Density dependence of the nuclear symmetry energy estimated from neutron skin thickness in finite nuclei

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Why is important the nuclear symmetry energy?

The nuclear symmetry energy is a fundamental quantity in Nuclear Physics and Astrophysics because it governs, at the same time, important properties of very small entities like the atomic nucleus (\( R \sim 10^{-15} \text{ m} \)) and very large objects as neutron stars (\( R \sim 10^4 \text{ m} \))

- **Nuclear Physics**: Neutron skin thickness in finite nuclei, stable nuclei, Heavy-Ion collisions, Giant Resonances...

- **High-Energy Physics**: Test of the Standard Model through atomic parity non-conservation observables.

- **Astrophysics**: Supernova explosion, Neutron emission and cooling of protoneutron stars, Mass-Radius relations in neutron stars, Composition of the crust of neutron stars...
Equation of State in asymmetric matter

\[ e(\rho, \delta) = e(\rho, 0) + c_{\text{sym}}(\rho)\delta^2 + O(\delta^4) \quad \left( \delta = \frac{\rho_n - \rho_p}{\rho} \right) \]

Around the saturation density we can write

\[ e(\rho, 0) \simeq a_v + \frac{1}{2} K_v \epsilon^2 \quad \text{and} \quad c_{\text{sym}}(\rho) \simeq J - L \epsilon + \frac{1}{2} K_{\text{sym}} \epsilon^2 \quad \left( \epsilon = \frac{\rho_0 - \rho}{3\rho_0} \right) \]

\[ \rho_0 \approx 0.16\text{fm}^{-3}, \quad a_v \approx -16\text{MeV}, \quad K_v \approx 230\text{MeV}, \quad J \approx 32\text{MeV} \]

However, the values of

\[ L = 3\rho \frac{\partial c_{\text{sym}}(\rho)}{\partial \rho}|_{\rho_0} \quad \text{and} \quad K_{\text{sym}} = 9\rho^2 \frac{\partial^2 c_{\text{sym}}(\rho)}{\partial \rho^2}|_{\rho_0} \]

which govern the density dependence of \( c_{\text{sym}} \) near \( \rho_0 \) are less certain and predictions vary largely among nuclear theories.
Recent research in heavy-ion collisions at intermediate energy is consistent with $c_{\text{sym}}(\rho) = c_{\text{sym}}(\rho_0)(\rho/\rho_0)^\gamma$ at $\rho < \rho_0$.

- Isospin diffusion $\gamma = 0.7–1.05$ ($L = 88 \pm 25$ MeV).
- Isoscaling $\gamma = 0.69$ ($L \sim 65$ MeV).
- Inferred from nucleon emission ratios $\gamma = 0.5$ ($L \sim 55$ MeV).
- The GDR of $^{208}$Pb analyzed with Skyrme forces suggests a constraint $c_{\text{sym}}(0.1 \text{ fm}^{-3}) = 23.3–24.9$ MeV ($\gamma \sim 0.5–0.65$).
- The study of the PDR in $^{68}$Ni and $^{132}$Sn predicts $L = 49–80$ MeV.
- The Thomas-Fermi model of Myers and Swiatecki fitted very precisely to binding energies of 1654 nuclei predicts an EOS that yields $\gamma = 0.51$.

NEUTRON SKIN THICKNESS?
Neutron skin thickness

Graph showing the relationship between quantal S in 208 Pb (fm) and L (MeV) for various nuclear interactions.
What is experimentally known about neutron skin thickness in nuclei?

- The neutron skin thickness is defined as $S = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$, where $\langle r_n^2 \rangle^{1/2}$ and $\langle r_p^2 \rangle^{1/2}$ are the rms radii of the neutron and proton distributions respectively.

- $\langle r_p^2 \rangle^{1/2}$ is known very accurately from elastic electron scattering measurements (e.g. $r_{ch}^{(208\text{Pb})} = 5.5010 \pm 0.0009$ fm [Angeli (2004)])..

- $\langle r_n^2 \rangle^{1/2}$ has been obtained with hadronic probes such as:
  (a) Proton-nucleus elastic scattering ($5.522$ fm $< r_n^{(208\text{Pb})} < 5.550$ fm [Clark (2003)])
  (b) Inelastic scattering excitation of the giant dipole and spin-dipole resonances ($r_n^{(208\text{Pb})} = 5.67 \pm 0.07$ fm [Krasznahorkay (1990)])
  (c) Antiprotonic atoms: Data from antiprotonic X rays and radiochemical analysis of the yields after the antiproton annihilation ($r_n^{(208\text{Pb})} = 5.66 \pm 0.02$ fm) [Trzcińska (2001)].
\[ S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02) \text{ fm} \]


CAN S OF 26 STABLE NUCLEI, FROM $^{40}\text{Ca}$ TO $^{238}\text{U}$, ESTIMATED USING ANTIPROTONIC ATOMS DATA HELP IN CONSTRAINING THE SLOPE AND CURVATURE OF $c_{\text{sym}}$?
Symmetry energy and neutron skin thickness
in the Liquid Drop Model

• Symmetry Energy

\[ a_{\text{sym}}(A) = \frac{J}{1 + x_A}, \quad x_A = \frac{9J}{4Q}A^{-1/3} \]

\[ E_{\text{sym}}(A) = a_{\text{sym}}(A)(l + x_A l_C)^2 A \]

where

\[ l = (N - Z)/A, \quad l_C = e^2 Z/(20JR), \quad R = r_0 A^{1/3} \]

• Neutron skin thickness

\[ S = \sqrt{\frac{3}{5}} \left[ t - e^2 Z/(70J) + \frac{5}{2R} (b_n^2 - b_P^2) \right] \]

where

\[ t = \frac{3r_0}{2} \frac{J/Q}{1 + x_A} (l - l_C) \]
Neutron skin thickness

\[ t = \frac{2r_0}{3J} \left[ J - a_{\text{sym}}(A) \right] A^{1/3} (I - I_C) \]
Table: Value of $a_{sym}(A)$ and density $\rho$ that fulfills $c_{sym}(\rho) = a_{sym}(A)$ for $A = 208, 116$ and 40 in MF models. $J$ and $a_{sym}$ are in MeV and $\rho$ is in fm$^{-3}$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$J$</th>
<th>$a_{sym}$</th>
<th>$\rho$</th>
<th>$a_{sym}$</th>
<th>$\rho$</th>
<th>$a_{sym}$</th>
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<tr>
<td>NL3</td>
<td>37.4</td>
<td>25.8</td>
<td>0.103</td>
<td>24.2</td>
<td>0.096</td>
<td>21.1</td>
<td>0.083</td>
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<td>NL-SH</td>
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<td>25.8</td>
<td>0.105</td>
<td>24.6</td>
<td>0.099</td>
<td>21.3</td>
<td>0.086</td>
</tr>
<tr>
<td>FSUGold</td>
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<td>25.4</td>
<td>0.098</td>
<td>24.2</td>
<td>0.090</td>
<td>21.9</td>
<td>0.075</td>
</tr>
<tr>
<td>TF-MS</td>
<td>32.6</td>
<td>24.2</td>
<td>0.093</td>
<td>22.9</td>
<td>0.085</td>
<td>20.3</td>
<td>0.068</td>
</tr>
<tr>
<td>SLy4</td>
<td>32.0</td>
<td>25.3</td>
<td>0.100</td>
<td>24.2</td>
<td>0.091</td>
<td>22.0</td>
<td>0.075</td>
</tr>
<tr>
<td>SkX</td>
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<td>25.7</td>
<td>0.102</td>
<td>24.8</td>
<td>0.096</td>
<td>22.8</td>
<td>0.082</td>
</tr>
<tr>
<td>SkM*</td>
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<td>SGII</td>
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<td>21.6</td>
<td>0.104</td>
<td>20.7</td>
<td>0.096</td>
<td>18.9</td>
<td>0.082</td>
</tr>
</tbody>
</table>
The $c_{\text{sym}}(\rho) - a_{\text{sym}}(A)$ correlation

- There is a genuine relation between the symmetry energy coefficients of the EOS and of nuclei: $c_{\text{sym}}(\rho)$ equals $a_{\text{sym}}(A)$ of heavy nuclei like $^{208}\text{Pb}$ at a density $\rho = 0.1 \pm 0.01 \text{ fm}^{-3}$.
- A similar situation occurs down to medium mass numbers, at lower densities.
- We find that this density can be very well simulated by
  \[ \rho \approx \rho_A = \rho_0 - \rho_0/(1 + cA^{1/3}) , \]
  where $c$ is fixed by the condition $\rho_{208} = 0.1 \text{ fm}^{-3}$.
- Using the equality $c_{\text{sym}}(\rho) = a_{\text{sym}}(A)$ and the LDM, the neutron skin thickness can be finally written as:
  \[ t = \sqrt{\frac{3}{5}} \frac{2r_0}{3} \frac{L}{J} \left( 1 - \epsilon \frac{K_{\text{sym}}}{2L} \right) \epsilon A^{1/3} (I - IC) \]
Neutron skin thickness

\[ t = \sqrt{\frac{3}{5}} \frac{2r_0 L}{J} \left( 1 - \epsilon \frac{K_{\text{sym}}}{2L} \right) \epsilon A^{1/3} (I - I_C) \]
Fitting procedure and results

• We optimize

\[ t = \sqrt{\frac{3}{5}} \frac{2r_0}{3} \frac{L}{J} \left(1 - \epsilon \frac{K_{sym}}{2L}\right) \epsilon A^{1/3} (I - I_C) \]

using

\[ c_{sym} = 31.6 \left(\frac{\rho}{\rho_0}\right) \gamma \text{MeV}, \quad \epsilon = \frac{1}{3(1 + cA^{1/3})}, \quad \rho_0 = 0.16 \text{fm}^{-3} \]

and taking as experimental baseline the neutron skins measured in 26 antiprotonic atoms.

• We predict \((b_n \approx b_p)\): \(L = 75 \pm 25 \text{ MeV}\)
\[ S = (0.9 \pm 0.15)I + (-0.03 \pm 0.02) \text{ fm} \]

Influence of the surface width \((b_n \neq b_p)\)

\[
S = \sqrt{3/5} \left[ t - \frac{e^2 Z}{70J} + \frac{5}{2R} (b_n^2 - b_p^2) \right]
\]

\(b_n\) and \(b_p\) are obtained at the ETF level.
Surface contribution to the neutron skin thickness

\[ \sqrt{\frac{3}{5}} \frac{5}{2R} (b_n^2 - b_p^2) = \sigma^{sw} l = (0.3 \frac{J}{Q} + c) l \]
Fit and results

\[ \frac{J}{Q} = 0.6 - 0.9 \]
Neutron skin thickness

$L = 30 - 80$ MeV
Constraints on the slope of the symmetry energy

$L \approx 61 \pm 11 \text{ MeV}$

- Antiprotonic Atoms
- Nuclear Model Fit
- Heavy Ion Collisions
- Giant Resonances
- $N\!-\!A$ scattering
- Charge Ex. Reac. Energy Levels
- BHF

- Neutron Skins
- Masses
- $n\!-\!p$ Emission Ratios
- Isoscaling
- Isospin Diffusion
- GDR
- PDR
- Optical Potentials
- Bare $N\!-\!N$ Potential

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Summary and Conclusions

- We have described a generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation.
- We take advantage of this relation to explore constraints on $c_{\text{sym}}(\rho)$ from neutron skins measured in antiprotonic atoms. These constraints point towards a soft symmetry energy.
- We discuss the $L$ values constrained by neutron skins in comparison with most recent observations from reactions and giant resonances.
- We learn that in spite of present error bars in the data of antiprotonic atoms, the size of the final uncertainties in $L$ is comparable to the other analyses.
- The generic relation between the symmetry energy in finite nuclei and in nuclear matter at subsaturation plausibly encompasses other prime correlations of nuclear observables with the density content of the symmetry energy as e.g. the constrains of $c_{\text{sym}}(0.1)$ from the GDR of $^{208}\text{Pb}$ (L. Trippa et al. Phys. Rev. C77, 061304(R) (2008)).
Thank you for your attention
Extra material
Some technical details

- The surface stiffness coefficient $Q$ and the surface widths $b_n$ and $b_p$ are obtained from self-consistent calculations of the neutron and proton density profiles in asymmetric semi-infinite nuclear matter.

- To this end one has to minimize the total energy per unit area with the constraint of conservation of the number of protons and neutrons with respect to arbitrary variations of the densities.

$$\frac{E_{\text{const}}}{S} = \int_{-\infty}^{\infty} \left[ \varepsilon(z) - \mu_n \rho_n(z) - \mu_p \rho_p(z) \right] dz,$$

where $\varepsilon(z)$ is the nuclear energy density functional.

- In the non-relativistic framework the densities $\rho_n$ and $\rho_p$ obey the coupled local Euler-Lagrange equations:

$$\frac{\delta \varepsilon(z)}{\delta \rho_n} - \mu_n = 0, \quad \frac{\delta \varepsilon(z)}{\delta \rho_p} - \mu_p = 0.$$

The relative neutron excess $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is a function of the $z$-coordinate. When $z \to -\infty$, the densities $\rho_n$ and $\rho_p$ approach the values of asymmetric uniform nuclear matter in equilibrium with a bulk neutron excess $\delta_0$. 
• From the calculated density profiles one computes:

\[ z_{0q} = \frac{\int_{-\infty}^{\infty} z \rho'_q(z) \, dz}{\int_{-\infty}^{\infty} \rho'_q(z) \, dz}, \]

\[ b^2_q = \frac{\int_{-\infty}^{\infty} (z - z_{0q})^2 \rho'_q(z) \, dz}{\int_{-\infty}^{\infty} \rho'_q(z) \, dz}. \]

• From the relation

\[ t = z_{0n} - z_{0p} = \frac{3r_0}{2} \frac{J}{Q} \delta_0, \]

one can evaluate \( Q \) from the slope of \( t \) at \( \delta_0 = 0 \).

• The distance \( t \) and the surface widths \( b_n \) and \( b_p \) in finite nuclei with neutron excess \( I = (N - Z)/A \) are obtained using \( \delta_0 \) given by:

\[ \delta_0 = \frac{I + \frac{3}{8} \frac{c_1}{Q} \frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4} \frac{J}{Q} A^{-1/3}}. \]