New Skyrme energy density functional for a better description of charge-exchange resonances

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\textit{Phys. Rev. C 86 031306(R) 2012}

19\textsuperscript{th} Nuclear Physics Workshop
“Marie & Pierre Curie” Kazimierz 2012
Table of contents:

- **Motivation**: charge exchange resonances, spin-isospin Landau-Migdal parameters, Spin-Orbit splittings, and Skyrme-EDFs.
- **Skyrme Interaction**: standard form with $J^2$ terms and two Spin-Orbit parameters.
- **Fitting Protocol**: experimental data and pseudo-data used in the fit.
- **Results**: EoS, $B/A$, $r_c$, $\Delta E_{SO}$, GMR, GDR, SDR and GTR.
- **Conclusions**
Motivation:

Spin-isospin properties

- **Skyrme HF+RPA** enables an effective description of the nuclear many-body problem
- Open problems need to be understood and eventually solved
  - Accurate determination of the spin-isospin properties of the Skyrme effective interaction \( \Rightarrow \) accurate description of charge exchange excitations such as the **Gamow Teller Resonance**
- **Gamow Teller**
  - transitions govern electron capture during the core-collapse of supernovæ
  - matrix el. are necessary for the study of **double-\(\beta\) decay** (in neutrinoless double-\(\beta\) decay is crucial for a precise determination of the neutrino mass).
  - matrix el. may be useful in the calibration of detectors used to measure electron-neutrinos coming from the Sun
Motivation:

**Gamow Teller Resonance I**

Neither the strength nor the $E_x$ properly described in HF+RPA

- SGII\textsuperscript{a} ⇒ earliest attempt to give a quantitative description of the GTR
- SkO’\textsuperscript{b} ⇒ accurate in ground state finite nuclear properties and improves the GTR
- Relativistic MF and Relativistic HF (PKO1\textsuperscript{c}) calculations are also available


Motivation:

Gamow Teller Resonance II: quenching of the strength

- Experimentally, the GTR exhausts 60–70% of the Ikeda sum rule: $\int [R_{\text{GT}^-}(E) - R_{\text{GT}^+}(E)]dE = 3(N - Z)$
- To explain the problem, two possibilities that go beyond RPA correlations have been drawn:
  - the effects of the second-order configuration mixing: 2p-2h correlations
  - within the quark model, a $n(p)$ can become a $p(n)$ or a $\Delta^+(\Delta^{++})$ under the action of the GT$^-$ operator and since there is no Pauli blocking for $\Delta^-h$ excitations $\Rightarrow$ it may contribute to the GTR.
- The experimental analysis of $^{90}\text{Zr} \Rightarrow$ quenching (2/3) has to be mainly attributed to 2p-2h coupling and not to $\Delta^-h$-isobar effects much smaller [T. Wakasa et. al., Phys. Rev. C 55, 2909 (1997)].
- $E_x$ GTR in nuclei mainly in the region of several tens of MeV and the $\Delta^-h$ states are hundreds of MeV above the GT $\Rightarrow$ hard to excite the $\Delta$ in the nuclear medium.
Motivation:

Which gs properties are important for describing the $E_{x}^{GTR}$?

A recent study\textsuperscript{a} on the GTR and the spin-isospin Landau-Migdal parameter $G_{0}'$ using several Skyrme sets,

\begin{itemize}
\item concluded that $G_{0}'$ is not the only important quantity in determining the excitation energy of the GTR in nuclei
\item spin-orbit splittings also influences the GTR
\end{itemize}

\begin{itemize}
\item Empirical indications\textsuperscript{b} suggest that $G_{0}' > G_{0} > 0$
\item Not a very common feature within available Skyrme forces\textsuperscript{c}
\end{itemize}

Why spin-orbit splittings are important?

\[ E_x^1 \approx \epsilon_{\pi 1g_{7/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^1 \]
\[ E_x^2 \approx \epsilon_{\pi 1g_{9/2}} - \epsilon_{\nu 1g_{9/2}} + \epsilon_{ph}^2 \]
\[ \Delta E_x \approx \Delta \epsilon_{\pi 1g} + \Delta \epsilon_{ph} \]

Schematic picture of single-particle transitions involved in the Gamow Teller Resonance of $^{90}$Zr. Transitions excited by $\sigma \tau_-$ operator.

F. Osterfeld, Rev. Mod. Phys. 64, 491 (1992)
Skyrme Model

Hamiltonian\(^a\)

Includes **central tensor terms** (**\(J^2\)** terms) due to the coupling of tensor and spin and gradients terms and **two spin-orbit parameters** (same as SkO and some SkI forces)

\[
\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}} + \mathcal{H}_{\text{SO}} + \mathcal{H}_{\text{sg}} + \mathcal{H}_{\text{Coul}}
\]

\[
\mathcal{K} = \hbar^2 \tau / 2m
\]

\[
\mathcal{H}_0 = (1/4)t_0[(2 + x_0)\rho^2 - (2x_0 + 1)(\rho_n^2 + \rho_p^2)]
\]

\[
\mathcal{H}_3 = (1/24)t_3 \rho^\alpha[(2 + x_3)\rho^2 - (2x_3 + 1)(\rho_n^2 + \rho_p^2)]
\]

\[
\mathcal{H}_{\text{eff}} = (1/8)[t_1(2 + x_1) + t_2(2 + x_2)]\tau \rho
\]

\[
+ (1/8)[t_2(2x_2 + 1) - t_1(2x_1 + 1)](\tau_n \rho_n + \tau_p \rho_p)
\]

\[
\mathcal{H}_{\text{fin}} = (1/32)[3t_1(2 + x_1) - t_2(2 + x_2)](\nabla \rho)^2
\]

\[
- (1/32)[3t_1(2x_1 + 1) + t_2(2x_2 + 1)][(\nabla \rho_n)^2 + (\nabla \rho_p)^2]
\]

\[
\mathcal{H}_{\text{SO}} = (1/2)W_0 \mathbf{J} \cdot \nabla \rho + (1/2)W'_0(\mathbf{J} \cdot \mathbf{n} \nabla \rho_n + \mathbf{J}_p \cdot \nabla \rho_p)
\]

\[
\mathcal{H}_{\text{sg}} = -(1/16)(t_1x_1 + t_2x_2)\mathbf{J}^2 + (1/16)(t_1 - t_2)(\mathbf{J}_n^2 + \mathbf{J}_p^2)
\]

\(^a\)E. Chabanat et al., Nucl. Phys. A 635, 231 (1998); E. Chabanat et al., ibid. 643, 441 (1998)
Fitting Protocol

\[ \chi^2 \text{ definition:} \quad \chi^2 = \frac{1}{N_{\text{data}}} \sum_{i}^{N_{\text{data}}} \left( \frac{O_{i}^{\text{theo.}} - O_{i}^{\text{data}}}{\Delta O_{i}^{\text{data}}} \right)^2 \]

**Landau-Migdal parameters** in infinite nuclear matter \( G_0 \) and \( G'_0 \) fixed to **0.15** and **0.35**, respectively, at \( \rho_0 \).

**Table:** Data and *pseudo*-data \( O_i \), adopted errors for the fit \( \Delta O_i \) and selected finite nuclei and EoS.

<table>
<thead>
<tr>
<th>( O_i )</th>
<th>( \Delta O_i )</th>
<th>( B )</th>
<th>( r_c )</th>
<th>( \Delta E_{SO} )</th>
<th>( e_n(\rho) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.00 MeV</td>
<td>0.01 fm</td>
<td>0.04 ( \times O_i )</td>
<td>0.20 ( \times O_i )</td>
</tr>
<tr>
<td>( 40,48 \text{ Ca}, \ 90 \text{ Zr}, \ 132 \text{ Sn} ) and ( 208 \text{ Pb} )</td>
<td>( 40,48 \text{ Ca}, \ 90 \text{ Zr} ) and ( 208 \text{ Pb} )</td>
<td>( \pi 1g ) in ( 90 \text{ Zr} ) and ( \pi 2f ) in ( 208 \text{ Pb} )</td>
<td>( \text{R. B. Wiringa et al., PRC 38, 1010 (1988)} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Skyrme Aizu Milano interaction: SAMi

Parameter set and nuclear matter properties:

**Table:** SAMi parameter set and saturation properties with the estimated standard deviations inside parenthesis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value $\sigma$</th>
<th>Value $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$</td>
<td>$-1877.75(75)$ MeV fm$^3$</td>
<td>$\rho_\infty$ 0.159(1) fm$^{-3}$</td>
</tr>
<tr>
<td>$t_1$</td>
<td>475.6(1.4) MeV fm$^5$</td>
<td>$e_\infty$ -15.93(9) MeV</td>
</tr>
<tr>
<td>$t_2$</td>
<td>$-85.2(1.0)$ MeV fm$^5$</td>
<td>$m_{IS}^*$ 0.6752(3)</td>
</tr>
<tr>
<td>$t_3$</td>
<td>10219.6(7.6) MeV fm$^{3+3\alpha}$</td>
<td>$m_{IV}^*$ 0.664(13)</td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.320(16)</td>
<td>$J$ 28(1) MeV</td>
</tr>
<tr>
<td>$x_1$</td>
<td>$-0.532(70)$</td>
<td>$L$ 44(7) MeV</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$-0.014(15)$</td>
<td>$K_\infty$ 245(1) MeV</td>
</tr>
<tr>
<td>$x_3$</td>
<td>0.688(30)</td>
<td>$G_0$ 0.15 (fixed)</td>
</tr>
<tr>
<td>$W_0$</td>
<td>137(11)</td>
<td>$G'_0$ 0.35 (fixed)</td>
</tr>
<tr>
<td>$W'_0$</td>
<td>42(22)</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.25614(37)</td>
<td></td>
</tr>
</tbody>
</table>
SAMi: spin and spin-isospin instabilities in NM

Imposing that spin and isospin dof at the Fermi surface are stable under generalized deformations [S.-O. Bäckman et al., Nucl. Phys. A 321, 10 (1979)]

\[ 1 + G_0 > 0 \quad 1 + G'_0 > 0 \]

![Graph showing stability regions for SAMi and SLy5 models with critical densities marked.](image-url)
Results
Equation of State: SAMi vs \textit{ab–initio} calculations

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\end{figure}
Results

Finite Nuclei: spherical double-magic nuclei

![Graph showing predicted and experimental data for spherical double-magic nuclei. The graph displays the mass number on the x-axis and 100x(O_{theo} - O_{exp})/O_{exp} on the y-axis. Various isotopes are plotted, including 16O, 40Ca, 56Ni, 48Ca, 90Zr, 100Sn, and 208Pb. The legend indicates symbols for N=Z, fitted data, and prediction.](image)

**Figure:** Finite nuclei properties as predicted by the HF SAMi (black circles) and some predictions (blue circles) for spherical double-magic nuclei. Experimental data taken from Refs. G. Audi et al., NPA 729, 337 (2003), I. Angeli, ADNDT 87, 185 (2004), M. Zalewski et al., PRC 77, 024316 (2008)
Results

Giant Monopole and Dipole Resonances in $^{208}\text{Pb}$

![Graph showing strength functions for GMR and GDR in $^{208}\text{Pb}$ predicted by SLy5 and the SAMi interaction.](image)

**Figure:** Strength function at the relevant excitation energies in $^{208}\text{Pb}$ as predicted by SLy5 and the SAMi interaction for GMR and GDR. A Lorentzian smearing parameter equal to 1 MeV is used. Experimental data for the centroid energies are also shown: $E_c(\text{GMR}) = 14.24 \pm 0.11$ MeV [D. H. Youngblood, et al., Phys. Rev. Lett. 82, 691 (1999)] and $E_c(\text{GDR}) = 13.25 \pm 0.10$ MeV [N. Ryezayeva et al., Phys. Rev. Lett. 89, 272502 (2002)].
Results

Gamow Teller Resonance in $^{48}$Ca, $^{90}$Zr and $^{208}$Pb

Operator:
$$\sum_{i=1}^{A} \sigma(i) \tau_{\pm}(i)$$

Results

Spin Dipole Resonances in $^{90}$Zr and $^{208}$Pb

Operator:

$$\sum_{i=1}^{A} \sum_{M} \tau_{\pm}(i) r_{i}^{L} [Y_{L}(\hat{r}_{i}) \otimes \sigma(i)]_{JM}$$

Sum Rule:

$$\int [R_{SD}^{-}(E) - R_{SD}^{+}(E)] dE = \frac{9}{4\pi} (N\langle r_{n}^{2} \rangle - Z\langle r_{p}^{2} \rangle)$$

Figure: Spin Dipole strength distributions in $^{90}$Zr as a function of the excitation energy $E_{x}$ in the $\tau_{-}$ channel (upper panel) and $\tau_{+}$ channel (lower panel) measured in the experiment [K. Yako et al., Phys. Rev. C 74, 051303(R) (2006)] and predicted by SAMi. Multipole decomposition is also shown. A Lorentzian smearing parameter equal to 2 MeV is used.

Figure: SDR strength distributions for $^{208}$Pb in the $\tau_{-}$ channel from experiment [T. Wakasa et al., Phys. Rev. C 85, 064606 (2012)] and SAMi calculations. Total and multipole decomposition of the SDR strength are shown: total (upper panel), $J^{\pi} = 0^{-}$ (middle-upper panel), $J^{\pi} = 1^{-}$ (middle-lower panel) and $J^{\pi} = 2^{-}$ (lower panel). A Lorentzian smearing parameter equal to 2 MeV is used.
Conclusions:

- we have **successfully determined a new Skyrme** energy density functional which **accounts** for the most relevant quantities in order to improve the description of **charge-exchange nuclear resonances**:
  - the **hierarchy** and **positive values** of the spin and spin-isospin Landau-Migdal parameters $G_0$ and $G'_0$
  - the **proton spin-orbit splittings** of different **high angular momenta** single-particle levels
- the **GTR** in $^{48}$Ca and the **GTR, IAR, and SDR** in $^{90}$Zr and $^{208}$Pb are predicted with **good accuracy** by SAMi
- **SAMi** does **not deteriorate** the description of other **nuclear observables**
- **applicability in nuclear physics and astrophysics**
Thank you for your attention!
Landau-Migdal vs Skyrme parameters

- **Within LDA**, at each density of the nucleus, $V_{ph} \approx V$ nuclear matter having the same density.

- Bulk properties of nuclear matter $\Rightarrow$ two-body interaction at the Fermi surface.

$$\langle k_1 k_2 | V | k_1 k_2 \rangle = \ldots \ (1)$$

- The p-h interaction at the Fermi surface is derived as the second functional derivative of the total energy with respect to density at the Fermi surface.

$$V_{ph} = \sum_{\alpha=1,\tau,\sigma,\tau\cdot\sigma} \frac{\delta H}{\delta \rho_\alpha \delta \rho_\alpha} = N_0^{-1}(F + F'_{\tau_1 \tau_2} + G_{\sigma_1 \sigma_2} + G'_{\tau_1 \tau_2 \sigma_1 \sigma_2}) \ (2)$$

- Comparing **Eqs. 1 and 2** one finds the relation between the Landau-Migdal and the Skyrme parameters: $G_0 N_0 = -\frac{1}{4} t_0 + \frac{1}{2} t_0 x_0 - \frac{1}{8} t_1 k_F^2 + \frac{1}{4} t_1 x_1 k_F^2 + \frac{1}{8} t_2 k_F^2 + \frac{1}{4} t_2 x_2 k_F^2 - \frac{1}{24} t_3 \rho^\alpha + \frac{1}{12} t_3 x_3 \rho^\alpha$

$$G'_{0} N_0 = -\frac{1}{4} t_0 - \frac{1}{8} t_1 k_F^2 + \frac{1}{8} t_2 k_F^2 - \frac{1}{24} t_3 \rho^\alpha$$

$N_0 = 2k_F m^*/\hbar^2 \pi^2$ is the density of states

**Note:** $k_1$ and $k_2$ are taken at the Fermi surface and, therefore, in homogeneous nuclear matter the Landau parameters are only functions of the angle between them and the Fermi momentum.
Empirical constraints on $G_0$ and $G'_0$


  - Landau-Migdal parameter $G'_0(N-N)$ dominates the excitation energy in the GTR as compared the contribution of to $G'_0(N-\Delta)$.
  - $G'_0(N-\Delta)$ influences more the quenching

- In our fit, **we do not use the obtained values as pseudodata** because our theoretical framework is different and the results are associated to different $m^*$ (our sp energies are based on HF calculations instead of a Wood-Saxon potential).

- **We use** the empirical result in which an hierarchy between spin and spin-isospin parameters is suggested:
  
  $$ G'_0 > G_0 > 0 $$
Covariance analysis: $\chi^2$ test

Observables $\mathcal{O}$ are used to calibrate the parameters $\mathbf{p}$ of a given model. The optimum parametrization $\mathbf{p}_0$ is determined by a least-squares fit with the global quality measure,

$$\chi^2(\mathbf{p}) = \sum_{i=1}^{m} \left( \frac{\mathcal{O}_i^{\text{theo.}} - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

Assuming that the $\chi^2$ is a well behaved (analytical) function in the vicinity of the minimum and that can be approximated by an hyper-parabola,

$$\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \approx \frac{1}{2} \sum_{i,j}^{n} (p_i - p_{0i}) \partial_{p_i} \partial_{p_j} \chi^2(p_j - p_{0j})$$

$$\equiv \sum_{i,j}^{n} (p_i - p_{0i}) \mathcal{M}_{ij} (p_j - p_{0j})$$

where $\mathcal{M}$ is the curvature matrix.
Covariance analysis: $\chi^2$ test

$\mathcal{M}$ provides us access to estimate the errors between predicted observables ($A(p)$),

$$\Delta A = \sqrt{\sum_{i}^{n} \partial_{p_i} A \mathcal{E}_{ii} \partial_{p_i} A}$$

(1)

$\mathcal{E} = \mathcal{M}^{-1}$ and the correlations between predicted observables,

$$c_{AB} \equiv \frac{C_{AB}}{\sqrt{C_{AA}C_{BB}}}$$

(2)

where,

$$C_{AB} = (A(p) - \overline{A})(B(p) - \overline{B}) \approx \sum_{ij}^{n} \partial_{p_i} A \mathcal{E}_{ij} \partial_{p_j} B$$
Covariance analysis: SLy5-min as an example
Covariance analysis: SLy5-min as an example

SLy5-min: correlation with GDR

SLy5-min: correlation with PDR

SLy5-min: correlation with $m_{-1}$

Figure: Pearson product-moment correlation coefficient for the IVGDR (left panel), IVPDR (middle panel) and $m_{-1}$(IVGDR) (right panel) with all other studied properties as predicted by the covariance analysis of SLy5.