for small oscillations

$$\alpha = \left(\frac{8\pi}{9}\right)e^2 m_{-1} = \left(\frac{8\pi}{9}\right)e^2 \sum_\nu |\langle \nu |F|0\rangle|^2 / E$$

where $m_{-1}$ is the inverse energy weighted moment of the strength function, defined as,

$$S(E) = \sum_\nu |\langle \nu |F|0\rangle|^2 \delta(E - E_\nu)$$
Isovector Giant Dipole Resonance:

Dipole polarizability: a macroscopic approach

Electric polarizability measures tendency of the nuclear charge distribution to be distorted \( \alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}} \)

- The dielectric theorem establishes that the \( m_{-1} \) moment can be computed from the expectation value of the Hamiltonian in the constrained ground state \( \mathcal{H}' = \mathcal{H} + \lambda D \).

Adopting the Droplet Model:

\[
m_{-1} \approx \frac{A\langle r^2 \rangle^{1/2}}{48J} \left( 1 + \frac{15}{4} \frac{J}{Q} A^{-1/3} \right)
\]

within the same model, connection with the neutron skin thickness:

\[
\alpha_D \approx \frac{A\langle r^2 \rangle}{12J} \left[ 1 + \frac{5}{2} \frac{\Delta r_{np}}{\langle r^2 \rangle^{1/2}(I - I_C)} + \sqrt{\frac{3}{5}} \frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}} \right]
\]
Isovector Giant Dipole Resonance in $^{208}$Pb:

Dipole polarizability: microscopic results HF+RPA

Experimental dipole polarizability $\alpha_D = 20.1 \pm 0.6 \text{ fm}^3$; A. Tamii et al., PRL 107, 062502 (RCNP).
Isovector Giant Dipole Resonance in $^{68}$Ni: Dipole polarizability: microscopic results HF+RPA

Experimental dipole polarizability $\alpha_D = 3.40 \pm 0.23$ fm$^3$ D. M. Rossi et al., PRL 111, 242503 (GSI). $\alpha_D = 3.88 \pm 0.31$ fm$^3$ “full” response D. M. Rossi, T. Aumann, and K. Boretzky.
208 Pb vs 68 Ni: Dipole polarizability: microscopic results \textit{HF+RPA}

Just an indication: $\alpha_D(A=208)/\alpha_D(A=68) \sim (208/68)^{5/3}$;
Cercled models predict $\Delta r_{np}^{(208 \text{ Pb})} = 0.17 \pm 0.03 \text{ fm}$ and $\Delta r_{np}^{(68 \text{ Ni})} = 0.18 \pm 0.02 \text{ fm}$; $J = 31 \pm 2 \text{ MeV}$; $L = 43 \pm 19 \text{ MeV}$.
Isovector Giant Quadrupole Resonance:

Quadrupole polarizability in $^{208}$Pb

$$\alpha_Q \approx \frac{A \langle r^4 \rangle}{16\pi J} \left[ 1 + \frac{7}{2} \frac{\Delta r_{np}}{\langle r^2 \rangle^{1/2}} + \frac{\sqrt{3/5} \frac{e^2 Z}{70J}}{\langle r^2 \rangle^{1/2}} - \Delta r_{\text{surface}}^{np} \right]$$

$$m_{-1} \frac{J}{\langle r^4 \rangle} = 21.8(1.6) + 109(9) \Delta r_{np}$$

$r=0.92$

\( E_{\text{excitation}}, \text{width and EWSR} \)
Giant Quadrupole Resonances

IVGQR experiments were experimentally known [R. Pitthan, proceedings of Giant Multiple Resonance conference, Oak Ridge 1980] but via a recent experimental technique, the accuracy has been improved [S. S. Henshaw, M. W. Ahmed, G. Feldman, A. M. Nathan, and H. R. Weller PRL107 (2011)].

**Key features** in the new polarized Compton scattering experiment:

- almost **monoenergetic and polarized** \( \gamma \)-ray beam
- \( E_1 - E_2 \) **interference** term has **opposite signs** in the forward and backward angles

Isovector Giant Quadrupole Resonance:

Within the Quantum Harmonic Oscillator approach

\[ E^\text{IV}_x = 2\hbar\omega_0 \sqrt{1 + \frac{5}{4}\frac{\hbar^2}{2m} \frac{V_{\text{sym}} \langle r^2 \rangle}{(\hbar\omega_0)^2 \langle r^4 \rangle}} \]

and connecting \( V_{\text{sym}} \) with the Droplet model,

\[ V_{\text{sym}} \approx 8(a_{\text{sym}}(A) - a_{\text{sym}}^{\text{kin}}) \]

for non-relativistic models

\[ a_{\text{sym}}^{\text{kin}} \sim \frac{\varepsilon_{F_0}}{3} \]

\[ a_{\text{sym}}(A) \sim \frac{\varepsilon_{F_0}}{3} \left\{ \frac{A^{2/3}}{8\varepsilon_{F_0}^2} \left[ \left( E^\text{IV}_x \right)^2 - 2 \left( E^\text{IS}_x \right)^2 \right] + 1 \right\} \]

Macroscopic and non-relativistic formula, estimate on \( a_{\text{sym}}(A) \), difficult to assess systematic errors.
\[
\frac{\Delta r_{np} - \Delta r_{np}^{\text{surf}}}{\langle r^2 \rangle^{1/2}} = \frac{2}{3} (I - I_C) \left\{ 1 - \frac{\varepsilon_{F_\infty}}{3J} - \frac{3}{7} \frac{I_C}{I - I_C} - \frac{A^{2/3}}{24\varepsilon_{F_\infty}} \left[ (E_x^{IV})^2 - 2(E_x^{IS})^2 \right] \right\}
\]

Mac-model predicted slope 0.025 MeV\(^{-1}\) fm; SAMi slope 0.027 MeV\(^{-1}\) fm; DD-ME slope 0.057 MeV\(^{-1}\) fm;

CONCLUSIONS
Conclusions:

- A precise and **model-independent** determination of $\Delta r_{np}$ in $^{208}\text{Pb}$ via PVES experiments *probes* the *symmetry energy*.

- We demonstrate a close **linear correlation** between $A_{pv}$ and $\Delta r_{np}$ within the same framework in which the $\Delta r_{np}$ is correlated with $L$ (expected to be better as heavier the nucleus).

- Other *experiments* fairly *agree* with the *correlation* between $A_{pv}$ and $\Delta r_{np}$ in $^{208}\text{Pb}$.

- EDFs show a linear correlation between $\alpha_{D,QJ}$ and $\Delta r_{np}$

- $A_{pv}$ and $\alpha_D$ are complementary *observables* that may set *tight constraints* on the *density dependence of the symmetry energy around saturation density*, *if precisely and/or systematically measured*. 
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