Elastic electron scattering as a revitalized experimental tool in modern nuclear physics: a theoretical point of view

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*Theoretical study of elastic electron scattering off stable and exotic nuclei* X. Roca-Maza, M. Centelles, F. Salvat, and X. Viñas


*Electron scattering in isotonic chains as a probe of the proton shell structure of unstable nuclei* X. Roca-Maza, M. Centelles, F. Salvat, and X. Viñas


*Neutron Skin of 208Pb, Nuclear Symmetry Energy, and the Parity Radius Experiment* X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda

Motivation

- In-medium nuclear (effective) interaction is not well understood for extreme values of isospin asymmetry, that is, far from the stability valley

- Experimental studies of elastic electron scattering by unstable nuclei:
  - will be feasible in rare ion beam facilities such as RIKEN (Japan) and GSI (Germany)
  - determine e-m charge distribution model independently
  - better understanding of nuclei under more extreme conditions
  - data on large isospin asymmetries

- Theoretical studies of elastic electron scattering by unstable nuclei:
  - Physical process well understood since many years ago.
  - Exact calculations available once the exact electromagnetic charge distribution is known
  - Theoretical guidance for future experiments
Motivation

- **In addition ...**
- **Experimental studies of inelastic electron scattering by unstable nuclei at forward angles that prominently measure the $E_1$ response:**
  - will be also **feasible** in facilities such as SCRIT (Japan)
  - determine the GDR in unstable nuclei (some mixing with other resonances will reduce the accuracy)
  - better understanding of the $E_1$ response of unstable nuclei
- **Theoretical studies on the GDR in unstable nuclei:**
  - Physical process **well understood**
  - **Calculations available**
  - **Theoretical guidance for experiments**
Motivation

- In-medium nuclear (effective) interaction for moderate values of isospin asymmetry, that is, close/within the stability valley is not precisely determined (neither)
- Experimental studies of parity violating elastic electron scattering by stable medium and heavy nuclei where isospin asymmetries are larger:
  - are feasible in facilities such as JLab (USA) and MAMI (Germany)
  - determine the weak charge distribution model independently
  - better understanding of neutron distribution in nuclei
- Theoretical studies of parity violating elastic electron scattering by stable nuclei:
  - Physical process well understood.
  - Exact calculations available once the exact electromagnetic and weak charge distributions are known
  - Theoretical guidance for experiments
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Elastic Scattering of Electrons by Nuclei

Theory: study of the nuclear charge distribution

- **$E_{\text{beam}} \sim 2\pi \frac{\hbar c}{\lambda_{\text{Nucl.size}}}$** where $\lambda_{\text{nucl.size}} \sim 2\langle r^2 \rangle^{1/2} \sim 2 - 10 \text{ fm}$
  \[ \Rightarrow 100 - 600 \text{ MeV}. \]
- **Relativistic treatment** is needed $m_e c^2 / E_{\text{beam}} \lesssim 0.005$.
- At these energies, effect of screening by the orbiting atomic electrons is limited to scattering angles smaller than 1 degree (we will not calculate them here).
- The interaction potential is $V_{\text{nucl.elec.}}$ calculated from $\rho_{\text{ch}}$ (parametrized, model, ... )
  \[
  V_{\text{nucl.elec.}} = 4\pi Z_0 e^2 \left\{ \frac{1}{r} \int_0^r \rho_{\text{ch}}(u) u^2 \, du + \int_r^\infty \rho_{\text{ch}}(u) u \, du \right\}
  \]
- spherical symmetry assumed
The scattering of relativistic electrons by a central field $V(r)$ is completely described by the direct scattering amplitude, $f(\theta)$, and the spin-flip scattering amplitude, $g(\theta)$.

$f(\theta)$ and $g(\theta)$ are complex functions solutions of the Dirac equation for $V(r)$ that behave asymptotically as a plane wave plus an outgoing spherical wave.

$f(\theta)$ and $g(\theta)$ admit the so called partial-wave expansion,

$$
f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} \left\{ (l+1) \left[ e^{2i\delta_{-l-1}} - 1 \right] + l \left[ e^{2i\delta_{l}} - 1 \right] \right\} P_l(\cos(\theta))$$

and,

$$
g(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} \left[ e^{2i\delta_{l}} - e^{2i\delta_{-l-1}} \right] P^1_l(\cos(\theta))$$

where $k$ is the projectile wave number ($\hbar k = p$), $P_l$ and $P^1_l$ are Legendre polynomials and $\delta_{\kappa}$ are the phase shifts induced by the central potential.
Theory: phase shifts $\delta_\kappa$

- The phase shifts $\delta_\kappa$ represent the large-$r$ behavior of the Dirac spherical waves, solution of the Dirac equation,

$$
\psi_{E\kappa m}(r) = \frac{1}{r} \begin{pmatrix}
P_{E\kappa}(r) \Omega_{\kappa,m}(\hat{r}) \\
iQ_{E\kappa}(r) \Omega_{-\kappa,m}(\hat{r})
\end{pmatrix},
$$

where $\Omega_{\kappa,m}(\hat{r})$ are the spherical spinors and $P_{E\kappa}(r)$ and $Q_{E\kappa}(r)$ satisfy,

$$
\frac{dP_{E\kappa}(r)}{dr} = -\frac{\kappa}{r} P_{E\kappa} + \frac{E - V + 2m_e c^2}{c\hbar} Q_{E\kappa},
$$
$$
\frac{dQ_{E\kappa}(r)}{dr} = -\frac{\kappa}{r} Q_{E\kappa} - \frac{E - V}{c\hbar} P_{E\kappa},
$$

where $\kappa = (l - j)(2j + 1)$ is the relativistic quantum number.

- $P_{E\kappa}(r \to \infty) \approx \sin(kr - l\pi/2 + \delta_\kappa)$ for finite range fields.
- Attractive (repulsive) potentials give positive (negative) phase shifts.
Theory: Basic quantities

- Elastic DCS per unit solid angle for spin unpolarized electrons
  \[
  \frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |g(\theta)|^2
  \]

- Spin polarization function of the electrons from an initially unpolarized beam (Sherman function)
  \[
  S(\theta) \equiv i \frac{f(\theta)g^*(\theta) - f^*(\theta)g(\theta)}{|f(\theta)|^2 + |g(\theta)|^2}
  \]
Theory: the Form Factor

\[ |F_{\text{DWBA}}(q)|^2 \equiv \frac{d\sigma/d\Omega}{d\sigma_{\text{point}}/d\Omega} \]

where \( d\sigma_{\text{point}}/d\Omega \) is the DWBA solution for a point nucleus and \( c\hbar q = 2E \sin(\theta/2) \).

- This definition, as compared to \( d\sigma/d\Omega \) \( d\sigma_{\text{Mott}}/d\Omega \), disentangles better the finite size effects of the nucleus.

- Nevertheless, it is found that the choice is not critical for the low momentum transfer regime.

Mott DCS: \( \frac{d\sigma_{\text{Mott}}}{d\Omega} = \left( \frac{Ze^2}{2E} \right)^2 \frac{\cos^2 \theta}{\sin^4 \theta} \); for small angles diverges as \( \theta^{-4} \)
**Theory: Energy Dependence in the e-m Form Factor**

**Test:** The form factor in DWBA is almost energy-independent in the low \( q \)-regime

\[ F_{DWBA}(q) \text{ is a good quantity for the study of the electromagnetic structure of the nucleus} \]
Stable Nuclei: Overview
Experiment versus Theory in stable nuclei

Nuclear Model (NM) provides:

NM+DWBA provides:
... and a more demanding test:

\[ D(A - B) \equiv (A - B)/(A + B) \]

\[ D(^{40}\text{Ca} - ^{42}\text{Ca}) \]
\[ D(^{40}\text{Ca} - ^{44}\text{Ca}) \]
\[ D(^{116}\text{Sn} - ^{118}\text{Sn}) \]

\[ D(^{40}\text{Ca} - ^{48}\text{Ca}) \]
\[ D(^{48}\text{Ca} - ^{48}\text{Ti}) \]
\[ D(^{118}\text{Sn} - ^{124}\text{Sn}) \]
### Ability test for the models

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$E_e$ MeV</th>
<th>$d_w^2$</th>
<th>Exp. fit</th>
<th>DD-ME2</th>
<th>G2</th>
<th>NL3</th>
<th>FSUGold</th>
<th>SLy4</th>
<th>SKM*</th>
<th>&quot;Best&quot; model</th>
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<tbody>
<tr>
<td>$^{16}O$</td>
<td>374.5</td>
<td></td>
<td>11.1$b$</td>
<td>88.7</td>
<td>13.1</td>
<td>38.6</td>
<td>206.</td>
<td>191.</td>
<td>194.</td>
<td>G2</td>
</tr>
<tr>
<td>$^{40}Ca$</td>
<td>250.0</td>
<td></td>
<td>7.18$b$</td>
<td>3.15</td>
<td>16.2</td>
<td>13.9</td>
<td>0.84</td>
<td>24.4</td>
<td>24.3</td>
<td>FSUGold</td>
</tr>
<tr>
<td>$^{48}Ca$</td>
<td>250.0</td>
<td></td>
<td>6.66$b$</td>
<td>4.85</td>
<td>9.74</td>
<td>7.14</td>
<td>4.08</td>
<td>14.9</td>
<td>13.6</td>
<td>FSUGold</td>
</tr>
<tr>
<td>$^{90}Zr$</td>
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<td>0.78$b$</td>
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<td>1.36</td>
<td>0.65</td>
<td>6.53</td>
<td>5.36</td>
<td>FSUGold</td>
</tr>
<tr>
<td>$^{118}Sn$</td>
<td>225.0</td>
<td></td>
<td>5.43$a$</td>
<td>18.4</td>
<td>34.8</td>
<td>25.5</td>
<td>31.8</td>
<td>2.75</td>
<td>4.20</td>
<td>SLy4</td>
</tr>
<tr>
<td>$^{208}Pb$</td>
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<td></td>
<td>30.6$b$</td>
<td>44.4</td>
<td>154.</td>
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<td>89.2</td>
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<tr>
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<td></td>
<td>21.2$b$</td>
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<td>61.1</td>
<td>95.9</td>
<td>76.5</td>
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</tr>
<tr>
<td>$D(^{40}Ca-^{42}Ca)$</td>
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<td>0.56$c$</td>
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<td>16.0</td>
<td>11.1</td>
<td>9.1</td>
<td>12.9</td>
<td></td>
<td>DD-ME2/SLy4</td>
</tr>
<tr>
<td>$D(^{40}Ca-^{44}Ca)$</td>
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<td>1.14$c$</td>
<td>4.5</td>
<td>29.6</td>
<td>12.2</td>
<td>3.88</td>
<td>7.08</td>
<td>9.13</td>
<td></td>
<td>FSUGold</td>
</tr>
<tr>
<td>$D(^{40}Ca-^{48}Ca)$</td>
<td>250.0</td>
<td>1.06$c$</td>
<td>16.4</td>
<td>4.89</td>
<td>7.74</td>
<td>38.5</td>
<td>94.1</td>
<td>49.3</td>
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<td>G2</td>
</tr>
<tr>
<td>$D(^{48}Ca-^{48}Ti)$</td>
<td>250.0</td>
<td>2.49$c$</td>
<td>18.0</td>
<td>19.6</td>
<td>31.0</td>
<td>37.8</td>
<td>71.8</td>
<td>64.9</td>
<td></td>
<td>DD-ME2</td>
</tr>
<tr>
<td>$D(^{116}Sn-^{118}Sn)$</td>
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<td>2.05$a$</td>
<td>8.05</td>
<td>7.80</td>
<td>9.00</td>
<td>10.1</td>
<td>13.2</td>
<td>18.5</td>
<td></td>
<td>G2</td>
</tr>
<tr>
<td>$D(^{118}Sn-^{124}Sn)$</td>
<td>225.0</td>
<td>4.03$a$</td>
<td>5.35</td>
<td>6.98</td>
<td>7.50</td>
<td>9.22</td>
<td>7.05</td>
<td>7.18</td>
<td></td>
<td>DD-ME2</td>
</tr>
</tbody>
</table>

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Conclusions

- Theory is well understood and calculations are feasible: exact solution of the scattering process once the nuclear e-m charge distribution has been provided.
- disagreement with the experiment due exclusively to the nuclear model
- The defined Form Factor
  - include all finite size effects
  - is nearly energy-independent at low momentum transfer
- Exist a quasi-model-independent $q$-regime: Up to $1 - 1.5$ fm$^{-1}$ for the studied models and scattering processes.
New experimental landscape: e - Rare Isotope Beams
ELISe@FAIR and SCRIT@RIKEN projects

- **Self-Confining Rare Isotope Target (e-RI scattering)** - SCRIT Operative
- **ELectron-Ion Scattering in a Storage Ring (eA collider)** - ELISe Still under development

At the beginning of next year, SCRIT collaboration will start measuring the e-m distribution of unstable Sn isotopes, from \( N=82 \) to \( N=62 \)
How can theory help in the experimental analysis? Could we find simple and general trends for $F(q)$ in exotic nuclei?
Helm Model: 2 parameters fitted to theoretical predictions to mimic future experimental analysis

- Helm Charge Form Factor: $R_0$ & $\sigma$

\[
F_H(q) = \int e^{i\vec{q}\cdot\vec{r}} \rho_H(\vec{r}) d\vec{r} = \frac{3}{R_0 q} j_1(qR_0) e^{-\sigma^2 q^2 / 2}
\]

where $\sigma$ measures the surface fall-off of the density distribution and $R_0$ measures its bulk extension.

- How we determine the parameters:
  - $R_0$: one requires that the first zero of $F_H$ occurs at the same $q$ of $F_{PWBA}$ (fourier transform of the self-consistent density). Therefore, it coincides with the sharp radius.
  - $\sigma$: is chosen to reproduce the height of the second maximum of $|F_{PWBA}|$
Results: $Z=50$ and $Z=20$ isotopic chains
Charge Form Factor

$F_{DWBA}$ increases and shifts towards smaller $q$ as the neutron number increases.

Methodology accurate for low-momentum transfer.
Correlations: the smaller the bulk part of the nuclear charge distribution and the compact the surface, the smaller the form factor.

**Calcium Isotopes**
500 MeV

- $r = 0.982$

**Tin Isotopes**
500 MeV

- $r = 0.992$
Therefore, if two or more isotopes have been measured ...

- linear correlations would provide, for an unknown nucleus of the chain, a hint on the value expected for the square of the experimental electric charge form factor at its first minimum
- if the value of the squared modulus of the form factor is determined experimentally at its first minimum, the charge density in the Helm model can be sketched from similar correlations
- use of more elaborated versions of the Helm model that take into account the central depression of the charge density should allow one to extend the domain of validity of our method up to larger values of the momentum transfer.
Results & Correlations: $N=82$, $N=50$ and $N=14$ isotonic chains
Differential cross sections and form factors

Methodology accurate for low-momentum transfer
Charge form factors $F_{DWBA}$ increases and shifts towards smaller $q$ as the neutron number increases.
The increasing rate of the form factor basically depends on the proton level which is being filled!!
Also in lighter isotonic chains...

N=14 Isotones
500 MeV

28Si, 26Mg, 24Ne, 30S, 32Ar, 34Ca

\[ 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, 1f_{5/2} \] also contribute

\[ \Delta q_{\text{min}} \]
The larger the number of protons, the larger the formfactor

...this was clear, less clear was that it is almost linear along isotonic chains
Conclusions: isotopic chains

- The described analysis is potentially useful for future electron-nucleus elastic scattering experiments,
  - the linear correlations shown would provide, for an unknown nucleus of a chain, a hint on the value expected for $|F_{\text{exp}}(q_{\text{min}})|^2$.
- The exact analysis of the Coulomb phase shifts applied to a exotic nuclei and compared with future measurements could, potentially, elucidate some aspects related with the isospin asymmetry of the nuclear force.
- The use of more elaborated versions of the Helm model should allow one to extend the domain of validity of our method up to larger values of $q$. 
Conclusions: isotonic chains

- Rate of change of the **electric charge form factor** is extremely sensitive on the proton level which is being filled
  - Levels with large \( n \) and small \( l \) contribute with opposite sign with respect to levels without radial nodes and large angular momenta.
  - Plotting \(|F(q)|^2\) against \( \sigma^2 q^2_{\text{min}} \) magnifies such effects.
- Therefore, **electron scattering** in isotonic chains can be a useful tool to probe the proton single-particle shell structure of exotic nuclei: filling order and occupancy of the different valence proton orbitals.
Conclusions: warning...

Extensive experimental investigations more difficult because of the limitations arising from small production rates, short half-lives, and small cross sections when one deals with unstable nuclei.
Parity violating electron scattering

Theory:

- The **electron interacts** with a **nucleus** by exchanging either a $\gamma$ or a $Z_0$ boson.

- $\gamma$ couples basically to protons, $Q_{em} = Z$, and $Z_0$ couples basically to neutrons,
  \[ Q_W = -N + (1 - 4 \sin^2(\theta_W))Z \approx -N + 0.1Z. \]

- Ultra-relativistic electrons interact with the Coulomb $+$ or $-$ the Weak potential depending on the helicity of the electrons,
  \[ V_{tot} = V_C \pm V_W \]
  This produces a parity-violating amplitude in the scattering process.

- The effect of the parity-violating part of the weak interaction may be isolated by measuring the parity-violating asymmetry,
  \[ A_{PV} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega} \]
  where $+/-$ indicates positive or negative helicity of $e$. 
Parity violating elastic scattering determine the nuclear weak charge distribution in a similar way as the electromagnetic charge distribution is determined in parity conserving elastic electron scattering.

The determination of $A_{PV}$ is model-independent.

Experiments at different angles are not planned for the near future ⇒ on cannot map the whole weak density in nuclei in a model independent way.
Theory:

Qualitatively,

- $A_{pv}$ within the Plane Wave Born Approximation,

$$A_{pv} = \frac{G_F q^2}{4\pi \alpha \sqrt{2}} \left[ 4 \sin^2 \theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)} \right]$$

- ... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$- \frac{q^2}{6} \left( \langle r_n^2 \rangle - \langle r_p^2 \rangle \right) = - \frac{q^2}{6} \left[ \Delta r_{np} (\langle r_n^2 \rangle^{1/2} + \langle r_p^2 \rangle^{1/2}) \right]$$

$$= - \frac{q^2}{6} \left( 2 \langle r_p^2 \rangle^{1/2} \Delta r_{np} + \Delta r_{np}^2 \right)$$

- variation of $A_{pv}$ at a fixed $q$ dominated by the variation of $\Delta r_{np}$. $F_p(q)$ well fixed by experiment.
Past and Future Experiments

**Past**
- PREx measured $A_{PV}$ in $^{208}$Pb @ 5deg and 1.063 GeV model-independently → first electro-weak probe of the existence of a weak charge radius larger than the electromagnetic charge radius in a heavy nucleus.

**Future**
- PREx II: improve accuracy of PREx (JLab)
- CREx: measure $A_{PV}$ in $^{48}$Ca @ 4deg and 2.2 GeV (JLab)
- Super PREx or PV-RAPTOR: $A_{PV}$ in $^{208}$Pb with better accuracy than PREx and PREx II (MAMI)*

* Open also to measure other nuclei if well motivated from the theory.
\[ ^{208}\text{Pb: direct correlations} \]

DWBA; no radiative corrections or strange quark effects included.


MF correlations allows to determine \( \Delta r_{np} \) and \( L \) without direct assumptions on \( \rho \), PREx-II and PV-RAPTOR expected accuracy \( \rightarrow \) constrain on \( L \)

Different experiments on proton elastic scattering and antiprotonic atoms agrees with the correlation.
48 Ca: direct correlations within MF including radiative corrections and strange quark effects

$A_{p\nu}$ decreases by around 0.005 ppm with an error of about 0.01 - 0.02 ppm when $G^s_E(Q^2)$ is included.

\[ 48 \text{Ca: estimation of spin-orbit effects} \]

In the two tested models, spin-orbit effects shifts to lower values the \( A_{pv} \) consistently by about 0.07 ppm. This predicts a reduction of \( \Delta r_{np} \) of about 0.05 fm.

Charge density distributions including spin orbit effects provided by J. Piekarewicz (FSU).
**48 Ca: Estimation of three-neutron forces effects in comparison with other corrections**

Shell Model calculations based on $\chi$EFT with NN to N3LO (fixed to scattering data) and 3N to N2LO (fixed to $B$ tritium and $R$ of alpha particle) provided by J. Menendez (TU Darmstadt).

Three-neutron forces used here shifts downwards the $A_{pv}$ by about **0.05 ppm (very similar to spin-orbit effect)**
Conclusions

- A precise and **model-independent** determination of $\Delta r_{np}$ in $^{48}$Ca and $^{208}$Pb via PVES experiments would **probe** at the same time the density dependence of the nuclear **symmetry energy** and the relevance of **three neutron-forces** in $^{48}$Ca. Eventually, it can also provide indirect indications on the impact of 3N in $^{208}$Pb.

- We demonstrate a close **linear correlation** between $A_{pv}$ and $\Delta r_{np}$ within the same framework in which the $\Delta r_{np}$ is correlated with $L$.

- Other **experiments** fairly **agree** with the **correlation** between $A_{pv}$ and $\Delta r_{np}$.
Collaborators:

Xavier Viñas
Mario Centelles
Francesc Salvat
Michal Warda
Extra material
Isotopes
Helm and self-consistent charge densities and charge radii

Calcium Isotopes

Tin Isotopes
Correlations: evolution of first minimum or inflection point

![Graphs showing correlations for Calcium and Tin Isotopes](image)

- **Calcium Isotopes**
  - $A^{-1/3}$ vs. $q_{\text{min}}$ (fm$^{-1}$)
  - $q_{\text{eff,min}}$ (fm$^{-1}$)
  - $R_{\text{ch}}$ (fm)

- **Tin Isotopes**
  - $A^{-1/3}$ vs. $q_{\text{IP}}$ (fm$^{-1}$)
  - $q_{\text{eff,min}}$ (fm$^{-1}$)
  - $R_{\text{ch}}$ (fm)

$r = 0.9997$ for Calcium Isotopes
$r = 0.997$ for Tin Isotopes
Isotones
Charge densities and proton single particle levels

![Graph showing charge densities and proton single particle levels](image)

- G2
- Helm

**122\(^{Z}\)Zr**, **140\(^{Z}\)Ce**, **146\(^{Z}\)Gd**, **154\(^{Z}\)Hf**

**\[ \rho_{ch}(\text{fm}^{-3}) \]**

**\( r \text{ (fm)} \)**

**\( \varepsilon_{nlj} \text{ (MeV)} \)**

Protons (b)