Deuteron and $\alpha$-particle semi-microscopic optical potentials

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Abstract

An analysis of the elastic scattering of deuterons on $^6$Li for energies from 3 to 12 MeV has been carried out by using a semi-microscopic optical potential. The microscopic real potential has been obtained via folding of the M3Y Paris effective interaction with d and $^6$Li density distributions extracted from experimental charge density measurements. An empirical non-locality correction to the microscopic real potential has been introduced which together with dispersion correction lead to a good agreement of the experimental and semi-microscopic elastic scattering angular distribution. At the same time, other limits of the optical potential knowledge which concern the $\alpha$-particles at low energies are considered on the basis of changes of the nuclear density at a finite temperature.

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1. Introduction

The description of deuteron-nucleus elastic scattering represents an important test for the quality of semi-microscopic optical models. However, the difficulties to interpret the data in terms of the usual optical-model potential (OMP) hampered a corresponding extensive analysis. The weak binding of the deuteron triggers significant contributions of the break-up channel and enhances a variety of reactions at low bombarding energy. In addition, the interaction between deuterons and light nuclei becomes more complex due to the pronounced cluster structure of some target nuclei, e.g. $^6$Li. Thus the presence of the deuteron as a sub-cluster structure of the $^6$Li target should be considered explicitly within a semi-microscopic analysis by including the elastic $\alpha$-transfer contribution in addition to the elastic-scattering angular distributions. While the elastic $\alpha$-transfer is indistinguishable experimentally from the elastic channel, it influences the scattering angular distributions at intermediate angles by interference effects and has larger effects at backward angles. The coupled reaction channels (CRC) calculations are requested in this case in order to describe the measured elastic scattering angular distributions [1,2]. Therefore, in order to draw conclusions regarding firstly the elastic scattering from the present analysis of the low energy deuterons scattering on $^6$Li, it has been restricted to 12 MeV where the contribution of the elastic $\alpha$-transfer is still low. Actually, a first attempt [1] for a better understanding of the elastic scattering of deuterons on $^6$Li, by means of both a semi-microscopic and a phenomenological OMP, was followed by the thorough use of the latter approach in the analysis [2] of experimental data up to 50 MeV. Lastly, the present work aims to point out the circumstances under which a suitable double-folding potential could be found for this case. At the same time, other limits of the optical
potential knowledge concern the α-particles at low energies, i.e. the difference between their behaviour within the incident and emergent channels respectively. The aspects which have to be taken into account in this respect are finally discussed.

2. The semi-microscopic DF analysis

The present analysis involves a semi-microscopic OMP which consists of a Coulomb term, the real double-folding (DF) potential [3], a phenomenological imaginary surface potential, and a spin-orbit component:

\[
U(r) = V_C(r) + U_{DF}(r) + iW_D g(r, R_D, a_D) + V_{SO}(h/m_e c)^2 \frac{1}{r} \frac{d}{dr} \left[ f(r, R_{SO}, a_{SO}) \right],
\]

where \(V_C(r)\) is the Coulomb potential of a uniformly charged sphere of radius \(R_C = r_C A^{1/3}\) and \(r_C = 1.30\) fm, \(f(r, r_0, a_0) = (1 + \exp[(r - r_A^{1/3})/a_0])^{-1}\), \(g(r, r_0, a_0) = -4a_0 d/dr \left[ f(r, r_0, a_0) \right]\), and \(h/m_e c\) is the pion Compton wavelength.

Fig. 1. Left: the deuteron s-state wave function (a), deuteron density distributions (b), and the \(^6\)Li density distribution (c). Right: the comparison of the DF potentials calculated by using the theoretical and experimental deuteron density as well as the Paris and Reid effective NN interactions.

The direct and exchange components of the real microscopic optical potential \(U_{DF}[2]\) are given in terms of the projectile and target nuclear densities, which are folded with the Paris M3Y effective NN interaction [4]. The nuclear density distribution of the target nucleus \(^6\)Li has been described by a Gaussian form with the parameters provided by the electron scattering data analysis of Bray et al. [5] (Fig. 1). The deuteron density distribution, shown in Fig. 1(b), has been obtained on the basis of the \(s\)-state wave function calculated with the Paris as well as CD-Bonn potentials [6] (also in Fig. 1), and the experimental charge form factors measured by Abbott et al. [7]. Since the two \(s\)-state wave functions are so close, the
corresponding density distributions lead to essentially identical folded potentials. On the other hand, the folded potential calculated with the deuteron density obtained on account of the experimental charge form factors [7] is also very close to the previous ones, as it is shown in the right part of Fig. 1. In the present analysis we have chosen the Abbott et al. deuteron density distribution for the DF potential calculation.

No normalization constant was involved in this work for the DF real potential, in order to emphasize the effects of further corrections requested by a satisfactory description of the experimental data. The differential elastic scattering cross sections have been calculated by a modified version of the code SCAT2 [8] which includes the DF model (DFM) results as an option for the OMP real part.

![Graph](image)

Fig. 2. Comparison of the experimental⁶Li(d,d⁶Li)⁶Li and calculated semi-microscopic angular distributions of the elastic-scattering of deuterons on Li between 3 and 12 MeV obtained by using the DF potential (dot-dashed curves), EELP potential (dashed curve), and the EELP potential with dispersion correction (solid curve).

The most striking feature of the available experimental elastic scattering differential cross sections [9,10] for the system d-⁶Li is a strong backward enhancement (Fig. 2). The same behavior was reported by Igo et al. [11] for the 11.8 MeV deuterons scattered on C, Al and Mg targets. In order to describe this peculiar behavior of angular distributions, Abramovich et al. [9] used strong spin-orbit terms with the depth VSO increased up to four times the typical values (∼8 MeV). In the previous semi-microscopic analysis [1] we obtained a similar strength of the spin-orbit potential in order to describe these experimental data. In the absence
of any polarization data there is no direct check of such a strong spin-orbit interaction. Most probable it only hides deficiencies of the semi-microscopic d-^6Li potential used in our analysis. Apart from the non-locality expected in the microscopic potential, the difficulty of the elastic-scattering OMP analysis is also increased by the concurrent break-up process of the target nucleus. The latter is favored by the cluster structure of the \(^{6}\text{Li}\) nucleus and the corresponding small separation energy of 1.48 MeV of the system \(^{6}\text{Li}=\text{d+}\alpha\). Therefore in the present work we did not study the role of the spin-orbit potential but kept constant both its depth and geometry, by using the values \(V_{\text{SO}}=8 \text{ MeV}, r_{\text{SO}}=0.86 \text{ fm}, \) and \(a_{\text{SO}}=0.25 \text{ fm}\) obtained from the previous phenomenological study [2].

Therefore only the parameters of the surface imaginary potential were obtained by fit of the experimental elastic scattering angular distributions shown in Fig. 2. A constant potential geometry has thus been proved suitable, through the values \(r_{\text{D}}=0.935 \text{ fm}\) and \(a_{\text{D}}=0.30 \text{ fm}\), while the depth of the potential has been found to increase with the incident energy. A first comparison of the experimental and calculated semi-microscopic elastic scattering differential cross sections is shown in Fig. 2. It seems that an overestimation of the cross sections occurs at forward angles for the lowest incident energy of 3 MeV, while a slight underestimation there is at the higher incident energies. At the same time, the calculated cross sections for the incident energies above 4 MeV show minima at \(\sim 136^\circ\), although the experimental data exhibit this feature only above 7 MeV. Hence the backward angles of the angular distributions are not properly described for incident energies up to 8 MeV.

3. Non-locality and dispersive corrections to the microscopic DF real potential

These results clearly indicate the need for corrections of the real potential. While an even energy-dependent normalization constant does not reveal the source of the related semi-microscopic potential deficiency, one may note firstly that a true microscopic treatment leads to a non-local optical potential. We use indeed to consider equivalent local potentials (ELP) within the available standard codes, in order to provide a simple description of elastic scattering. However, the original non-local and the equivalent local wave functions usually differ in the region of nuclear interaction. Thus their ratio, the Perey factor [12] is in a sense a measure of the non-locality of the original interaction. The Perey factor was originally obtained based on Pauli exchange effects, which could appear in our specific analysis of the \(^{6}\text{Li}\) elastic scattering from the elastic \(\alpha\)-transfer. Another type of non-locality discussed by Fiedeldey et. al. [13] arises from the dynamical coupling of the elastic to non-elastic channels, which may be important in the processes involving the deuteron mainly due to its great probability of break-up.

Therefore, in order to obtain a more accurate Empirical Equivalent Local Potential (EELP) on the DFM basis, we applied the Fiedeldey [14] correction to the DF real potential:

\[
U_{DF}^{\text{EELP}}(r) \approx U_{DF}(r) + c_{\text{non}} \frac{d^2}{dr^2} U_{DF}(r),
\]

where the empirical non-locality parameter \(c_{\text{non}}\) has been found by the fit of the experimental elastic scattering angular distributions. The values obtained in this way are shown in Fig. 3(a), and the calculated semi-microscopic angular distributions corresponding to the use of the EELP are also shown in Fig. 2 in comparison with the experimental data and the previous results. The change of sign of the non-locality parameter for the energies from 3 to 4 MeV accounts for the opposite directions of change proved necessary by the corresponding data fit,
in order to decrease the cross sections at forward angles for the energy of 3 MeV, respectively to increase them at the higher energies. An improved description of the differential cross sections has thus been obtained mainly at forward angles, for the low incident energies, and at backward angles for incident energies between 9-12 MeV. On the other hand, the pronounced minimum of the angular distributions at \( \sim 136^\circ \) remains almost unchanged.

Finally we have taken into account, in addition to the use of the EELP in the analysis of the elastic scattering data, also the dispersion correction of this potential provided by the phenomenological imaginary potential part. The analytical solution and linear schematic model of Mahaux et al. [15] has been involved in this respect by using the average energy dependence of the surface potential depth values shown in Fig. 3(b). While these values have been obtained by the former data fit, the constant value for the incident energies above 12.6 MeV has been considered in agreement with the trend found within the previous phenomenological analysis up to 50 MeV [2]. Moreover, the reference energy \( E_c = 12.6 \) MeV and the segment limits \( E_a = 1.18 \) MeV, \( E_b = 12.6 \) MeV, and \( E_c = 30 \) MeV were used, leading to the energy dependence of the real-potential dispersive contribution shown in Fig. 3(c). The corresponding calculated angular distributions are also compared in Fig. 2 with the experimental data as well as with the results of both the semi-microscopic potential and EELP. Conclusively, a better description of the experimental angular distributions at the incident energies up to 12 MeV has been obtained. The most important improvement of this description corresponds to the lower incident energies of 3-7 MeV, namely at the forward angles and around the above-mentioned minimum at \( \sim 136^\circ \). Moreover, the agreement of the calculated and experimental angular distribution is better within the whole angular range for the incident energies from 8 to 12 MeV.

We have found worthy of note comparatively the effects on the folded potential of the non-locality correction and the dispersive correction, shown in Fig. 4. The maximum result of the non-locality correction is localized at \( r \)-values lower than 1.5 fm, in agreement with Doleschall et al. [16]. Nevertheless, the related effects on the semi-microscopic potential are still present at larger \( r \)-values as it follows from Eq. (2) and can be also seen in the upper part of Fig. 4. On the other hand, the contribution of dispersion correction to the EELP has its maximum at 1.7 fm, while it becomes negligible above 3 fm where the full corrected microscopic potential is almost identical with EELP.
The bell shape behaviour of the empirical non-locality parameter $c_{non}$ versus the incident energy can be noticed in the energy dependence of the volume integral of the non-locality corrected potential, also shown in Fig. 4. Furthermore, it is preserved as well by the dispersive contribution to the real potential. This behaviour, striking different from the monotonic decrease with the energy of the volume integral of the DF potential recalls the “bell shape anomaly” at low energies predicted by Mahaux et al. [15] for the nucleus-nucleus potential. It may be correlated with an increased coupling between the elastic and non-elastic channels. The same anomaly, which in the present analysis is the results of the non-locality correction, has been found in the previous work [2] devoted to the phenomenological analysis of elastic scattering of deuterons on $^6$Li.

![Graph showing the effects of empirical non-locality and dispersion corrections on DF potential](image)

Fig. 4. The effects of the empirical non-locality correction (dot-dashed curve) and the dispersion correction (solid curve) on the DF potential (dashed curve) and the corresponding volume integral (bottom part).

4. DF and equivalent Woods-Saxon potentials for $\alpha$-particle emission

A different kind of still open questions of the optical potentials also provided by the DFM concerns the $\alpha$-particles at low energies, where it is known that the statistical $\alpha$-particle emission is underestimated by the OMPs that account for elastic scattering on the (cold) ground-state nuclei. First, a significant effective barrier reduction between ground-state and excited states of nuclei resulted from comparison of calculated to measured $\alpha$-particle evaporation spectra [17-19], while the need for new physics in potentials to describe nuclear de-excitation within the statistical model calculations was next pointed out [20]. It was thus
suggested that particle evaporation occurs from a transient nuclear stratosphere of the emitter nucleus, with a density that differs from cold nuclei and which has not yet relaxed to the density profile expected for complete equilibration [21].

Similar points concerning the difference between the $\alpha$-particle behaviour within the incident and emergent channels respectively have also been achieved within the DFM. The former DF semi-microscopic analysis of the $\alpha$-particle elastic scattering on $A\sim100$ nuclei at energies below 32 MeV made also possible to establish a related phenomenological regional optical potential (ROP) for low-energy $\alpha$-particles [3]. The microscopic DF real potential and the regional Woods-Saxon (WS) forms had different radial dependences, with a larger volume integral for the latter, but the two potentials proved almost identical shapes in the potential tail region, i.e., for absolute depth values $<10$ MeV as shown in Fig. 5(a-c). A similar tail behaviour was obtained with the global potential of McFadden and Satchler [22] while a larger real-potential diffuseness $a_R$ is needed for description of $\alpha$-particle emission from excited compound nuclei [23]. On the other hand, the global potential [23] has a real depth value and surface radial dependence very close to the DF real potential, e.g. Fig. 5(d). Thus, the main points deduced from the comparison of the work of Refs. [3] and [23] seem to be at variance with earlier conclusion that different inherent sensitivities of the scattering and reaction data to the potentials at various radii explain the differences between the optical potentials obtained from analyses of either scattering data or fusion cross sections [17].

![Fig. 5](image_url)

**Fig. 5.** Comparison between the radial dependence of the microscopic DF and phenomenological (ROP) real potentials for $\alpha$-particle scattering on $^{90}$Zr at 21, 23.4 and 25 MeV (a-c), and also at 15 MeV including global predictions [3,22] (d), the DF potentials corresponding to nuclei in g.s. ($\gamma=1$) and excited states ($\gamma=0.8$), and the equivalent WS potential for the latter case (e), also in addition to the Coulomb and centrifugal potentials (f).
Moreover, the use of ROP for calculation of \( (n,\alpha) \) reaction cross sections for the target nuclei \(^{92,95,98,100}\)Mo \([24]\) do not describe the available experimental data \([25]\), with the exception of the \(^{95}\)Mo\((n,\alpha)^{92}\)Zr reaction data \([26]\) in the low energy range from 1 to 500 keV (Fig. 6). The results range between those obtained by using the global OMP parameter set of McFadden and Satchler \([22]\), which generally underestimates the reaction cross sections, and the global OMP established especially for emitted \(\alpha\)-particles \([23]\). The case of the lightest isotope \(^{92}\)Mo could be considered especially illustrative due to the well increased PE contribution for the heaviest isotopes \(^{98,100}\)Mo, including the effect of the OMP imaginary part which enters in the calculation of the corresponding intranuclear transition rates. On the other hand it becomes particularly possible to focus in this case on uncertainties of the OMP parameters and their improvement on the basis of microscopic models since a consistent local parameter set, adopted through the analysis of various independent experimental data, has been used within the rest of a recent analysis of all available fast-neutron activation cross sections for Mo isotopes \([27]\). There were thus avoided the significant uncertainties of calculated cross sections due to the remaining parameters needed in statistical models \([28]\).

The need for an increased diffuseness for the OMP which are able to describe the \((n,\alpha)\) reaction cross sections, is supported by the previously mentioned suggestions \([17,20,21]\) that the temperature dependence of the nuclear density distribution function could be the missing degree of freedom that have to be included in the conventional statistical model calculations.

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**Fig. 6.** Comparison of total reaction cross sections for \(\alpha\)-particles incident on \(^{89,92}\)Zr calculated by using the DF-equivalent WS potentials corresponding to nuclei in g.s. \((\gamma=1)\) and excited states \((\gamma=0.8)\), as well as global \([3,22]\) predictions (top), and of the experimental \([25,26]\) and calculated cross sections of the \((n,\alpha)\) reaction on the target nuclei \(^{92,95}\)Mo by using the same potentials (bottom).
Therefore we have looked for the effect of the temperature dependence of the nuclear density used for calculation of the DF real potential, being guided in this respect by the Fermi-type local density distribution $\rho(r,T)$ determined within the extended Thomas-Fermi (ETF) method by Antonov et al. [29]:

$$\rho_{ETF}(r,T) = \rho_0(T)/[1 + \exp[(r - R(T))/\alpha(T)]]^{\gamma(T)},$$

(3)

Since their values of the temperature-dependent parameters $\rho_0(T)$, $R(T)$, and $\alpha(T)$, calculated only for two kinds of nucleons in the nucleus $^{208}$Pb and temperatures up to $T=4$ MeV have a variation generally below 5%, while the exponent $\gamma(T)$ decreases by ~30-50% with increasing temperature, we have chosen to adopt within the above-mentioned DFM formalism a modified Fermi form:

$$\rho(r) = \rho_0/[1 + \exp(r-c)/a]^{\gamma},$$

(4)

where only the additional exponent $\gamma$ has been introduced as a free parameter, while the same parameterization within DFM was used for the constants $c$ and $a$, and the central density $\rho_0$ was found by normalization to the mass number $A$. The microscopic DF real potentials developed so far for cold target nuclei correspond to the parameter value $\gamma=1$, whilst the DF potentials obtained by using various $\gamma$-values decreasing from unity were involved in the calculation of $(n,\alpha)$ reaction cross sections for the target nuclei $^{92,95,98,100}$Mo [24]. Actually this was carried on by means of DF-equivalent Woods-Saxon potentials shown in Fig. 5(e), found by the fit of the DF potentials including the dispersion correction, with also fixed volume integrals. Then, these real DF-equivalent potentials have been used together with the imaginary part of the ROP [3] for the calculation of the $(n,\alpha)$ reaction cross sections. The general trend is an increase of the diffuseness parameter with the decrease of the $\gamma$-value, and consequently an increase of the $\alpha$-particle transmission coefficients for the $(n,\alpha)$ reaction cross sections. Finally, we have found for $\gamma=0.8$, that the DF potential as well as DF-equivalent WS are in close agreement with the previous global potential for the $\alpha$-particle emission [23]. The corresponding $(n,\alpha)$ reaction cross sections also agree with the experimental data for the $^{92,95,100}$Mo isotopes [24]. Furthermore, the particular description of the $^{95}$Mo$(n,\alpha)^{92}$Zr reaction data [26] by the ROP provided by elastic-scattering analysis is found [24] to be a result of the lowest $\alpha$-particle energies (<5-6 MeV) and partial waves (Fig. 6) which are mainly involved only in the case of this reaction.

5. Conclusions

We tried in the present work to find a convenient non-locality correction, easy to be incorporated in the practical calculations [30] performed with semi-microscopic folded potentials. The improved description of the elastic-scattering differential cross sections obtained with the empirical equivalent local potential emphasized both the suitable action over the semi-microscopic potential and the sensitivity of the calculated cross sections towards this correction. A further comparative analysis involving folded potentials built on effective NN interactions with different range non-locality [31] may be useful also for the NN effective interactions as long as one does not yet definitely know how much and which type of non-locality can occur [16,30,31]. On the other hand, it is shown that the temperature dependence of the nuclear density distribution function, properly also considered within the DFM, can be an important aspect that has to be included in statistical model calculations even for a nuclear temperature smaller than 2 MeV. The difference between the $\alpha$-particles
behaviour within the incident and emergent channels, respectively, can be accounted for on this basis.

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