Main Interactions of Charged Particles

Dominant type of interaction: inelastic collisions with electrons

Collisions with nuclei:
\[
\frac{\sigma_{\text{nucl}}}{\sigma_{\text{atom}}} \sim \frac{\pi R_{\text{nucl}}^2}{\pi a_Z^2} \sim \frac{Z^2 \cdot 10^{-26} \text{cm}^2}{10^{-16} \text{cm}^2} \sim Z^2 \cdot 10^{-10} \sim 10^{-6}
\]

Atomic excitation
Ionization
Fluorescence
Phosphorescence

Most interactions of charged particles with material components occur with atomic electrons.

Nuclear collisions are noticeable only at low kinetic energies.
Multiple Scattering and Straggling

U ions, 60 keV, straggling in range and angle
Target of layers ("absorbers" Be, Au, Si)

Range is diffuse:
"Range straggling"
Range and Stopping Power

Scattering angle $\theta_s$ path variables

Stochastic multiple scattering process produces straggling in range, energy loss, angle

\[
R(E) = \int ds \langle \cos \theta_s \rangle = \int_0^E dE' \left[ \frac{dE'}{ds} \right]^{-1} \cdot \langle \cos \theta_s \rangle \quad \text{Range}
\]

\[
\left[ \frac{dE}{ds} \right] \quad \text{Stopping power}
\]

\[
S(E) = \int ds \geq R(E) \quad \text{Path length of trajectory}
\]
Phenomenological Model Of Energy Loss in Matter

Bethe et al. (1930-1953), Lindhardt’s electron theory describes energy loss through ionization, incoming ions are fully stripped.

Estimate of trends/Order of magnitude
E=particle kinetic energy, e⁻ ≈ at rest

\[ \Delta p_e = F_{Coul} \cdot \Delta t_{coll} \approx \left( \frac{e^2 Z_p}{r^2} \right) \cdot \left( \frac{2r}{v} \right) \]

\[ -dE(r, x) \sim \left[ 2\pi r \, dr \, dx \, N_e \right] \cdot \left[ \frac{(\Delta p_e)^2}{2m_e} \right] \]

\[ -\frac{dE(r, x)}{dr \, dx} \sim \left[ \frac{4\pi e^4 Z_p^2}{m_e v^2} \, N_e \right] \cdot \frac{1}{r} \]
Phenomenological Model Of Energy Loss in Matter

Integrate over radial coordinate:

\[
\frac{dE(x)}{dx} = \int_{r_{\text{min}}}^{r_{\text{max}}} dr \frac{dE(r, x)}{dr \, dx}
\]

\[
-\frac{dE(x)}{dx} \sim \left[ \frac{4\pi e^4 Z_p^2}{m_e v^2} N_e \right] \cdot \ln \left( \frac{r_{\text{max}}}{r_{\text{min}}} \right)
\]

Estimate of radial limits: \( N_e = \text{e}^- \) density \( \lambda_e = \text{electronic wave length} \), \( I_E = \text{ionization energy} \)

\[
r_{\text{min}} \geq \lambda_e = \frac{h}{p_e} = h\sqrt{1 - \beta^2}/m_e v
\]

non-adiabatic motion

\[
r_{\text{max}} / v \leq \Delta t_{\text{coll}} = T_e / \sqrt{1 - \beta^2} = 1 / \left[ \sqrt{1 - \beta^2} \langle v_e \rangle \right]
\]

\( \langle v_e \rangle = I_E / h = \text{average e}^- \text{ orbital frequency} \)
Phenomenological Model Of Energy Loss in Matter

Further:
\[- \frac{dE}{dx} \sim \left[ \frac{4\pi e^4 Z_p^2}{m_e v^2 N_e} \right] \cdot \ln \left( \frac{2m_e v^2}{h \langle v_e \rangle \left( 1 - \beta^2 \right)} \right)\]

Insert: \( \rho = \) atomic density, \( Z_T = \) atomic number of target
\( N_e = Z_T \cdot \rho \)

\[- \frac{1}{\rho} \frac{dE}{dx} \approx 4\pi \frac{e^4 Z_p^2}{m_e v^2} Z_T \ln \left( \frac{2m_e v^2}{IE\left( 1 - \beta^2 \right)} \right)\]

\( IE = h \langle v_e \rangle \approx \begin{cases} (12 \cdot Z_T + 7) \text{ eV} & \text{for } Z_T < 13 \\ (9.76 \cdot Z_T + 58.8 \cdot Z_T^{-0.19}) \text{ eV} & \text{for } Z_T \geq 13 \end{cases} \)
Bethe-Bloch Equation

Quantum mechanical calculation (for heavy particles \( M \gg m_e \)):

\[
- \frac{1}{\rho} \frac{dE}{dx} \approx 0.1535 \frac{Z_p^2 Z_T}{\beta^2 A_T} \left[ \ln \left( \frac{2m_e c^2}{IE(1 - \beta^2)} \right) \right]^2 - 2 \beta^2 \frac{MeV}{g/cm^2}
\]

\( \rho \) = atomic density
\( Z_T \) = atomic number of target
\( A_T \) = mass number of target

\( IE = h\langle \nu_e \rangle \approx \begin{cases} (12 \cdot Z_T + 7) \text{ eV} & \text{for } Z_T < 13 \\ (9.76 \cdot Z_T + 58.8 \cdot Z_T^{-0.19}) \text{ eV} & \text{for } Z_T \geq 13 \end{cases} \)
Stopping Power & Isotopic Scaling Laws

\[ \frac{dE}{dx} = \frac{d}{dx} \left( \frac{m_p v^2}{2} \right) = Z_p \cdot f \left( \frac{E}{A_p} \right) \]

\[ \frac{dv}{dx} = \left( \frac{Z_p^2}{A_p} \right) \cdot g(v) \rightarrow \]

\[ R(v) \approx \int dv \left( \frac{dv}{dx} \right)^{-1} \approx \left( \frac{A_p}{Z_p^2} \right) \cdot h(v) \]

Describes well the difference of \( R \) for different isotopes of a given element, but:

\[ \frac{A_p}{Z_p^2} = 0.12 \]

\[ R(\text{Be})/R(\text{Ar}) = 2.97 \text{ expt } 4.67 \text{ theo} \]

\[ Z_{\text{eff}} \neq Z_p \text{ effective charge} \]
Stopping Power in Silicon

Important for applications in radiation detection

Atomic density = $5 \cdot 0.10^{22}$ atoms/cm$^3$

Mass density $\rho = 2.35$g/cm$^3$

$1\mu$m $\Delta = 0.235$ mg/cm$^2$ Si

$\Delta E/ip = 3.62$ eV/ip

Heavier ions have higher stopping power ($dE/dx$)
Range in Silicon

Important for applications in radiation detection

Mass density $\rho = 2.35\, \text{g/cm}^3$

$1\mu\text{m} \Delta 0.235\, \text{mg/cm}^2\, \text{Si}$

$\Delta E/\text{ip} = 3.62\, \text{eV/ip}$

To reach a certain depth, heavier ions must have a higher energy, since they have a higher $dE/dx$. 
Range and Specific Ionization

E-loss in Air: 1 atm, 15°C

Stopping power $dE/dx$ (specific energy loss) depends on energy $E$ and therefore on $x$

Bragg Curve

Highest E loss close to end of path $\rightarrow$

Bragg maximum

\[
\frac{dE}{dx} \triangleq \frac{\#(e^- - \text{ion pairs})}{\text{unit length}}
\]

$\alpha$ particles:

\[
\frac{dE}{dx} \leq \frac{7 \cdot 10^3 \text{ pairs}}{mm}
\]

Main E-loss mechanism: ionization, production of $\delta$ electrons, electron-ion pairs
Transmission Functions

Transmission \( T(x) = \frac{N(x)}{N_0} \)

Heavy particles (\( \alpha, p, \ldots, HI \)) have a well-defined range, \( R(E) \).

Multiple scattering at small angles only, because of \( M \gg m_e \)

Light particles (electrons) and photons are scattered off original path by large angles.

Range is not well defined.

W. Udo Schröder, 2003
Stopping Power of Relativistic Particles

\[ E^2 = (pc)^2 + \left( m_0 c^2 \right)^2 \]

\( m_0 = \text{particle rest mass} \)
\( v = \text{particle velocity} \)
\( c = 2.9979 \times 10^8 \text{ m s}^{-1} \)
\( \beta = \frac{v}{c} \)
\( \gamma = \left( \sqrt{1 - \beta^2} \right)^{-1} \)
\( pc = (\gamma m_0) vc = \gamma \beta m_0 c^2 \)

Note: here \( p \frac{dE}{dx} \rightarrow \frac{dE}{dx} \)
Čerenkov Effect

Similar to supersonic boom in acoustic medium. Occurs for particle velocity
\[ v > c_{medium} = \frac{c_{vacuum}}{n} \quad n = \text{index of refraction} \]

Atoms become electrically polarized by Coulomb field (field velocity \( v_{Coul} = c/n \)) of fast particle, atoms radiate coherently

\[ \cos \theta = \frac{c}{nv} = \frac{1}{\beta n} \quad \text{Emission of Č light} \]

Light spectrum:
\[ \frac{dN(v)}{dv} = \frac{4\pi^2}{hc^2} (eZ_p)^2 \left[ 1 - \frac{1}{(\beta n)^2} \right] \]
γ-Induced Processes

γ-rays (photons) come from electromagnetic transitions between different energy states of a system → important structural information

Detection principles are based on:
- Photo-electric absorption
- Compton scattering
- Pair production
- γ-induced reactions

1. Photo-electric absorption (Photo-effect)

\[ E_{\text{kin}} = \hbar \omega - E_n; \quad E_n = \text{binding energy} \]

\[ E_n = Rhc \cdot \frac{(Z - \sigma)^2}{n^2} \quad \text{Moseley's Law} \]

\[ Rhc = 13.6 \, eV \quad \text{Rydberg constant} \]

\[ \sigma_K \approx 3, \quad \sigma_L \approx 5, \quad \text{different subshells} \]

Electronic vacancies are filled by low-energy "Auger" transitions of electrons from higher orbits
Absorption coefficient \( \rightarrow \mu \, (1/\text{cm}) \)

"Mass absorption" is measured per density \( \rho \)
\[ \rightarrow \mu/\rho \, (\text{cm}^2/\text{g}) \]

"Cross section" is measured per atom
\[ \rightarrow \sigma \, (\text{cm}^2/\text{atom}) \]

Absorption of light is quantal resonance phenomenon: Strongest when photon energy coincides with transition energy (at K,L,... "edges")

Probabilities for independent processes are additive:
\[ \mu_{\text{PE}} = \mu_{\text{PE}(K)} + \mu_{\text{PE}(L)} + ... \]

\[ \sigma_{\text{PE}}(E_\gamma, Z) \propto Z^5 \cdot E_\gamma^{-\gamma/2} \]
low \( E_\gamma \)

\[ \sigma_{\text{PE}}(E_\gamma, Z) \propto Z^5 \cdot E_\gamma^{-\gamma/4} \]
high \( E_\gamma \)


**Photon Scattering (Compton Effect)**

Relativistic \( E^2 = (pc)^2 + (m_0c^2)^2 \)  photons: \( m_0 = m_\gamma = 0 \)

\[ E_\gamma = \hbar \omega_\gamma = p_\gamma c \]

\[ \lambda' - \lambda = \lambda_c \cdot (1 - \cos \theta) \]

"Compton wave length \( \lambda_c \)"

\[ \lambda_c = \frac{2\pi}{m_e c} = 2.426 \text{ pm} \]

Momentum balance:

\[ \vec{p}_e = \vec{p}_\gamma - \vec{p}'_\gamma \rightarrow |\vec{p}_e c|^2 = |(\vec{p}_\gamma - \vec{p}'_\gamma) c|^2 \]

\[ p_e^2 c^2 = E_\gamma^2 + E'_\gamma^2 - 2E_\gamma E_{\gamma'} \cdot \cos \theta \]

Energy balance:

\[ E_\gamma + m_e c^2 = E'_\gamma + \sqrt{(p_e c)^2 + (m_e c^2)^2} \]

\[ E_{\gamma'} = \frac{E_\gamma}{1 + \left(\frac{E_\gamma}{m_e c^2}\right)(1 - \cos \theta)} \]

\[ m_e c^2 = 0.511 \text{ MeV} \]
Compton Angular Distributions

Intensity as function of $\theta$

Klein-Nishina-Formula ($\alpha = E_\gamma/m_e c^2$)

Forward scattering for high-energy photons, symmetric about $90^0$ for low-energy

\[
\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2} \left( \frac{E_{\gamma'}}{E_\gamma} \right)^2 \left\{ \frac{E_\gamma}{E_{\gamma'}} + \frac{E_{\gamma'}}{E_\gamma} \right\} \sin^2\theta
\]

*Classical e~ radius* $r_0 = 2.818$ f.m.

Alternative formulation:

\[
\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2} \left[ 1 + \cos\theta \right] \left\{ \frac{1}{1 + \alpha (1 - \cos\theta)} \right\}^3 \times
\]

\[
\times \left\{ \frac{\alpha^2 (1 - \cos\theta)^2}{\left(1 + \cos^2\theta\right)\left[1 + \alpha (1 - \cos\theta)\right]} \right\}
\]

Total scattering probability: $\sigma_c = Z$ (number of e~)
Compton Electron Spectrum

Actually, not photons but recoil-electrons are detected

Scattered photon energy:

\[
E' = \frac{E_\gamma}{1 + \left(\frac{E_\gamma}{m_ec^2}\right)(1 - \cos \theta)}
\]

Scattered recoil electron energy:

\[
E_{\text{kin}} = E_\gamma - E' = \frac{E_\gamma \left(\frac{E_\gamma}{m_ec^2}\right)(1 - \cos \theta)}{1 + \left(\frac{E_\gamma}{m_ec^2}\right)(1 - \cos \theta)}
\]

Minimum photon energy: \(\theta = 180^0\)

\[
E_{\gamma'} = \frac{E_\gamma}{1 + 2 \frac{E_\gamma}{m_ec^2}}
\]

Maximum electron energy (Compton Edge):

\[
E_{\text{kin}} \leq E_{\text{CE}} = E_\gamma \frac{2 \left(\frac{E_\gamma}{m_ec^2}\right)}{1 + 2 \left(\frac{E_\gamma}{m_ec^2}\right)}
\]
Pair Creation by High-Energy $\gamma$-rays

$\gamma$-rays

{$e^+, e^-, e^-$} triplet and one doublet in H bubble chamber

Magnetic field provides momentum/charge analysis

Event A) $\gamma$-ray (photon) hits atomic electron and produces {$e^-, e^+$} pair

Event B) one photon converts into a {$e^-, e^+$} pair

In each case, the photon leaves no trace in the bubble chamber, before a first interaction with a charged particle (electron or nucleus).
Dipping into the Fermi Sea: Pair Production

Dirac theory of electrons and holes:

World of normal particles has positive energies, $E \geq +mc^2 > 0$

Fermi Sea is normally filled with particles of negative energy, $E \leq -mc^2 < 0$

Electromagnetic interactions can lift a particle from the Fermi Sea across the energy gap $\Delta E = 2mc^2$ into the normal world $\rightarrow$ particle-antiparticle pair

Holes in Fermi Sea: Antiparticles

Minimum energy needed for pair production (for electron/positron)

$$E_\gamma > E_{\text{Threshold}} = 2m_e c^2 = 1.022 \text{MeV}$$
The Nucleus as Collision Partner

\[ E_\gamma > E_{\text{Threshold}} = 2m_c^2 \]

Actually converted: \[ E_\gamma = 2m_c^2 + E_{kin}^+ + E_{kin}^- + \ldots \]

Excess momentum requires presence of additional charged body, the nucleus

\[
\frac{d\sigma_{pp}}{dE_{kin}^+} = Z^2 \frac{1}{137} \left( \frac{e^2}{m_c^2} \right)^2 \frac{P(Z, E_\gamma)}{E_\gamma - 2m_c^2} \]

\[ P \text{ slowly varying} \]

Increase with \( E_\gamma \) because interaction sufficient at larger distance from nucleus

Eventual saturation because of screening of charge at larger distances
\textbf{\textgamma-Induced Nuclear Reactions}

Real photons or "virtual" electromagnetic quanta of high energies can induce reactions in a nucleus:

\((\gamma, \gamma'), (\gamma, n), (\gamma, p), (\gamma, \alpha), (\gamma, f)\)

Nucleus can emit directly a high-energy secondary particle or, usually sequentially, several low-energy particles or \gamma-rays.

\gamma-induced nuclear reactions are most important for high energies, \(E_\gamma \gtrsim (5 - 8)\text{MeV}\)

Can heat nucleus with (one) \gamma-ray to boiling point, nucleus thermalizes, then "evaporates" particles and \gamma-rays.
**Efficiencies of \( \gamma \)-Induced Processes**

Different processes are dominant at different \( \gamma \) energies:

- **Photo** absorption at low \( E_\gamma \)
- **Pair** production at high \( E_\gamma \)
- **Compton** scattering at intermediate \( E_\gamma \)

Z dependence important: Ge\((Z=32)\) has higher efficiency for all processes than Si\((Z=14)\). Take high-Z for large photo-absorption coefficient.

Response of detector depends on

- detector material
- detector shape
- \( E_\gamma \)
A real photomultiplier tube has about 10 dynodes, and about a million electrons reach the anode for each electron ejected from the photocathode.