Stellar weak interaction rates and shape coexistence for A~70 proton-rich nuclei within beyond-mean-field approach

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Outline

- complex EXCITED VAMPIR beyond-mean-field model

- terrestrial and stellar weak interaction rates for A~70 proton-rich nuclei:
  - isospin-symmetry-breaking and shape-coexistence effects on superallowed Fermi $\beta$ decay of the $Z=N+2$ isotopes $^{70}$Kr and $^{74}$Sr
  - Gamow-Teller $\beta$ decay and shape-coexistence for
    - $^{70}$Kr and $^{74}$Sr
    - $^{68}$Se and $^{72}$Kr $rp$-process waiting points
A~70 proton-rich nuclei manifest exotic structure and dynamics generated by the interplay of

- shape coexistence and shape mixing
- competing $T=0$ and $T=1$ pairing correlations
- isospin-symmetry-breaking interactions

responsible for

drastic changes in structure with number of nucleons, spin, and excitation energy

Challenges for theory

- realistic effective Hamiltonians in adequate model spaces, beyond-mean-field methods
- unitary treatment of structure phenomena and $\beta$-decay properties

Goals:

- tests of fundamental symmetries and interactions
- reliable predictions on stellar weak interaction rates 
  based on self-consistent description of experimentally accessible properties
complex VAMPIR model family – beyond-mean-field approaches

• the model space is defined by a finite dimensional set of spherical single particle states

• the effective many-body Hamiltonian is represented as a sum of one- and two-body terms

• the basic building blocks are Hartree-Fock-Bogoliubov (HFB) vacua

• the HFB transformations are essentially complex and allow for proton-neutron, parity and angular momentum mixing being restricted by time-reversal and axial symmetry ($T=1$ and $T=0$ neutron-proton pairing correlations included already at the mean-field level)

• the broken symmetries ($s=N, Z, I, p$) are restored by projection before variation

* The models allow to use rather large model spaces and realistic effective interactions
Beyond-mean-field variational procedure

complex Vampir

$$E^s[F^s_1] = \frac{\langle F^s_1 | \hat{H} \Theta^s_{00} | F^s_1 \rangle}{\langle F^s_1 | \Theta^s_{00} | F^s_1 \rangle}$$

$$|\psi(F^s_1); sM\rangle = \frac{\Theta^s_{M0} |F^s_1\rangle}{\sqrt{\langle F^s_1 | \Theta^s_{00} | F^s_1 \rangle}}$$

complex Excited Vampir

$$|\psi(F^s_i); sM\rangle = \Sigma_{j=1}^{i} |\phi(F^s_j)\rangle \alpha^i_j \quad \text{for} \quad i = 1, \ldots, n - 1$$

$$|\phi(F^s_i); sM\rangle = \Theta^s_{M0} |F^s_i\rangle$$

$$|\psi(F^s_n); sM\rangle = \Sigma_{j=1}^{n-1} |\phi(F^s_j)\rangle \alpha^n_j + |\phi(F^s_n)\rangle \alpha^n_n$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi^{(n)}_\alpha; sM\rangle = \sum_{i=1}^{n} |\psi_i; sM\rangle f^{(n)}_{i\alpha}, \quad \alpha = 1, \ldots, n$$
$A \sim 70$ mass region

$^{40}\text{Ca} - \text{core}$

*model space for both: protons and neutrons*

\[ 1p_{1/2} \ 1p_{3/2} \ 0f_{5/2} \ 0f_{7/2} \ 1d_{5/2} \ 0g_{9/2} \]

(charge-symmetric basis + Coulomb contributions to the $\pi$-spe from the core)

\[ 1p_{1/2} \ 1p_{3/2} \ 0f_{5/2} \ 0f_{7/2} \ 2s_{1/2} \ 1d_{3/2} \ 1d_{5/2} \ 0g_{7/2} \ 0g_{9/2} \ 0h_{11/2} \ (\text{ext-model space}) \]

renormalized $G$-matrix (OBEP- \textbf{Bonn A/ CD})

- **pairing properties enhanced by short range Gaussians for:**
  
  \( T = 1 : \) \( pp \ ( -35 \text{ MeV} ), \ np \ ( -20 \text{ MeV} ), \ nn \ ( -35 \text{ MeV} ) \)
  
  \( T = 0, S = 0 \) and \( S = 1 \ ( -35 \text{ MeV} ) \)

- **onset of deformation influenced by monopole shifts:**

  \[ <0g_{9/2} \ 0f; T=0 \ |G| \ 0g_{9/2} \ 0f; T=0> \ (0f_{5/2}, \ 0f_{7/2}) \]

  \[ <1d_{5/2} \ 1p; T=0 \ |G| \ 1d_{5/2} \ 1p; T=0> \ (1p_{1/2}, \ 1p_{3/2}) \]

- **Coulomb interaction between valence protons added**
Self-consistent terrestrial and stellar weak interaction rates for \( A \approx 70 \) nuclei

**Fermi transition probabilities**

\[
B_{i\rightarrow f}(F) = \frac{1}{2J_i + 1} \frac{g_V^2}{4\pi} |M_F|^2
\]

\[
M_F \equiv (\xi_f J_f || \hat{1} || \xi_i J_i)
= \delta_{J_i J_f} \sum_{ab} M_F(ab)(\xi_f J_f || [c_a^\dagger \tilde{c}_b]_0 || \xi_i J_i)
\]

\[
M_F(ab) = (a||\hat{1}||b)
\]

**Gamow-Teller transition probabilities**

\[
B_{i\rightarrow f}(GT) = \frac{1}{2J_i + 1} \frac{g_A^2}{4\pi} |M_{GT}|^2
\]

\[
M_{GT} \equiv (\xi_f J_f || \hat{\sigma} || \xi_i J_i)
= \sum_{ab} M_{GT}(ab)(\xi_f J_f || [c_a^\dagger \tilde{c}_b]_1 || \xi_i J_i)
\]

\[
M_{GT}(ab) = 1/\sqrt{3}(a||\hat{\sigma}||b)
\]
Weak interaction rates and shape coexistence for the Z=N+2 isotopes $^{70}$Kr and $^{74}$Sr

Isospin-symmetry-breaking and shape-coexistence effects on superallowed Fermi $\beta$-decay

$A = 70 : \quad ^{36}$Kr$_{34} - ^{35}$Br$_{35} - ^{34}$Se$_{36}$

$A = 74 : \quad ^{38}$Sr$_{36} - ^{37}$Rb$_{37} - ^{36}$Kr$_{38}$

$B(\Gamma)(g.s.) = \frac{K}{2G^2_F(1 + \Delta_R^2)}$ \hspace{1cm} $\delta c$ -- isospin-symmetry-breaking correction

Shape coexistence and deformation revealed by spectroscopic quadrupole moments

<table>
<thead>
<tr>
<th>$I(\pi)$</th>
<th>$^70$Se</th>
<th>$^70$Br</th>
<th>$^70$Kr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+_1$</td>
<td>-7</td>
<td>-18</td>
<td>-25</td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>4</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>-7</td>
<td>-30</td>
<td>-42</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$I(\pi)$</th>
<th>$^74$Kr</th>
<th>Exp</th>
<th>$^74$Rb</th>
<th>$^74$Sr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+_1$</td>
<td>-54</td>
<td>-53(24)</td>
<td>-57</td>
<td>-50</td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>49</td>
<td>24(21)</td>
<td>53</td>
<td>48</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>-74</td>
<td>-80(40)</td>
<td>-77</td>
<td>-70</td>
</tr>
</tbody>
</table>

$^{70}$Kr \hspace{1cm} $Q_{EC} = 10.480$ MeV

$0^+_{gs} \rightarrow 0^+$

Nonanalog branches:
$0^+_IV, 0^+_V \leq 0.4\%$

$2^+_\text{yrast} \rightarrow 2^+$

Nonanalog branches:
$2^+_IV \leq 1.3\%$
$^{74}\text{Sr} \quad Q_{EC} = 11.090 \text{ MeV}$

$1\% \leq \delta_c \leq 3\%$  \hspace{1cm} Nonanalog branches:

$0^+_{II} , 0^+_{VI} \leq 0.8\%$

Nonanalog branches:

$1\% \leq \delta_c \leq 3.6\%$  \hspace{1cm} $2^+_{II} \leq 1.3\% , 2^+_{IV} \leq 0.8\%$

Relevant for astrophysical scenarios on rp-process path in X-ray burst environment: 0$^+_\text{exc}$ and 2$^+_{\text{sec}}$ decay
Gamow-Teller β decay and shape coexistence for $^{70}$Kr and $^{74}$Sr


Independent chains of variational calculations in parent and daughter nuclei

Large variety of deformations in daughter states revealed by spectroscopic quadrupole moments
Gamow-Teller strength distributions for the decay of low-lying 0\(^+\) and 2\(^+\) states in \(^{70}\)Kr

Specific shape mixing for each parent state influences the strength distributions.

Contributions from \(p^{\nu(\pi)}_{1/2}p^{\pi}_{3/2}\), \(p^{\nu}_{3/2}p^{\pi}_{3/2}\), \(f^{\nu}_{5/2}f^{\pi}_{5/2}\), \(f^{\nu(\pi)}_{5/2}f^{\pi(\nu)}_{7/2}\), \(g^{\nu}_{9/2}g^{\pi}_{9/2}\) matrix elements (coherent/cancelling effect)
Gamow-Teller strength distributions for the decay of low-lying $0^+$ and $2^+$ states in $^{74}\text{Sr}$
Terrestrial half-lives

\[
\frac{1}{T_{1/2}} = \frac{1}{D} \sum_{0 \leq E_f < Q_{EC}} f(Z, E_f)[B_{if}(GT) + B_{if}(F)]
\]

\(T_{1/2}^{\exp} = 52(17)\) ms

\(T_{1/2}^{GT} = 258\) ms \quad \(T_{1/2}^{F} = 63\) ms

\(T_{1/2}^{\text{EXVAM}} = 51\) ms

Terrestrial half-lives

\(T_{1/2}^{\exp} = 27(8)\) ms

\(T_{1/2}^{GT} = 137\) ms \quad \(T_{1/2}^{F} = 48\) ms

\(T_{1/2}^{\text{EXVAM}} = 36\) ms
Weak interaction rates in X-ray burst astrophysical environment

In the X-ray burst stellar environment at densities ($\sim 10^6 \text{ mol/cm}^3$) and temperatures ($\sim 10^9 \text{K}$) typical for the rp-process the contribution of thermally populated low-lying $0^+$ and $2^+$ states may be relevant.


$$\lambda^\alpha = \frac{\ln 2}{K} \sum_i \frac{(2J_i + 1)e^{-E_i/(kT)}}{G(Z, A, T)} \sum_j B_{ij} \phi_{ij}^\alpha$$

$$G(Z, A, T) = \sum_i (2J_i + 1) \exp(-E_i/(kT))$$

$$B_{ij} = B_{ij}(F') + B_{ij}(GT)$$

$$\phi^{ec}_{ij} = \int_{w_1}^{\infty} wp(Q_{ij} + w)^2 F(Z, w) S_e(w)(1 - S_\nu(Q_{ij} + w)) dw$$

$$\phi^{\beta^+}_{ij} = \int_{1}^{Q_{ij}} wp(Q_{ij} - w)^2 F(-Z + 1, w)(1 - S_p(w))(1 - S_\nu(Q_{ij} - w)) dw$$
Stellar rates for $^{70}$Kr : $\beta^+$ - decay

$0^+_gs$ and $2^+_yrast$ - parent states
$\beta^+$ and electron capture rates for $^{70}\text{Kr}$

\[ \lambda \left( \text{s}^{-1} \right) \]

\[ \rho Y_e = 10^6 \]

\[ \rho Y_e = 10^6 \]

\[ \rho Y_e = 10^7 \]

\[ t_{1/2} \text{(s)} \]
Stellar rates for $^{74}$Sr : $\beta^+$ - decay

$E_{0^+_{\text{exc}}}^{\text{th}} = 0.564$ MeV  \hspace{1cm}  $E_{2^+_{\text{yrast}}}^{\text{th}} = 0.471$ MeV  \hspace{1cm}  $E_{2^+_{\text{sec}}}^{\text{th}} = 0.823$ MeV
\[ \lambda \left( \text{s}^{-1} \right) \]

\( \beta^+ \) and electron capture rates for \(^{74}\text{Sr}\)
Self-consistent terrestrial and stellar weak interaction rates for $^{68}$Se and $^{72}$Kr waiting points


Shape coexistence and mixing in both parent and daughter nuclei

$^{68}$Se:  $E_{2+}^{yrast} = 0.854$ MeV  $Q^{sp}_{2+}^{yrast} = 3.5$ efm$^2$(A);  -7.1 efm$^2$(CD)  $B(E2;2^+\rightarrow 0^+) \sim 500$ e$^2$fm$^4$ (Exp.: 430(60) e$^2$fm$^4$)

$^{68}$As

$T_{1/2}^{exp} = 35.5(7)$ ms  $T_{1/2}^{EXVAM} = 48.8$ ms (BonnA)  $T_{1/2}^{EXVAM} = 33.5$ ms (BonnCD)
$^{72}$Kr: \[ E_{0^+_{gs}} = 0.0 \text{ MeV} \ [60/40(\%) - p/o mixing] \]

$E_{0^+_{exc}} = 0.671 \text{ MeV} \ [38/62(\%) - p/o mixing]$

$E_{2^+_{yrast}} = 0.710 \text{ MeV} \ [41/59(\%) - p/o mixing] \text{ (BonnA-ext-space)}$

Contributions: 
- $p^{v(\pi)}\frac{3}{2} p^{\pi(\nu)}\frac{3}{2}$, $p^{v(\pi)}\frac{3}{2} P^{\pi(\nu)}\frac{3}{2}$, $f^{v(\pi)}\frac{5}{2} f^{\pi(\nu)}\frac{5}{2}$, $f^{v(\pi)}\frac{5}{2} f^{\pi(\nu)}\frac{7}{2}$, $g^{v(\pi)}\frac{9}{2} g^{\pi(\nu)}\frac{9}{2}$ matrix elements (decay to 1$^+$ states)
- $p^{v(\pi)}\frac{3}{2} p^{\pi(\nu)}\frac{1}{2}$, $p^{v(\pi)}\frac{3}{2} P^{\pi(\nu)}\frac{3}{2}$, $f^{v(\pi)}\frac{5}{2} f^{\pi(\nu)}\frac{7}{2}$ matrix elements (decay to 3$^+$ states)

$^{72}$Br

$E_{0^+_{gs}} = 0.0 \text{ MeV} \ [60/40(\%) - p/o mixing]$

$E_{0^+_{exc}} = 0.671 \text{ MeV} \ [38/62(\%) - p/o mixing]$

$E_{2^+_{yrast}} = 0.710 \text{ MeV} \ [41/59(\%) - p/o mixing] \text{ (BonnA-ext-space)}$

$T_{1/2}^{\text{exp}} = 17.1(2) \text{ ms}$

$T_{1/2}^{\text{EXVAM}} = 20.8 \text{ ms (BonnA)}$

$T_{1/2}^{\text{EXVAM}} = 20.7 \text{ ms (BonnA-ext-space)}$

$T_{1/2}^{\text{EXVAM}} = 18.9 \text{ ms (BonnCD)}$
Significant continuum electron capture contribution
Stellar rates for $^{72}$Kr: $\beta^+$ and continuum electron capture

Significant continuum electron capture contribution
Summary and outlook

Complex EXCITED VAMPIR model self-consistently describes shape-coexistence effects on

- superallowed Fermi $\beta$-decay of the $Z=N+2$ isotopes $^{70}$Kr and $^{74}$Sr
- terrestrial and stellar weak interaction rates for $A\sim70$ proton-rich nuclei

Currently we are extending the investigations to $N=Z$ nuclei up to $^{100}$Sn